GUIDANCE NOTES ON

SPECTRAL-BASED FATIGUE ANALYSIS FOR FLOATING OFFSHORE STRUCTURES

MARCH 2005

American Bureau of Shipping
Incorporated by Act of Legislature of
the State of New York 1862

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Foreword

These Guidance Notes provide additional information and suggestions about particular aspects of the ABS Classification criteria. The present Guidance Notes serve to illustrate an envisioned scope and method to perform spectral fatigue analysis for various types of floating offshore structures.

The Rules and Guides for Classification to which these Guidance Notes are considered to be most relevant are:

- ABS *Rules for Building and Classing Mobile Offshore Drilling Units*
- ABS *Guide for Building and Classing Offshore Installations*
- ABS *Guide for Building and Classing Floating Production Installations*

Additionally, the use of these Guidance Notes relies on reference to:

- ABS *Guide for the Fatigue Assessment of Offshore Structures*

These Guidance Notes specifically relate to the latest editions of the above-mentioned Rules and Guides. The use and relevancy of these Guidance Notes to other editions of these references, or with other ABS criteria, should be established in consultation with ABS.

These Guidance Notes are based on an earlier publication entitled:

- ABS *Guidance Notes on Spectral-Based Fatigue Analysis for Floating Production, Storage and Offloading (FPSO) Systems* (January 2002)

The present document supersedes the earlier one.

ABS welcomes comments and suggestions for the improvement of these Guidance Notes. Comments or suggestions can be sent electronically to *rdd@eagle.org*.

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SPECTRAL-BASED FATIGUE ANALYSIS FOR FLOATING OFFSHORE STRUCTURES

CONTENTS

SECTION 1 Introduction

SECTION 1 Introduction

1 Background and Applicability

These Guidance Notes serve to illustrate an envisioned scope and method to perform spectral fatigue analysis for various types of floating offshore structures.

Spectral fatigue analysis relies on the presumed linearity of wave-induced loads with respect to waves. This condition is sufficiently satisfied for ship-type offshore installations; such as Floating Production, Storage and Offloading (FPSO) and Floating Storage and Offloading (FSO) systems. For a ship-type hull, linear diffraction forces are the dominant component of wave load. The presentation given herein illustrates the application of the spectral fatigue analysis method to a ship-type hull. For other structural types composed of members with relatively large cross sectional dimensions; such as a Tension Leg Platform, Spar, or Column Stabilized hull, a spectral approach is often employed; but with appropriate modifications to account for the influence of nonlinear drag forces that tend to become more important as cross sectional member dimensions decrease. The presentation given herein assumes that any required modification to linearize the wave-induced load effects has been satisfactorily accomplished.

These Guidance Notes employ basic concepts and terminology that were defined in the ABS *Guide for the Fatigue Assessment of Offshore Structures* (2003). In that reference it is stated that,

> *Fatigue assessment* denotes a process where the *fatigue demand* on a structural element (e.g., a connection detail) is established and compared to the predicted *fatigue strength* of that element**.** One way to categorize a fatigue assessment technique is to say that it is based on a direct calculation of *fatigue damage* or expected *fatigue life*. Three important methods of assessment are called the *Simplified Method*, the *Spectral Method* and the *Deterministic Method*. Alternatively, an indirect fatigue assessment may be performed by the *Simplified Method*, based on limiting a predicted (probabilistically defined) stress range to be at or below a permissible stress range. There are also assessment techniques that are based on *Time Domain* analysis methods that are especially useful for structural systems that are subjected to nonlinear structural response or nonlinear loading.

In these Guidance Notes, the fatigue assessment technique being presented is a direct calculation method based on the spectral analysis method, which can produce a fatigue assessment result in terms of either expected damage or life. The fatigue strength of structural details is established using the S-N curve approach that is specified in the referenced Guide.

It should be borne in mind that for the hull structure of an Offshore Installation, wave-induced loading is usually the dominant source of fatigue damage. However some types of floating offshore structures may also be subjected to significant fatigue damage from other loading sources. This can be true for hull types that undergo frequent loading and discharge of produced fluids. For example, in FPSO and FSO systems, such load changes can induce large ranges of hull girder stress and secondary stress (albeit at lower cycles than direct wave loads), and because of changing hull drafts, different locations of the hull's external shell will be exposed to direct wave loads. Significant fatigue damage may also be induced by the operation of equipment associated with the function of the Offshore Installation. The extent to which the spectral-based fatigue method can or will be adapted to take into account these "non-wave" sources of fatigue damage must be further considered by the designer.

The classification criteria issued for Floating Offshore Installations indicate the structural details for which a fatigue assessment should be performed. Typically, there are a large number of details that are candidates for a fatigue assessment. Because of this, use may be made of "screening" procedures based on a simplified method to sort, and possibly exempt, some details from the full rigor of the spectral-based assessment technique. Good judgment, guided by experience and analytical results, may also be applied to comparable details to decide if the results for one detail can be applied to others.

3 FPSO Areas for Fatigue Assessment

There are two general categories of FPSO structural details for which fatigue assessments are required. The first type relates to conventional tanker hull details and is indicated in 1/3.1. For some of these details, in addition to wave loads, low cycle produced fluid (cargo) loading and offloading induced loads should be considered in the fatigue assessment. The second type of details is specific to an FPSO as indicated in 1/3.3. For some this latter type, other kinds of loads, e.g., low-frequency loads or operational dynamic loads, should be included in the fatigue assessments.

3.1 Hull Structure

General guidance on areas of the hull where fatigue assessment should be performed is as follows:

3.1.1 Connections of Longitudinal Stiffeners to Transverse Web/Floor and to Transverse Bulkhead

3.1.1(a) 2 to 3 selected side longitudinal stiffeners in the region from the 1.1 draft to about $^1/_3$ draft in the midship region and also in the region between 0.15L and 0.25L from F.P., respectively.

3.1.1(b) 1 to 2 stiffeners selected from each of the following groups:

- Deck longitudinals, bottom longitudinals, inner bottom longitudinals and longitudinals on side longitudinal bulkheads.
- One longitudinal on the longitudinal bulkheads within 0.1D from the deck is to be included.

For these structural details, the fatigue assessment is to be first focused on the flange of the longitudinal stiffener at the rounded toe welds of attached flat bar stiffeners and brackets.

Then, the critical spots on the web plate cut-out, on the lower end of the stiffener, as well as the weld throat, are also to be checked for the selected structural detail.

Where the stiffener end bracket arrangements on two sides of a transverse web are different, both configurations are to be checked.

3.1.2 Shell, Bottom, Inner Bottom or Bulkhead Plating at Connections to Webs or Floors (for Fatigue Assessment of Plating)

3.1.2(a) 1 to 2 selected locations of side shell plating near the summer LWL amidships and between 0.15L and 0.25L from F.P. respectively.

3.1.2(b) 1 to 2 selected locations in way of bottom and inner bottom amidships.

3.1.2(c) 1 to 2 selected locations of lower strakes of side longitudinal bulkhead amidships.

3.1.3 Connections of the Slope Plate to Inner Bottom and Side Longitudinal Bulkhead Plating at the Lower Cargo Tank Corners

One selected location amidships at transverse web and between webs, respectively.

3.1.4 End Bracket Connections for Transverses and Girders

1 to 2 selected locations in the midship region for each type of bracket configuration

3.1.5 Other Regions and Locations

Other regions and locations, highly stressed by fluctuating loads, as identified from structural analysis

3.3 FPSO-Specific Structural Areas

The adequacy of the following FPSO-specific areas for fatigue should be suitably demonstrated:

- Position mooring/hull interface, if spreading moored (4-2/15.1 of the ABS *Guide for Building and Classing Floating Production Installations*)
- Turret and its interface with hull, if turret moored (5-4/13 and 5-4/15 of the ABS *Guide for Building and Classing Floating Production Installations*)
- Riser porches
- The details, below and on the deck of the hull, comprising the supports of the topside structures. (The interface details between the hull structure and equipment skids and support frames deserve particular attention.)
- Additional areas, as applicable, including: flare tower foundation, crane pedestals, helideck to deck connections and deck penetrations.

5 Tanker Conversion

When an FPSO is converted from a trading tanker, the fatigue damage accumulated during the ”trading tanker” phase is to be deducted when establishing the remaining fatigue life for future service as an FPSO.

When calculating the fatigue damage for past services, the wave conditions of specific routes the ship has experienced in past service can be employed, instead of using the wave condition representing unrestricted service as may have been done for classification as a tanker.

When calculating the fatigue damage accumulated during the ”trading tanker” phase, the effects of vessel speed (encounter frequency) should be included (i.e., in the evaluation of stress RAOs and the number of stress cycles).

7 General Comments about the Spectral-based Method

Spectral-based Fatigue Analysis is a complex and numerically intensive technique. As such, there is more than one variant of the method that can be validly applied in a particular case. ABS does not wish to preclude the use of any valid variant of a Spectral-based Fatigue Analysis method by “over specifying” the elements of an approach. However, there is a need to be clear about the basic minimum assumptions that are to be the basis of the method employed and some of the key details that are to be incorporated in the method to produce results that will be acceptable to ABS. For this reason, most of the remainder of these Guidance Notes is a presentation on these topics.

As for the main assumptions underlying the Spectral-based Fatigue Analysis method, these are listed below:

i) Ocean waves are the main source of the fatigue-inducing loads acting on the structural system being analyzed. The fatigue damage from other loading sources can be considered separately.

ii) In order for the frequency domain formulation and the associated probabilistically-based analysis to be valid, load analysis and the associated structural analysis are assumed to be linear. Hence, scaling and superposition of stress transfer functions from unit amplitude waves are considered valid.

iii) Nonlinearities, brought about by nonlinear roll motions and intermittent application of loads such as wetting of the side shell in the splash zone, are treated by correction factors.

iv) Structural dynamic amplification, transient loads and effects such as springing are insignificant for a typical FPSO hull structure, and hence, use of quasi-static finite element analysis is valid, and the fatigue inducing stress variations due to these types of load effects can be ignored.

Also, for the particular method presented in Appendix 1, it is assumed that the short-term stress variation in a given sea-state is a random narrow-banded stationary process. Therefore, the short-term distribution of stress range can be represented by a Rayleigh distribution.

The key components of the Spectral-based Fatigue Analysis method for the selected structural locations can be categorized into the following components:

- Establish Fatigue Demand
- Determine Fatigue Strength or Capacity
- Calculate Fatigue Damage or Expected Life

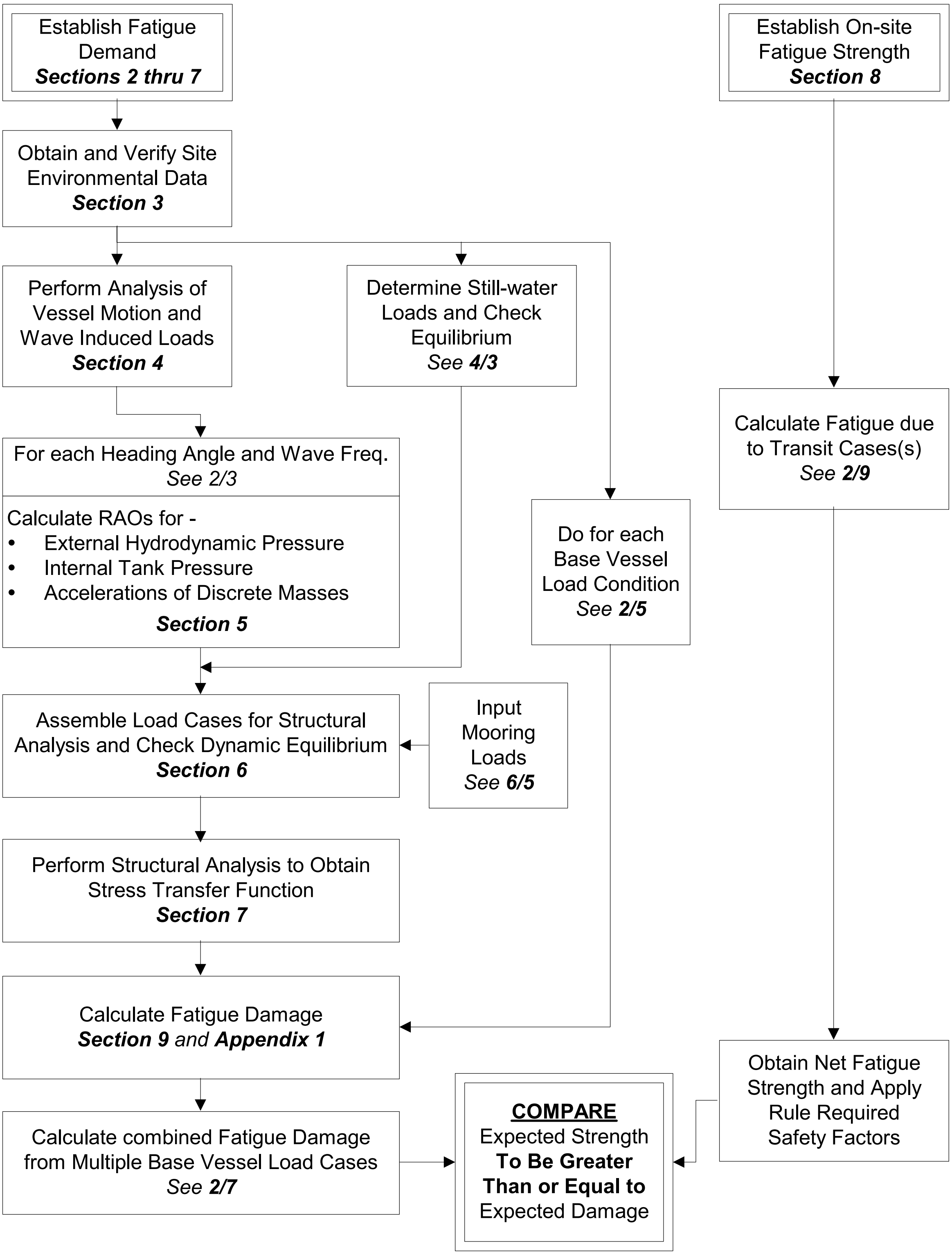
These analysis components can be expanded into additional topics, as follows, which become the subject of particular Sections in the remainder of these Guidance Notes.

The topic, "Establish Fatigue Demand" is covered in Sections 2 through 7 and part of Appendix 1. The topics of "Determine Fatigue Strength or Capacity" and "Calculate Fatigue Damage or Expected Life" are the subjects of Sections 8 and 9, respectively. Reference can be made to Section 1, Figure 1 for a schematic representation of the Spectral-based Fatigue Analysis Procedure.

A purposeful effort is made in these Guidance Notes to avoid complicated formulations, which will detract from the concepts being presented. The most complex formulations are those relating to the calculation of fatigue damage resulting from the predicted stress range. These formulations are presented in Appendix 1. It is often at this formulation level that valid variations of a method may be introduced, and for that reason, it is emphasized that the contents of Appendix 1 are provided primarily to illustrate principle, rather than as mandatory parts of the Spectral-based Fatigue method.

FIGURE 1
Schematic Spectral-based Fatigue Analysis Procedure

(For Each Location or Structural Detail)



SECTION 2 Establishing Fatigue Demand

1 Introduction

Sections 2 through 7 address the procedures used to estimate the stress transfer functions or stress RAOs at a structural location that is the object of the fatigue assessment.

3 Stress Transfer Function

With ocean waves considered the main source of fatigue demand, the fundamental task of a spectral fatigue analysis is the determination of the stress transfer function, $H_\sigma(\omega|\theta)$, which expresses the relationship between the stress at a particular structural location and the unit amplitude wave of wave frequency (ω) and wave heading (θ).

It is preferred that a structural analysis be carried out at each frequency, heading angle, "Base Vessel Loading Condition" (see Subsection 2/5) and vessel speed, if applicable, employed in the spectral analysis, and that the resulting stresses are used to directly generate the stress transfer function.

Normally, the frequency range to be used is 0.2 to 1.80 radians/second, in increments not larger than 0.1 rad/s. However, depending on the characteristics of the response, it may be necessary to consider a different frequency range. The wave heading range is 0 to 360 degrees, in increments not larger than 30 degrees.

Note: The pertinent frequency range and increment applicable to other types of floating offshore installations may be different.

5 Base Vessel Loading Conditions

The Base Loading Conditions relate to the probable variations in loading that the hull structure of the FPSO will experience during its on-site service life and for the transit case(s). The main parameters defining a Base Loading Condition are tank loading/ballast arrangements, hull draft and trim, and significant variations in topside equipment loads. These parameters have a direct influence on the "static" stress components of the hull's response, but they also affect the wave-induced cyclic stress experienced at a structural location. There are two direct ways that this influence is felt. First, this influence is felt in the magnitudes and distributions of masses and restoring forces in the determination of global and local accelerations and rigid body displacements, which in turn affect the wave-induced load effects employed in the structural analysis. Secondly, the variation of draft affects the areas of the hull that will be subjected to direct external pressures, and the magnitude and distribution of these pressures.

7 Combined Fatigue from Multiple Base Vessel Loading Conditions

Because of the variability in Base Vessel Loading Conditions and its effects on the fatigue damage predictions, it is necessary to consider more than one base case in the fatigue analysis. As a minimum for the analysis of post-installation on-site conditions, two cases are to be modeled and used in the Spectral-based Fatigue Analysis process. The two required cases are ones resulting from, and representing, the probable deepest and shallowest drafts, respectively, that the vessel is expected to experience during its on-site service life.

Note: Suggested Approach: In some (so-called "Closed Form") formulations to calculate fatigue demand, the fraction of the total time on-site for each Base Vessel Loading Condition is used directly. In this case, potentially useful information about the separate fatigue damage from each vessel loading condition is not obtained. Therefore, it is suggested that the fatigue damage from each vessel loading condition be calculated separately. Then the "combined fatigue life" is calculated as a weighted average of the reciprocals of the lives resulting from considering each case separately. For example, if two base loading conditions are employed, and the calculated fatigue life for a structural location due to the respective base vessel loading conditions are denoted L_1 and L_2, and it is assumed that each case is experienced for one-half of the FPSO's on-site service life, then the combined fatigue life, L_C is:

$$L_C = 1/[0.5(1/L_1)+ 0.5(1/L_2)].$$

As a further example, if there were three base vessel loading conditions L_1, L_2, L_3 with exposure time factors of 40, 40 and 20 percent, respectively; then the combined fatigue life, L_C is:

$$L_C = 1/[0.4(1/L_1) + 0.4(1/L_2) + 0.2(1/L_3)].$$

9 Transit Cases

The fatigue demand arising from anticipated FPSO transit cases (usually only the FPSO voyage to the installation site) is to be determined.

During assessment of the fatigue damage accumulated during transit cases, the effects of vessel speed are to be included in the evaluation of stress RAOs and the number of stress cycles.

Note: As in the previous Note, it is suggested that the fatigue demand produced by the transit case(s) be calculated separately.

The extent to which fatigue analysis is to be performed for the transit cases associated with an FPSO with a classification notation **Disconnectable** (see 1-3/5.1 of the ABS *Guide for Building and Classing Floating Production Installations*) will be specially considered.

SECTION 3 Environmental Conditions

1 General

Fatigue damage of a structure is caused by fluctuating loads that occur during the structure's service life. For offshore structures, the most dominant source of fluctuating load is waves. However, in some particular cases other sources, such as vortex induced vibrations (VIV), wind, current and operational loads may become significant.

A structure will experience various fluctuating loads during its service life. To describe these sources adequately is the first, crucial step in the fatigue assessment of the structure. Obviously, it is impossible for one to predict or describe with certainty the expected environmental conditions the structure will experience during its service life. However, one can define a series of conditions and establish statistically the probability of each condition happening to the structure in its life. A fatigue analysis can then be performed based on this kind of statistical description of environmental conditions.

3 Waves

During the service life of an offshore structure, it will experience a huge number of waves, from very small wavelets to possibly giant waves. A practical way to describe these unceasingly changing waves is to divide them into various categories (sea states), and use short-term wave statistics to depict each sea state and long-term wave statistics, usually in the form of a wave scatter diagram and rosette, to delineate the rate at which a sea state occurs.

In a similar way, there are two levels in the description of wave directionality, i.e., wave directional spectrum or wave spreading for short-term, and wave rosette for long-term, respectively.

3.1 Wave Spectra (Short-term Wave Statistics)

3.1.1 Unidirectional Spectra

A wave spectrum describes the energy distribution among wave components of different frequencies of a sea state. Wave spectra can be obtained directly from measured data. However, various mathematical formulae of wave spectra have been available based on analysis of measured data, such as ISSC Wave Spectrum, Bretschneider Spectrum (or Pierson-Moskowitz (P-M) spectrum), JONSWAP spectrum and Ochi's six-parameter spectrum. These spectrum formulae are suitable for different sea states.

A fully-developed sea is a sea state that will not change if wind duration or fetch is further increased (for a fixed wind speed). The Bretschneider spectrum is applicable to fully-developed seas. For most of the ships and offshore structures in ABS's classification, either the Bretschneider spectrum for open ocean areas with fully-developed seas, or the JONSWAP spectrum for fetch-limited regions are used, respectively. For example, the Bretschneider wave spectrum is usually employed to describe tropical storm waves, such as those generated by hurricanes in the Gulf of Mexico or typhoons in the South China Sea. The JONSWAP wave spectrum is used to describe winter storm waves of the North Sea. In some cases, it can also be adjusted to represent waves in Offshore Eastern Canada and swells, such as those in West Africa and Offshore Brazil. A suitable wave spectrum should be chosen based on a partially or fully developed sea state for fatigue strength assessment. In general, the Bretschneider spectrum has a greater frequency bandwidth than the JONSWAP spectrum. Therefore, the selection of a spectrum should be based on the frequency characteristics of the wave environment.

The above-described two spectra are single-modal spectra, which are usually used to represent pure wind waves or swell-only cases. When wind waves co-exist with swells (i.e., there are multi-modes in the spectrum), no single-modal spectrum can match the spectral shape very well. In this case, recourse can be made to the use of the Ochi-Hubble 6-Parameter Spectrum.

3.1.2 Directional Spectra (Wave Spreading)

When wave spreading is considered, the cosine squared spreading function may be used to simulate the short-crestedness of the wind waves and the cosine 4th or higher power spreading function to represent swells. The directional spectrum of a sea state can be expressed as the product of the energy spreading function, $D(\theta)$, and the unidirectional spectrum, $S(\omega)$, i.e.:

$$S(\omega, \theta) = S(\omega)\, D(\theta)$$

where

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} D(\theta) d\theta = 1$$

3.3 Wave Scatter Diagram and Rosette (Long-term Wave Statistics)

Long-term descriptions of the wave environment in the form of “wave scatter diagram” and “rosette” are required for long-term statistical analysis of structural response, such as the predictions of extreme response and the fatigue assessment.

3.3.1 Wave Scatter Diagram

A wave scatter diagram consists of a table of the probabilities of occurrence of various “sea states”. Each cell in the table contains information on three data items, namely (1) the significant wave height, H_s, (2) the characteristic wave period, T, and (3) the fraction of the total time or probability of occurrence for the sea state defined by spectrum with parameters H_s and T. The characteristic wave period usually can be given in peak period, average period or zero up-crossing periods. Attention should be paid to which characteristic wave period is specified in a wave scatter diagram so that it will be consistent with the wave period in the wave spectrum formulation.

3.3.2 Wave Rosette

A wave rosette (also called long-term wave directionality) describes the probability of each heading angle (the main wave direction) of a site. Directional convention should be noticed in using the rosette (e.g., for NOAA wave data, index 1 represents wave coming from true north and as the index increases, the wave direction changes clockwise). Directionality has significant effects on structural response. It is recommended that a realistic wave rosette be used in the fatigue analysis. In case the wave rosette is not available, it is reasonable to assume equal probability of all heading angles in open ocean conditions. However, for a moored offshore structure, the waves may have strong directional characteristics that should be accounted for.

5 Currents

Current in the ocean can be any or any combination of wind-driven, thermohaline, tidal and storm surge currents. At a location (installation site), the current is defined by its speed and directional profiles through the water depth. Currents may change with time, from hourly to seasonally.

Although current itself can be treated as producing essentially static loads, it can induce or intensify certain kinds of dynamic loads and fatigue damage (especially for slender structures such as mooring lines and risers). There are mainly two ways current can affect fatigue: 1) the presence of a current can increase cyclic drag loading of waves due to the nonlinear coupling of current velocity and wave orbital velocity. It is recommended that current be considered if its magnitude is comparable with the wave orbital velocity for those waves that make the greatest contributions to the fatigue damage; 2) Current may also create cyclic “lifting” loads due to vortex shedding, which can cause significant fatigue damage.

7 Wind

Another source of fatigue-inducing loads is wind. Wind produces low-frequency drift motions, which can produce low-frequency fatigue effects in mooring lines and risers. These fatigue-inducing effects of wind can be included in the fatigue assessment as described in 4/5.5. Also wind gusts, vortex shedding and other dynamic wind effects can create significant fatigue damage, especially to superstructure components. However, these wind gust and dynamic effects on fatigue are not typically considered in classification.

SECTION 4 Motion Analysis and Wave-induced Loads

1 General

This Section gives general criteria on the parameters to be obtained from the vessel motion analysis and the calculation of wave-induced load effects. In the context of a Spectral-based Fatigue Analysis, the main objective of motion and load calculations is the determination of *Response Amplitude Operators* (RAOs), which are mathematical representations of the vessel responses and load effects to unit amplitude sinusoidal waves. The motion and load effects RAOs are to be calculated for ranges of wave frequencies and wave headings, as indicated in Subsection 2/3.

Aside from vessel motions, the other wave-induced load effects that need to be considered in the Spectral-based Fatigue Analysis of an FPSO are the external wave pressures, internal tank pressures due to tank fluid accelerations, inertial forces on the masses of structural components and significant items of equipment and mooring loads. Additionally, there may be situations where partial models of the FPSO's structural system are used. In such a case, hull girder shear forces and bending moments are to be determined to appropriately represent the boundary conditions at the ends of the partial models.

Note: Fatigue damage due to the sloshing of fluid in partially-filled tanks is not within the scope of the **SFA** classification notation. However, the designer is encouraged to perform and submit such calculations if deemed important for the FPSO.

3 Still-water Loads

The motion and load calculations are to be performed with respect to static initial conditions representing the vessel geometry and loadings, (see Subsection 2/5). With the input of hull loadings, the hull girder shear force and bending moment distributions in still water are to be computed at a sufficient number of transverse sections along the hull's length, in order to accurately take into account discontinuities in the weight distribution. A recognized hydrostatic analysis program is to be used to perform these calculations. By iteration, the convergence of the displacement, Longitudinal Center of Gravity (LCG) and trim should be checked to meet the following tolerances:

Displacement:		± 1%
Trim:		± 0.5 degrees
Draft:		
	Forward	± 1 cm
	Mean	± 1 cm.
	Aft	± 1 cm
LCG:		± 0.1% of length
SWBM:		± 5%

Additionally, the longitudinal locations of the maximum and the minimum still-water bending moments and, if appropriate, that of zero SWBM should be checked to assure proper distribution of the SWBM along the vessel's length.

5 Essential Features of Motion and Wave Load

5.1 General Modeling Considerations

The representation of the hull should include the masses of the topside equipment and their supporting structure. The model should also consider the interaction with the mooring system, and as appropriate, the effects of risers, the effects of the Dynamic Positioning system and the operations of offloading or support vessels. The motion analysis should appropriately consider the effect of shallow water on vessel motions. There should also be sufficient compatibility between the hydrodynamic and structural models so that the application of fluid pressures onto the finite element mesh of the structural model can be done appropriately.

For the load component types and structural responses of primary interest, analysis software formulations derived from linear idealizations are deemed to be sufficient. However, the use of enhanced bases for the analysis, especially to incorporate nonlinear loads (for example hull slamming) is encouraged, if this proves to be necessary for the specific design being evaluated. The adequacy of the employed calculation methods and tools is to be demonstrated to the satisfaction of ABS.

5.3 Diffraction-Radiation Methods

Computations of the wave-induced motions and loads should be carried out using appropriate, proven methods. Preference should be given to the application of seakeeping analysis codes utilizing three-dimensional potential flow-based diffraction-radiation theory. All six degrees-of-freedom rigid-body motions of the vessel should be accounted for, and effects of water depth should be considered. These codes, based on linear wave and motion amplitude assumptions, make use of boundary element methods with constant or higher order sink-source panels over the entire wetted surface of the hull on which the hydrodynamic pressures are computed.

5.5 Low Frequency Motions

The low frequency motions of a floating structure are induced by higher-order wave forces and wind loads. The low frequency drift motion of an FPSO is an example of this kind of motion. Although low frequency motions usually have negligible effects on most structural details of the hull, they become significant, and may even dominate the fatigue of structural components such as the mooring system, risers and their interfaces with the hull.

The low frequency motions of a floating structure are to be analyzed for each sea state using a recognized hydrodynamic analysis program. All motion constraints imposed by, for instance, mooring lines, risers, control umbilical and positioning thrusters, should be considered. Although the low-frequency motions may be calculated in either time domain or frequency domain, the time domain simulations are recommended in the sense that the frequency domain method cannot account for the nonlinearity and time-varying drift force.

There are typically two approaches, which may be employed when the time domain method is used to evaluate the low frequency motions.

i) *Approach I.* The calculations of wave-frequency and low-frequency motions are completely decoupled. The time domain global performance analysis is carried out over the drift frequency range only and the wave frequency responses are filtered out. The overall motions are obtained by combining the low-frequency motions with the wave-frequency motions, which are calculated using diffraction-radiation methods (RAOs of the motions) for the given sea states. A relatively large time step may be used in the time domain analysis. Subsection 9/3 discusses the combination of wave-frequency and low-frequency responses for the fatigue damage calculation.

ii) *Approach II.* Another approach, which is theoretically and numerically more rigorous, is to perform a time domain simulation including both wave-frequency and low-frequency motions simultaneously. The rainflow counting method is used to obtain the number of cycles for each stress range. The disadvantage of this approach, however, is that a much smaller time step should be used to achieve the sufficient accuracy.

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SECTION 5 Wave-induced Load Components

1 General

Wave-induced loads on a buoyant structure are complicated because, in addition to producing direct forces (e.g., wave pressures on the external surface of the hull), there are indirect force components produced by the rigid body motions of the vessel. The motions result in inertial forces and rotational components of the (quasi-statically considered) loads. These two motion-related load components are referred to below as the "inertial" and "quasi-static" load components. For a moored, buoyant structure such as an FPSO, added complexity arises from the mooring system, which produces reaction load components.

The treatment of the various load and motion effects is typically done through the use of their real and imaginary parts that are employed separately in structural analyses. In a physical sense, the real and imaginary parts correspond to two wave systems that are 90 degrees out of phase relative to each other.

The following Subsections list the primary wave-induced load components that are to be considered in the Spectral-based Fatigue Analysis of an FPSO. Using the methods and calculation tools that are described in Section 4, the Response Amplitude Operators (RAOs) for the listed components are obtained.

3 External Pressure Component

3.1 Total Hydrodynamic Pressures

The total hydrodynamic pressure includes the direct pressure components due to waves and the components due to hull motions. The components of the hydrodynamic pressure should be determined from the model and calculation procedure mentioned in Section 4.

3.3 Intermittent Wetting

Ship motion analysis based on linear theory will not predict the nonlinear effects near the mean waterline due to intermittent wetting. In actual service, this phenomenon is manifested by a reduction in the number of fatigue cracks at side shell plating stiffeners located near the waterline compared to those about four (4) or five (5) bays below. To take into account the pressure reduction near the mean waterline due to this nonlinearity, the following reduction factor can be used:

$$RF = 0.5[1.0 + \tanh(0.35d)]$$

where d is the depth, in meters, of the field point below the still-water waterline.

The pressure distribution over a hydrodynamic panel model may be too coarse to be used directly in the structural FEM analysis. Therefore, as needed, the pressure distribution is to be interpolated (3-D linear interpolation) over the finer structural mesh.

5 Internal Tank Pressure Component

As stated in Subsection 5/1, the vessel motion-related internal tank pressure is composed of quasi-static and inertial components. The quasi-static component results from gravity for the instantaneous roll and pitch of the vessel. The inertial component is due to the acceleration of the fluid caused by vessel motion in six degrees of freedom. The vessel motion should be obtained from analysis performed in accordance with Section 4.

The internal tank pressure (quasi-static + inertial) for each of the tank boundary points is calculated by the following:

$$P = P_0 + \rho h_t[(g_x - a_x)^2 + (g_y - a_y)^2 + (g_z - a_z)^2]^{0.5}$$

where

P = total internal tank pressure at a tank boundary point

P_o = value of the pressure relief valve setting

ρ = density of the fluid cargo or ballast

h_t = total pressure head defined by the height of the projected fluid column in the direction to the total acceleration vector

a_x, a_y, a_z = longitudinal, lateral and vertical motion wave-induced accelerations relative to the vessel's axis system at a tank boundary point.

g_x, g_y, g_z = longitudinal, lateral and vertical instantaneous gravitational accelerations relative to the vessel's axis system at a tank boundary point.

The internal pressure at the tank boundary points are to be linearly interpolated and applied to all of the nodes of the structural analysis model defining the tank boundary.

7 Loads from the Motions of Discrete Masses

Vessel motions produce loads acting on the masses of "lightship" structure and equipment. The motion induced acceleration, A_t, is determined for each discrete mass from the formula:

$$A_t = (R \times \Phi)\omega_e^2 + a$$

where

R = distance vector from the hull's CG to the point of interest

Φ = rotational motion vector

× = cross product between the vectors

a = translation acceleration vector

ω_e = encounter frequency

It might be noted that the nonlinear term due to the centripetal acceleration of rotational motion is neglected in the above equation.

Using the real and imaginary parts of the complex accelerations calculated above, the motion-induced inertial load is computed by:

$$F = -m\ (A_t)$$

where m is the discrete mass under consideration.

The real and imaginary parts of the motion-induced loads from each discrete mass in all three directions are calculated and applied to the structural model.

SECTION 6 Loading for Global Finite Element Method (FEM) Structural Analysis Model

1 General

For each heading angle and wave frequency at which the structural analysis is performed (see Subsection 2/3), two load cases corresponding to the real and imaginary parts of the frequency regime wave-induced load components are to be analyzed. Then, for each heading angle and wave frequency, the frequency-dependent wave-induced cyclic stress transfer function, $H_\sigma(\omega|\theta)$, is obtained for the Base Vessel Loading Condition and vessel speed, if applicable.

When inputting the pressure loading components, care is to be taken in the interpolation near regions where pressure changes sign.

3 Number of Load Cases

In order to generate the stress transfer function, the number of combined load cases for each Base Vessel Loading Condition can be relatively large. When the structural analysis is performed for 33 frequencies (0.2 to 1.80 rad/s at a 0.05 increment) and 12 wave headings (0 to 360 degree at a 30-degree increment), the number of combined Load Cases is 792 (considering separate real and imaginary cases). If there are three (3) Base Vessel Loading Conditions, the total number of load cases is $(3 \cdot 792) = 2376$.

However, a significant reduction in the number of heading angles, hence load cases, to be analyzed is possible in the "on-site" analysis of an FPSO system with a "weathervaning" turret mooring. Where justified by the environmental data, a minimum of 5 heading angles, head-on and 30 and 60 degrees off either side of head-on, will be considered sufficient (refer to Section 2, Appendix 1 of the ABS *Guide for Building and Classing Floating Production Installations*). In this case, with 3 Base Vessel Loading Conditions the number of load cases for analysis is $(33 \cdot 2 \cdot 5 \cdot 3) = 990$.

Note: The term "head-on" does not imply a stern-mounted turret is treated differently.

5 Mooring Loads

Mooring loads are primarily elastic reactions resisting the combined effects of wave-induced forces and motions, and current, wind, etc., effects on the FPSO hull. The effects of mooring can be considered in three regimes of hull motion: *first-order* (wave frequency), *second-order* (low frequency or slowly varying) and *steady offset* due to wind and wave. These frequency-related components are to be obtained using a recognized vessel mooring analysis method.

The results of the mooring analysis and the environmental data on the directionality of the prevalent load effects are to be used to establish the mooring loads to be included in the structural analyses of Section 7.

7 Equilibrium Check

The applied hydrodynamic external pressure and the mooring loads should be in equilibrium with all other loads applied. The unbalanced forces in three global directions for each load case should be calculated and checked. For the head sea condition, the unbalanced force should not exceed one percent of the displacement. For oblique and beam sea conditions, it should not exceed two percent of the displacement. These residual forces could be balanced out by adding suitably distributed inertial forces (so called "inertial relief") before carrying out the FEM structural analysis.

SECTION 7 Structural Modeling and Analysis

1 General

The stress transfer function, $H_\sigma(\omega|\theta)$, for a location where the fatigue strength is to be evaluated, should be determined by the finite element method (FEM) of structural analysis using a three dimensional (3-D) model representing the entire hull structure, the topside equipment support structure and the interface with the mooring system, and as applicable, the risers. The Load Cases to be used in the analysis should be those obtained in accordance with Section 6. Special attention should be paid to the modeling of the stiffness of a turret mooring system and the transmission of mooring loads into the hull.

As necessary to evaluate the fatigue strength of local structure, finer mesh FEM analyses should also be performed. Results of nodal displacements or forces obtained from the overall 3-D analysis model should be used as boundary conditions in the subsequent finer mesh analysis of local structures.

Specialized fine mesh FEM analysis is required in the determination of stress concentration factors associated with the "hot-spot" fatigue strength evaluation procedures (see Subsection 7/9).

Note: Reference should be made to additional ABS Guidance on the expected modeling and analysis of vessel structure; e.g., the *FEA in ABS SafeHull Online Manuals* that is provided with the SafeHull software. While there is a significant difference in the extent of structural model described here and the partial hull model allowed in SafeHull, numerous detailed modeling considerations are shared; such as expected element types, mesh sizes, dependence between local and global models, etc.

3 Areas for Fatigue Strength Evaluations

Refer to Subsection 1/3.

5 3-D Global Analysis Modeling

The global structural and load modeling should be as detailed and complete as practicable. For the Spectral-based Fatigue Analysis of a new-build structure, gross or as-built scantlings are ordinarily used.

In making the model, a judicious selection of nodes, elements and degrees of freedom is to be made to represent the stiffness and inertial properties of the hull. Lumping of plating stiffeners, use of equivalent plate thickness and other techniques may be used to keep the size of the model and required data generation within manageable limits.

The finite elements, whose geometry, configuration and stiffness closely approximate the actual structure, are of three types:

i) Truss or bar elements with axial stiffness only

ii) Beam elements with axial, shear, bending and torsional stiffness, and

iii) Thin plate and shell elements, either triangular or quadrilateral.

Mesh design, the discretization of a structure into a number of finite elements, is one of the most critical tasks in finite element modeling and often a difficult one. The following parameters need to be considered in designing the layout of elements: mesh density, mesh transitions and the stiffness ratio of adjacent elements. As a general rule, a finer mesh is required in areas of high stress gradient. The performance of elements degrades as they become more skewed. If the mesh is graded, rather than uniform, as is usually the case, the grading should be done in a way that minimizes the difference in size between adjacent elements.

In modern FEA software, most analysts rely on preprocessors to develop the finite element mesh. Automatic mesh generators yield adequate meshes. However, in very demanding configurations, the mesh generator may produce a poor mesh. In such situations, the mesh should be manually produced to improve the mesh quality.

In modeling complex structural assemblies, there is a possibility of constructing models where adjacent structural elements have very different stiffness. To prevent large numerical errors, a conservative stiffness ratio of the order of 10^4 or more between members making up a model should be avoided.

7 Analyses of Local Structure

More refined local stress distributions should be determined from the fine mesh FEM analysis of local structure. In the fine mesh models, care is to be taken to accurately represent the structure's stiffness, as well as its geometry. Boundary displacements obtained from the 3-D global analysis are to be used as boundary conditions in the fine mesh analysis. In addition to the boundary constraints, the pertinent local loads should be reapplied to the fine mesh models.

The models are to be constructed with linear quadrilateral and triangular elements (shell elements being flat bending plate elements arranged in the mid-plane of the structural components) and one-dimensional rod and beam elements. As shown in Section 7, Figure 1, the areas around the expected stress concentrations in each model (e.g., bracket heel, bracket toe, etc.) are to be carefully meshed with quadrilateral shell elements of approximate size $t \times t$, where t is the minimum plate thickness in the vicinity of a particular stress concentration. Edges of brackets and cutouts are to be meshed with dummy rod elements of approximate length t and cross-sectional area 0.01 cm^2.

The welds are usually not modeled as illustrated in Section 7, Figure 1, except for special cases where the results are affected by high local bending, e.g., due to an offset between plates, such as doubling, or due to a small free plate length between adjacent welds such as at lug (or collar) plates, as shown in Section 7, Figure 2. In this case, the weld may be included by vertical or inclined plate elements having appropriate stiffness or by introducing multiple constrained elements to couple node displacements.

9 Hot Spot Stress Concentration

The differences between a Nominal Stress Approach and a Hot-Spot Stress Approach and the selection of S-N curves in the respective approaches are described in Section 2 of the ABS *Guide for the Fatigue Assessment of Offshore Structures.*

When employing the so-called "Hot-Spot" Stress Approach (for example, to determine the fatigue strength at the toe of a fillet weld), it is necessary to establish a procedure to be followed to characterize the expected fatigue strength. The two major parts of the procedure are (a) the selection of an S-N Data Class (see Section 8) that applies in each situation; and (b) specifying the fine mesh FEM model adjacent to the weld toe detail and how the calculated stress distribution is extrapolated to the weld toe (hot-spot) location. The figure below shows an acceptable method that can be used to extract and interpret the "near weld toe" element stresses and to obtain the (linearly) extrapolated stress at the weld toe. When stresses are obtained in this manner, the use of the E class S-N curve for plated details is considered to be most appropriate.

FIGURE 1
Fine Mesh FEM Model

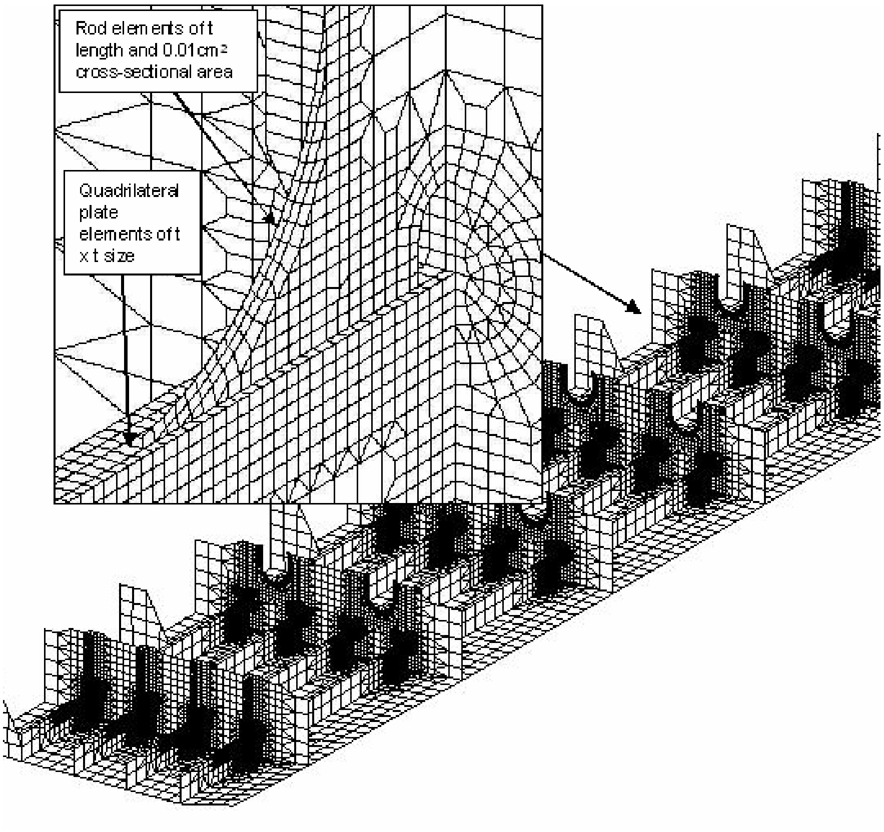
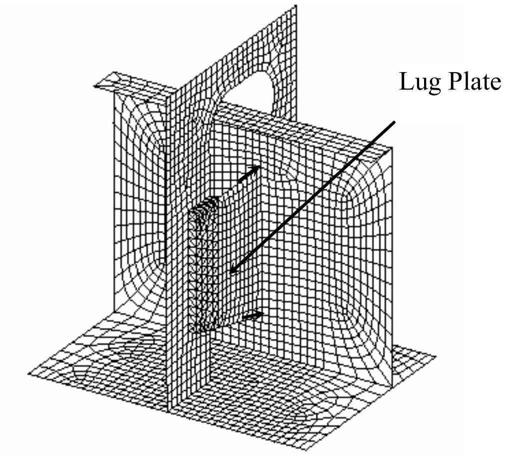


FIGURE 2
Local Structural FE Model: Welds modeled



The algorithm described below may be used to obtain the hot spot stress for the point at the toe of a weld, as shown in Section 7, Figure 3.

Consider the four points, P_1 to P_4, measured by the distances X_1 to X_4 from the weld toe, designated as the origin of the coordinate system (Section 7, Figure 4). These points are the centroids of four neighboring finite elements, the first of which is adjacent to the weld toe. Assuming that the applicable surface component stresses, S_i, at P_i have been determined from FEM analysis, the corresponding stresses at "hot spot" (i.e., the stress at the weld toe) can be determined by the following procedure:

(1) Select two points, A and B, such that points A and B are situated at distances $3t/2$ and $t/2$ from the weld toe; i.e.:

$$X_A = 3t/2, \qquad X_B = t/2$$

where t denotes the thickness of the member to which elements 1 to 4 belong.

(2) For a given point X, compute the values of four coefficients, as follows:

$$C_1(X) = [(X - X_2)(X - X_3)(X - X_4)] / [(X_1 - X_2)(X_1 - X_3)(X_1 - X_4)]$$

$$C_2(X) = [(X - X_1)(X - X_3)(X - X_4)] / [(X_2 - X_1)(X_2 - X_3)(X_2 - X_4)]$$

$$C_3(X) = [(X - X_1)(X - X_2)(X - X_4)] / [(X_3 - X_1)(X_3 - X_2)(X_3 - X_4)]$$

$$C_4(X) = [(X - X_1)(X - X_2)(X - X_3)] / [(X_4 - X_1)(X_4 - X_2)(X_4 - X_3)]$$

(3) The corresponding stress at the given point can be obtained by interpolation as:

$$S_L = C_1S_1 + C_2S_2 + C_3S_3 + C_4S_4$$

(4) Apply step (2) and (3) to Point A and Point B. The stress at Point A and Point B can be obtained by interpolation, i.e.:

$$S_A = C_1(X_A)S_1 + C_2(X_A)S_2 + C_3(X_A)S_3 + C_4(X_A)S_4$$

$$S_B = C_1(X_B)S_1 + C_2(X_B)S_2 + C_3(X_B)S_3 + C_4(X_B)S_4$$

(5) The corresponding stress at hot spot, S_{hot}, is given by

$$S_{hot} = (3S_B - S_A)/2$$

FIGURE 3
Weld Toe Extrapolation Points

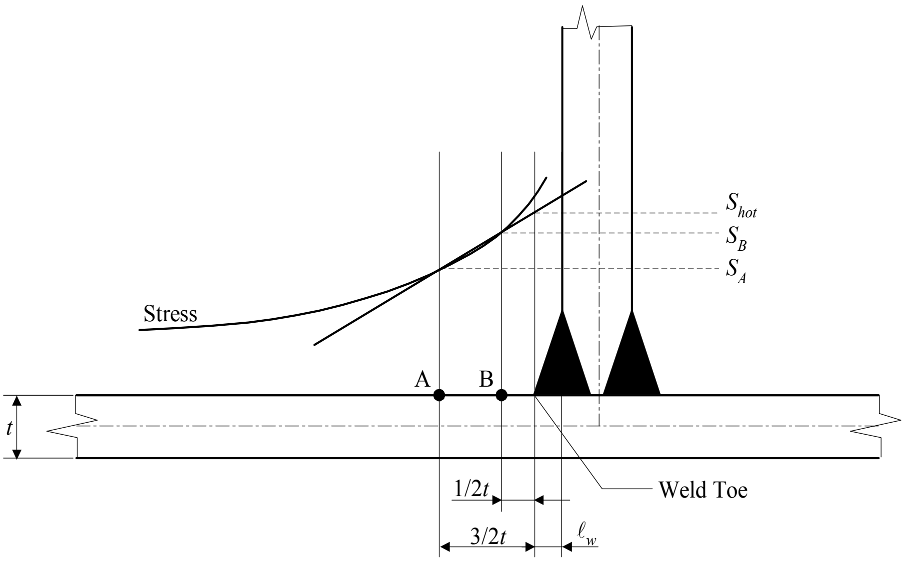
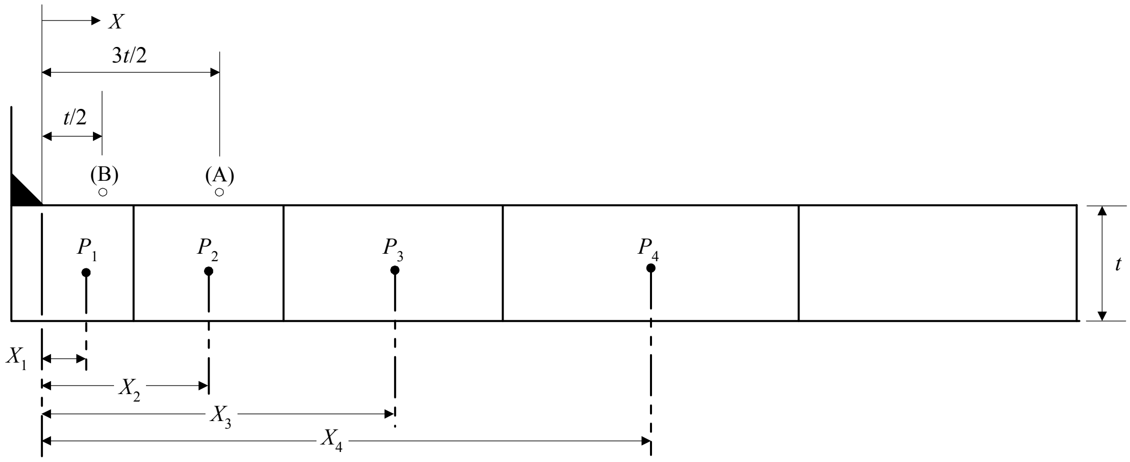


FIGURE 4
Elements Adjacent to Weld Toe



SECTION 8 Fatigue Strength

1 General

The previous Sections of these Guidance Notes have addressed establishing the stress transfer function or stress RAOs for locations in the structure for which the adequacy of fatigue life is to be evaluated. The following steps to evaluate the stress range distribution are described in Appendix 1 of these Guidance Notes. The capacity of a location to resist fatigue damage is characterized by the use of S-N Data, which are described in Sections 2 and 3 of the ABS *Guide for the Fatigue Assessment of Offshore Structures*.

Using the S-N approach, fatigue strength (capacity) is usually characterized in one of two ways. One way is called a *nominal stress approach.* In this approach, the acting cyclic stress (demand) is considered to be obtained adequately from the nominal stress distribution in the area surrounding the particular location for which the fatigue life is being evaluated. The other way of characterizing fatigue strength (capacity) at a location is the *"hot-spot" approach* (see Subsection 7/9). The hot-spot approach is needed for locations where complicated geometry or relatively steep local stress gradients would make the use of the nominal stress approach inappropriate or questionable. Reference should be made to Section 2 of the ABS *Guide for the Fatigue Assessment of Offshore Structures* for further explanation and application of these two approaches.

3 S-N Data

Section 3 of the ABS *Guide for the Fatigue Assessment of Offshore Structures* provides S-N curves for non-tubular details, tubular joints and cast steel components. Each set of curves can be adjusted for thickness effect and adjustments are provided to reflect corrosion effects. Three corrosive conditions are considered: in-air, cathodically protected in seawater, and free corrosion in seawater. For cast steel components "in-air" S-N curve is given.

There are other adjustments that could be considered to increase fatigue capacity above that portrayed by the cited S-N data. These include adjustments for "mean stress" effects, a high compressive portion of the acting cyclic stress and the use of "weld-improvement" techniques. The first two of these adjustments are not permitted, primarily because they may violate the calibration and validation that was performed in the development of the recommended fatigue strength criteria. The use of a weld-improvement technique, such as weld toe grinding or peening to relieve ambient residual stress, can be effective in increasing fatigue life. However, credit should not be taken for such a weld improvement in the design phase of the structure. Consideration for granting credit for the use of weld-improvement techniques should be reserved for situations arising during construction, operation or future reconditioning of the structure.

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SECTION 9 Fatigue Life (Damage) Calculation and Acceptance Criteria

1 General

Mathematically, spectral-based fatigue analysis begins after the determination of the stress transfer function. Wave data are then incorporated to produce stress response spectra, which are used to derive the magnitude and frequency of occurrence of local stress ranges at the locations for which fatigue damage is to be calculated. Wave data are represented in terms of a wave scatter diagram and a wave energy spectrum. The wave scatter diagram consists of sea states, which are short-term descriptions of the sea in terms of joint probability of occurrence of a significant wave height, H_s, and a characteristic period.

An appropriate method is to be employed to establish the fatigue damage resulting from each considered sea state. The damage resulting from individual sea states is referred to as "short-term". The total fatigue damage resulting from combining the damage from each of the short-term conditions can be accomplished by the use of a weighted linear summation technique (i.e., Miner's rule).

Appendix 1 contains a detailed description of the steps involved in a suggested Spectral-based Fatigue Analysis method that follows the basic elements mentioned above. ABS should be provided with background and verification information that demonstrate the suitability of the analytical method employed.

3 Combination of Wave-Frequency and Low-Frequency Responses

When the process that induces variable stresses in a structural detail contains wave-frequency and low-frequency components, the process is considered to be wide-banded. Although the Wirsching's rainflow counting correction can be applied to account for a wide band process, the formulae are calibrated only to a wave frequency process.

When wave-frequency and low-frequency stress responses are obtained separately, simple summation of fatigue damage from the two frequency bands does not count the effects of simultaneous occurrence of the two frequency bands processes. This method is therefore non-conservative and should not be used.

There is an alternative method, which is both conservative and easy to use, that is known as the combined spectrum method. In this method, the stress spectra for the two frequency bands are combined. The RMS and the mean up-crossing frequency of the combined stress process are given, respectively, as

$$\sigma_c = (\sigma_w{}^2 + \sigma_\ell{}^2)^{1/2}$$

$$f_{0c} = (f_{0w}^2\sigma_w^2 + f_{0\ell}^2\sigma_\ell^2)^{1/2} / \sigma_c$$

where

σ_w = RMS of the wave-frequency stress components

σ_ℓ = RMS of the low-frequency stress components

f_{0w} = mean up-crossing frequency of the wave-frequency stress components

$f_{0\ell}$ = mean up-crossing frequency of the low-frequency stress components

For each sea state, the fatigue damage for the combined wave-frequency and low-frequency process is obtained by substituting the above quantities for the combined process into the closed-form formula of spectral fatigue given in Appendix 1.

However, if both frequency components of stress range are significant, the above-mentioned combination method may be too conservative since the wave-frequency contribution is expected to dominate, thus controlling the mean up-crossing frequency of the combined stress process. To eliminate the conservatism, a correction factor given below can be applied to the calculated fatigue damage of the sea state:

$$\frac{f_{0p}}{f_{0c}}\left[\lambda_l^{m/2+2}\left(1-\left(\lambda_w/\lambda_l\right)^{1/2}\right)+\left(\pi\lambda_w\lambda_l\right)^{1/2}\frac{m\Gamma(m/2+1/2)}{\Gamma(m/2+1)}\right]+\frac{f_{0w}}{f_{0c}}\lambda_w^{m/2}$$

where

λ_ℓ = σ_ℓ^2/σ_c^2

λ_w = σ_w^2/σ_c^2

f_{0p} = $(\lambda_\ell^2 f_{0\ell}^2 + \lambda_\ell \lambda_w f_{0w}^2)^{1/2}$

m = slope parameter of the lower cycle segment of the S-N curve

$\Gamma()$ = complete gamma function

An alternative, more accurate method of fatigue damage calculation is to simulate the combined stress process in the time domain, and employ rainflow counting to count the stress cycles for each sea state. The accumulative fatigue damage is the weighted summation of the damages from all sea states considering the probability of occurrence of each sea state, as given in the wave scatter diagram.

5 Fatigue Damage due to Loading/Offloading of Produced Fluids

The fatigue damage due to low-cycle, high-stress-range situations needs to be taken into account, as caused by the loading and unloading of produced fluids (cargo). This is especially true in the fatigue assessment of bulkhead and side shell details, if significant fatigue damage is produced by these operations (see 4-2/13.5 of the ABS *Guide for Building and Classing Floating Production Installations*).

It is noted that these low-cycle loads can create a large stress range, which sometimes may be greater than the yield stress of the material. In this case, ordinary S-N curves are not valid at this level of stress ranges. It is recommended that S-N data or Δε-N data (total strain range vs. cycles to failure) suitable for large stress ranges be provided to ABS and used in the computation of damage instead.

Fatigue damage induced by these low-cycle loads is usually estimated with the cycle counting for the stress range and can be added to that induced by wave loads via linear summation. The analysis procedure usually includes:

i) Selection of structural details

ii) Definition of the loading configurations that characterize a typical complete loading and offloading cycle experienced by the vessel in the field

iii) Calculation of the still-water stresses for each of the loading conditions

iv) Definition of the stress sequence in a complete loading and offloading cycle according to ii) and iii); and evaluation of stress ranges and their cycles using the rainflow counting method

v) Fatigue damage (per complete loading and offloading cycle) based on S-N curve(s) if the stress ranges do not exceed the yield stress of the material

vi) Fatigue damage for design life is based on the number of loading and offloading cycles in design life multiplied by the result of *v)*

Alternatively, fatigue damage induced by these low-cycle loads can also be estimated by determining the RMS value and the mean up-crossing frequency of the stress based on either the loading manual or from observational data (since the crew may deviate from the typical loading cases in the Operating Manual by using computerized control of loading/offloading that keeps the vessel on an even keel without exceeding permissible stress limits). The RMS of the stress can be combined with that induced by wave and drift loads via the square root of the sum of the squares (SRSQ) method (see Subsection 9/3). The analysis procedure usually includes:

i) Selection of structural details

ii) Definition of the loading configurations that characterize a typical complete loading and offloading cycle experienced by the vessel in the field

iii) Calculation of the stresses for each of the loading conditions

iv) Definition of the stress sequence in a complete loading and offloading cycle according to ii) and iii); and use of a statistical method to calculate the RMS and the mean up-crossing frequency of stress due to loading/offloading

7 Acceptance Criteria

The required fatigue strength can be specified in several ways, primarily depending on the evaluation method employed. For the Spectral-based approach, it is customary to state the minimum required fatigue strength in terms of a damage ratio (*D*) or minimum target Life (*L*). The latter is employed in these Guidance Notes and the ABS *Guide for Building and Classing Floating Production Installations.*

See Section 4 of the ABS *Guide for the Fatigue Assessment of Offshore Structures* regarding target life and reference to additional factors, which are to be applied to the design fatigue life in some situations.

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APPENDIX 1 Outline of a Closed Form Spectral-based Fatigue Analysis Procedure

Notes:

(1) This Appendix is referred to in Section 9. It is provided to describe the formulations comprising a Spectral-based Fatigue Analysis approach. However, it is often at this formulation level that valid variations of a method may be introduced. For this reason, it is emphasized that the contents of this Appendix are provided primarily to illustrate principle, rather than to give mandatory steps for the Spectral-based Fatigue method.

(2) The procedure described below considers the use of a wave scatter diagram that represents long-term wave data at the installation site that have been "normalized" to represent a period of one-year. Where a different base period for the wave scatter diagram is employed, the procedure must be suitably modified.

1 General

In the "short-term closed form" approach described below, the stress range is normally expressed in terms of probability density functions for different short-term intervals corresponding to the individual cells or bins of the wave scatter diagram. These short-term probability density functions are derived by a spectral approach based on the Rayleigh distribution method, whereby, it is assumed that the variation of stress is a narrow-banded random Gaussian process. To take into account effects of swell, which are not accounted for when the wave environment is represented by the scatter diagram, Wirsching's "rainflow correction" factor is applied in the calculation of short-term fatigue damage. Having calculated the short-term damage, the total fatigue damage is calculated through their weighted linear summation (using Miner's rule). Mathematical representations of the steps of the Spectral-based Fatigue Analysis approach just described are given next.

3 Key Steps in Closed Form Damage Calculation

1. Determine the complex stress transfer function, $H_\sigma(\omega|\theta)$, at a structural location of interest for a particular load condition. This is done in a direct manner where structural analyses are performed for the specified ranges of wave frequencies and headings, and the resulting stresses are used to explicitly generate the stress transfer function. See Sections 2 to 7.

2. Generate a stress energy spectrum, $S_\sigma(\omega|H_s, T_z, \theta)$, by scaling the wave energy spectrum $S_\eta(\omega|H_s, T_z)$ in the following manner:

$$S_\sigma(\omega|H_s, T_z, \theta) = |H_\sigma(\omega|\theta)|^2 \cdot S_\eta(\omega|H_s, T_z) \qquad (1)$$

3. Calculate the spectral moments. When vessel speed *V* is considered (i.e., transit case or past service of tanker conversion), the *n*-th spectral moment, m_n, is calculated as follows:

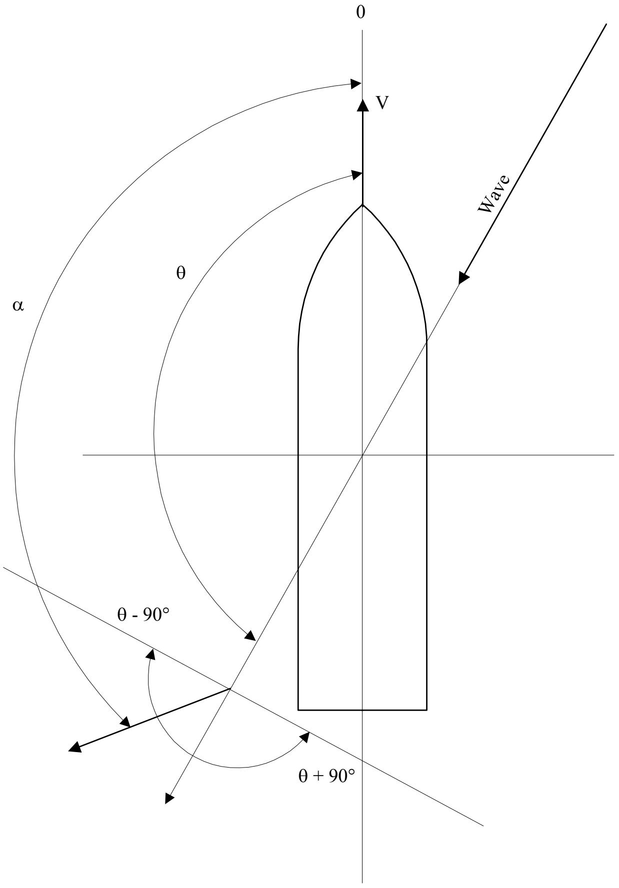
$$m_n = \int_0^\infty (\omega - V\omega^2 \cos\theta / g)^n S_\sigma(\omega \,|\, H_s, T_z, \theta) d\omega \text{ .... (2)}$$

Most fatigue damage is associated with low or moderate seas, hence confused short-crested sea conditions must be allowed. Confused short-crested seas result in a kinetic energy spread which is modeled using the cosine-squared approach, $(2/\pi)\cos^2\alpha$. Generally, cosine-squared spreading is assumed from -90 to +90 degrees on either side of the selected wave heading (refer to Appendix 1, Figure 1). Applying the wave spreading function modifies the spectral moment as follows:

$$m_n = \int_{\theta-90}^{\theta+90} \left(\frac{2}{\pi}\right) \cos^2(\alpha - \theta) \cdot \left(\int_0^\infty (\omega - V\omega^2 \cos\alpha / g)^n S_\sigma(\omega \,|\, H_s, T_z, \alpha) d\omega \right) d\alpha \text{ .... (3)}$$

The above integral is usually performed for each cell in the wave scatter diagram. However, the number of times to perform integration can be dramatically reduced if it is noted that for the cells of the same T_z, the *n*-th spectral moments are scalable to H_s^2 since the wave spectra are proportional to H_s^2.

FIGURE 1
Spreading Angles Definition



4. Using the spectral moments, the Rayleigh probability density function (pdf) describing the short-term stress-range distribution, the zero up-crossing frequency of the stress response and the bandwith parameter used in calculating Wirsching's "rainflow correction" are calculated as follows:

   Rayleigh pdf:

   $$g(S) = \frac{S}{4\sigma^2}\exp\left[-\frac{S^2}{8\sigma^2}\right] \quad \text{.......... (4)}$$

   Zero-up crossing frequency:

   $$f = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} \quad \text{.......... (5)}$$

   Bandwidth Parameter:

   $$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \quad \text{.......... (6)}$$

   where

   S = stress range (twice the stress amplitude)

   σ = $\sqrt{m_0}$

   m_0, m_2 and m_4 are the spectral moments.

5. Calculate cumulative fatigue damage based on Palmgren-Miner's rule, which assumes that the cumulative fatigue damage (D) inflicted by a group of variable amplitude stress cycles is the sum of the damage inflicted by each stress cycle (d_i), independent of the sequence in which the stress cycles occur:

   $$D = \sum_{i=1}^{N_{total}} d_i = \sum_{i=1}^{N_{total}} \frac{n_i}{N_i} \quad \text{.......... (7)}$$

   where

   n_i = number of stress cycles of a particular stress range

   N_i = average number of loading cycles to failure under constant amplitude loading at that stress range according to the relevant S-N curve

   N_{total} = total number of stress cycles.

   Failure is predicted to occur when the cumulative damage (D) over N_{total} loading cycles exceeds a critical value equal to unity. The short-term damage incurred in the *i*-th sea-state assuming an S-N curve of the form $N = KS^{-m}$ is given by:

   $$D_i = \left(\frac{T}{K}\right)\int_0^\infty S^m f_{0i} p_i g_i ds \quad \text{.......... (8)}$$

   where

   D_i = damage incurred in the *i*-th sea-state

   m, K = physical parameters describing the S-N curve

T = target fatigue life

f_{0i} = zero-up-crossing frequency of the stress response

p_i = joint probability of H_s and T_z

g_i = probability density function governing S

S = stress range

Summing D_i over all of the sea-states in the wave scatter diagram leads to the total cumulative damage, D. Therefore:

$$D = \left(\frac{f_0 T}{K}\right)\int_0^\infty S^m \left[\sum f_{0i} p_i g_i / f_0\right] ds \quad \text{.......} (9)$$

where

D = total cumulative damage

f_0 = "average" frequency of S over the lifetime

= $\Sigma_i \, p_i f_{0i}$

Introducing the long-term probability density function, $g(S)$, of the stress range as:

$$g(S) = \frac{\sum_i f_{0i} p_i g_i}{\sum_i f_{0i} p_i} \quad \text{.......} (10)$$

and N_T equal to the total number of cycles in life time = $f_0 T$, the expression for total cumulative damage, D can be re-written as:

$$D = \left(\frac{N_T}{K}\right)\int_0^\infty S^m g(S) dS \quad \text{.......} (11)$$

The minimum target fatigue life is twenty years. Having calculated the damage, fatigue life would then be equal to 20/D. Changing the minimum target fatigue life to higher values is done accordingly.

5 Closed Form Damage Expression

For all one-segment linear S-N curves, the closed form expression of damage, D as given by Equation 9, is as follows:

$$D = \frac{T}{K}(2\sqrt{2})^m \Gamma(m/2+1) \sum_i \lambda(m, \varepsilon_i) f_{0i} p_i (\sigma_i)^m \quad \text{.......} (12)$$

where

σ_i = σ in Equation (4)

λ = rainflow factor of Wirsching and is defined as

$$\lambda(m, \varepsilon_i) = a(m) + [1 - a(m)][1 - \varepsilon_i]^{b(m)} \quad \text{.......} (13)$$

where

$a(m)$ = $0.926 - 0.033m$

$b(m)$ = $1.587m - 2.323$

ε_i = Spectral Bandwidth (Equation 6)

For bi-linear S-N curves where the negative slope changes at point $Q = (S_q, 10^q)$ from m to $m' = m + \Delta m$ ($\Delta m > 0$) and the constant K changes to K', the expression for damage, as given in Equation 12, is as follows:

$$D = \frac{T}{K}(2\sqrt{2})^m \Gamma(m/2+1)\sum_i \lambda(m,\varepsilon_i)\mu_i f_{0i} p_i (\sigma_i)^m \qquad (14)$$

where μ_i is the endurance factor having its value between 0 and 1 and measuring the contribution of the lower branch to the damage. It is defined as:

$$\mu_i = 1 - \frac{\int_0^{S_q} S^m g_i ds - \left(\frac{K}{K'}\right)\int_0^{S_q} S^{m+\Delta m} g_i ds}{\int_0^{\infty} S^m g_i ds} \qquad (15)$$

If $g(S)$ is a Rayleigh distribution, then μ_i is:

$$\mu_i = 1 - \frac{\gamma(m/2+1,\nu_i) - (1/\nu_i)^{\Delta m/2}\gamma(m'/2+1,\nu_i)}{\Gamma(m/2+1)} \qquad (16)$$

where

ν_i = $(1/8)\,[S_q/\sigma_i]^2$

γ = incomplete gamma function

= $\gamma(a,x) = \int_0^x u^{a-1}\exp(-u)du$