

GUIDE FOR

VESSEL MANEUVERABILITY

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American Bureau of Shipping Incorporated by Act of Legislature of the State of New York 1862

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Foreword

This Guide is intended to assist users in applying IMO maneuvering standards and to allow the Owner, designer and builder to rate the vessel's maneuvering performance relative to statistical data of vessel maneuvering characteristics. The Guide summarizes the procedures to be used in assessing a vessel's maneuvering performance.

ABS welcomes comments and suggestions for improvement of this Guide. Comments or suggestions can be sent electronically to rsd@eagle.org.



GUIDE FOR

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SECTION 1 Introduction

1 General Description

The International Maritime Organization (IMO) approved Standards for Ship Maneuverability [IMO 2002a and IMO 2002b] and encouraged the application of these standards for vessels constructed after 2004. The IMO standards specify the type of standard maneuvers and associated criteria. Some port and flag states already have adopted some of IMO standards as their national requirements. An International Standard also exists for planning, conducting and reporting sea trials [ISO 2005].

ABS provides this Guide to help its clients prepare for implementation of the IMO standards and application of the relevant procedures. Minimum requirements given in this Guide are consistent with IMO standards. An optional class notation, **MAN**, offered for compliant vessels could be used as evidence of adherence to the IMO standards (see Section 1, Table 1).

ABS may assign another optional class notation, **MAN-A**, as defined in Section 1, Table 1. This optional class notation is in compliance with IMO Standards and signifies demonstration of maneuvering performance superior to IMO Standards.

Maneuvering performance of a vessel is judged based on maneuvering criteria which are characteristic of several maneuvers. These maneuvers and their criteria, as well as the required numerical values, are described in Section 2 of this Guide.

This Guide summarizes the procedures to be used in assessing a vessel's maneuvering performance with an explanation of requirements in Subsection 1/2. Criteria of the maneuvering performance are described in Section 2.

There are two ways to assess maneuvering performance and demonstrate compliance with IMO Standards. See Section 1, Figure 1.

Full scale sea trials in full load conditions could be used for the demonstration of compliance with IMO standards. The requirements for the sea trials are described in Section 4. Appendix 5 contains forms for the results of sea trials and Appendix 6 describes a procedure of environmental correction for sea trials.

Prediction of maneuverability validated by full scale sea trials could be used for the demonstration of compliance with IMO standards. Prediction of maneuverability performance in the design stage enables a designer to take appropriate measures in good time to achieve compliance with IMO standards. The prediction of the maneuvering could be carried out with the following methods: using existing data, or scaled model test, or numerical simulation or any combination of the three listed methods. See Section 3 for details. The prediction method is to be validated with the full scale trials. If full scale sea trials are used for the validation of maneuvering prediction methods, the trials do not have to be carried out in full load conditions. Once the prediction method is validated it may be used to demonstrate compliance with IMO standard for a vessel in full load conditions.

Results of maneuvering prediction and sea trials are also used for the wheelhouse poster recommended by IMO Resolution A.601(16) [IMO 1987]. Contents and source of maneuverability data needed for the wheelhouse poster and pilot card are described in Section 5. A sample of the wheelhouse poster and pilot card is given in Appendix 7.

The Guide also contains a large amount of reference material along with practical recommendations and samples.

Prediction of maneuverability

Prediction of maneuverability

Validation with full scale trials

If compliance is demonstrated, considered to meet IMO standard. Optional cless notation MAN may be assigned by the Bureau.

If ABS maneuvering rating equals or exceeds 2.5, optional class notation MAN-A my be assigned by the Bureau

FIGURE 1
General Sequence of Maneuverability Assessment

Appendix 1 contains a detailed sample of the numerical simulation with all the necessary equations of motion, data on hydrodynamic forces and intermediate results. Appendix 3 reviews the background information on physical phenomena behind vessel maneuvering. Appendix 2 is a detailed nomenclature for the mathematical model used in Appendices 1 and 3.

The Guide contains some sample methods for *Empirical assessment of turning and stopping ability in the early design stage*. Their advantage is to indicate possible problems with maneuvering performance as soon as principal dimensions, hull form coefficients, speed, power and type of engine are known. Methods of empirical assessment are based on statistical processing of model test and past sea trials data. Application of the empirical methods is limited to the range of parameters of the vessel included in the statistics. The empirical methods provide simple formulae and therefore do not require detailed calculations. The procedure for empirical assessment of turning and stopping ability in the early design stage is described in Appendix 4.

Appendix 8 provides a review of the Human Factors aspect of maneuverability. Finally, the references are found in Appendix 9

The maneuverability standards included in this Guide are applicable to vessels equipped with all types of steering and propulsion devices of 100 m in length and over and chemical and gas carriers regardless of length. These standards are not applicable for high-speed craft as defined in the relevant Code.

2 Description and Requirements of Optional Class Notations

In recognition of demonstrated compliance with the requirement of the Guide, ABS may assign an optional class notation, MAN or MAN-A.

These optional class notations can be used to certify that maneuverability of the vessel complies with IMO Standards for Ship Maneuverability, Resolution MSC.137(76) [IMO 2002a]. The general sequence of these procedures is shown in the form of a flowchart in Section 1, Figure 1

The optional class notations may be assigned based on the results of full scale sea trials or maneuvering prediction.

Loading conditions during the full scale sea trials are further referred to as "full load" if they correspond to full load at summer load line draft and at even keel. Any other loading conditions during the sea trials are referred to as "trial load".

If full scale sea trials are performed in "full load", the optional class notations may be assigned solely based on the results of such trials.

Upon successful validation, the optional class notations may be assigned based on the results of prediction of maneuvering in full load conditions. Validation of prediction is to be done by performing full scale sea trials and comparing the results of these trials with the results of maneuvering prediction performed for "trial load" condition. Full scale sea trials carried out for validation purposes only do not have to be performed in full load conditions. However, all other requirements of Section 4 are applicable. To demonstrate compliance, the prediction of maneuvering is to be done for a vessel in full load conditions. See Section 3 for details.

The optional class notations are assigned based on a Maneuverability Rating that consists of the rating levels one to five of all the rated maneuverability criteria, as described in Section 2. Minimum rating equal to one indicates adherence to the IMO standards. Any rating above one shows that maneuverability performance exceeds the IMO standards [IMO 2002a].

- Rtd tactical diameter rating
- $Rt\alpha$ rating for the first overshoot angle
- Rti initial turning ability rating
- Rts stopping ability rating

The overall rating is an average of the above ratings:

$$Rt = 0.25 \cdot (Rtd + Rt\alpha + Rti + Rts). \tag{1.1}$$

The other criteria in Section 2 are not rated. They are essentially a "pass-fail" type:

- Advance
- Second overshoot angle in the 10/10 zig-zag maneuver
- Track reach
- Width of unstable loop (applicable only for straight-line unstable vessels; recommended, not mandatory)

Requirements for optional class notations include development of maneuverability information and pilot cards that must be available onboard as recommended by [IMO 1987].

The optional class notation **MAN** may be assigned if all non-rated criteria are satisfied and rating of all the rated criteria is 1 or more.

The optional class notation **MAN-A** may be assigned if all non-rated criteria are satisfied and rating of all the rated criteria is 1 or more and the overall rating is 2.5 or more

The requirements for the optional class notations are described in Section 1, Table 1.

Although prediction of maneuverability is not required for assigning optional class notations if sea trials are carried out in full load conditions, it is recommended that it be performed during the design stage to check for compliance with IMO standards.

TABLE 1
Requirements for Optional Class Notations

Optional	Description	Requir	rements
Class Notation		Maneuvering assessment is based on sea trials in full load conditions	Maneuvering assessment is based on prediction
MAN	Demonstrated Maneuverability performance in compliance with IMO standards	 Sea trials are performed in full load conditions, as required in Section 4. Based on sea trials, maneuverability rating is not less than 1 for each of rated criteria as required in Section 2. Based on sea trials, all mandatory non-rated maneuverability criteria are satisfied as required in Section 2. Maneuverability information is posted on the navigation bridge and pilot card is available onboard as described in Section 5. 	 Maneuvering prediction is performed for full load conditions as required in Section 3 Based on maneuvering prediction in full load conditions, maneuverability rating is not less than 1 for each of rated criteria as required in Section 2 Based on maneuvering prediction in full load conditions, all mandatory non-rated maneuverability criteria are satisfied as required in Section 2. Sea trials are performed in trial load conditions; all other requirements of Section 4 are met. Maneuvering prediction is performed for trial load conditions as required in Section 3. Method used for maneuvering prediction is successfully validated using data of sea trials in trial load conditions as required in Section 3 Maneuverability information is posted on the navigation bridge and pilot card is available onboard as described in Section 5.
MAN-A	Demonstrated Maneuverability Performance Superior to IMO standards	In addition to the requirement above, maneuverability rating is 2.5 or more.	In addition to the requirement above, maneuverability rating is 2.5 or more.

3 Submittals, Surveys and Maintenance of Optional Class Notation

The following information is to be submitted in case an optional class notation is requested on the basis of full load sea trials:

- General ship data, including information on propulsion and steering devices. Exact set of data is to be determined by ABS on a case-by-case basis
- Report on sea trials in full load as required in Section 4
- Assessment of maneuvering based on trials as described in Section 2
- Copy of wheelhouse poster and pilot card as required in Section 5

The following information is to be submitted in case an optional class notation is requested on the basis of maneuvering prediction:

- General ship data, including information on propulsion and steering devices. Exact set of data is to be determined by ABS on a case-by-case basis
- Description of prediction method and results of prediction of maneuvering in full load as required in Section 3
- Assessment of maneuvering based on prediction for the vessels in full load conditions as described in Section 2
- Report on full scale sea trials in trial load as required in Section 4.

- Results of maneuvering prediction in trial load as required in Section 3
- Report on validation of maneuvering prediction method using results of prediction and sea trials in trial conditions as required in Section 3
- Copy of wheelhouse poster and pilot card as required in Section 5

Optional class notation **MAN** or **MAN-A** may by assigned upon the completion of the survey after construction. During the survey after construction, the Surveyor will verify that

- Wheelhouse poster is displayed in the wheelhouse
- Copies of pilot card are available on-board
- Information included in wheelhouse poster and pilot card is consistent with submittals approved by ABS.

Maintenance of the optional class notation shall be assured by Annual Surveys.

In general, a modification that will alter the vessel's maneuverability performance includes, but is not limited to: any change in hull form length or shape, change in appendages (such as changing the rudder or installation of anti-rolling fins, increasing/decreasing the size of bilge keels or changing the vessel speed), replacing or modifying the main engine, propeller, the steering machinery (e.g., changing the operating pressure), etc. If any such modification has been made, the optional class notation is to be reviewed.

During the Annual Survey, the Surveyor verifies that

- Modifications that may affect the vessel maneuverability performance have not been made to the vessel
- Wheelhouse poster is displayed in the wheelhouse
- Pilot card are available onboard

4 General Definitions and Nomenclature

The following contains nomenclature and definitions for most of the values used in this Guide. For the user's convenience, they are subdivided into several groups. This Subsection does not include the definitions and nomenclature for values only related to numerical simulation and equations of motion, which are given separately in Appendix 2.

- Test Speed or Approach Speed. The test speed or the approach speed is defined as a speed at the moment when the maneuver begins. The test speed or the approach speed for all criteria and corresponding test maneuvers either at early stage design prediction, numerical simulation, or sea trials must be within 90-100% of the speed corresponding to 85% of maximum continuous rating (MCR) of the engine.
- Full Load Conditions. Full load conditions correspond to full load at summer load line draft and at even keel. Deviation of draft within 5% is allowed.
- *Trial Load Conditions*. Trial loading conditions are loading conditions during sea trials other than operational loading conditions.
 - The following symbols are used for vessel dimensions and other general values:

L = length of the vessel, in m, measured between perpendiculars

B = molded breadth, in m

T = design draft (summer) at full load, in m

 C_B = block coefficient V = test speed, in m/s V_S = test speed, in knots

Trim = static trim, in m, positive when trimmed by stern

 A_B = submerged bow profile area, in m², considered positive when projecting forward of the fore perpendicular, see Section 1, Figure 2

 T_L = draft, in m, at which the turning circle is estimated

 δ_R = rudder angle, in degrees (positive to starboard)

Sp = span of rudder, in m

Ch = chord of rudder, in m, defined as mean chord (average of root and tip chords), see

Section 1, Figure 2

ST = stern profile type (1 = Closed 2 = Open, see Section 1, Figure 3)

NR = number of rudders

STD = steady turning diameter, in m

TD = tactical diameter, in m

Ad =advance, in m Tr =transfer, in m

 V_T = velocity in a turn, in knots

HR = head reach, in m TR = track reach, in m

 α_U = width of instability loop, in degrees

FIGURE 2
Definitions of Rudder and Bow Profile Dimensions

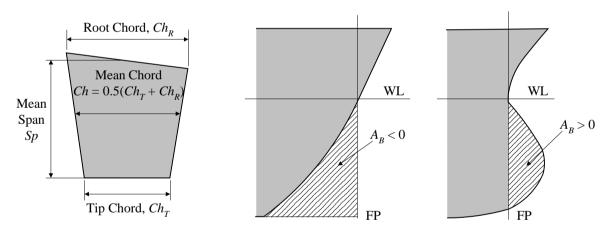
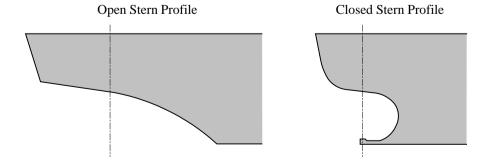


FIGURE 3
Types of Stern Profiles



5 Waiver of Full Scale Tests for Sister Vessels (15 February 2017)

For vessels seeking **MAN** notation, full scale maneuvering tests required for the notation may be waived for sister vessels in the same series. For this the vessel for which tests were carried out and sister vessels should have identical:

- Hull form
- Design draft and speed
- Steering/propulsion systems (rudder, propeller, steering gear)
- Main engine (make, model, power output)
- Energy saving devices and other appendages that may influence the inflow to propeller and rudder, if applied

Tests may be waived for sister vessels under the conditions given above and if the attained overall rating for the tested vessel is 3.0 or higher. However, an initial turning test is required even for the sister vessel seeking **MAN** or **MAN-A** notation.

Flag Administration requirements are to be checked concerning maneuvering tests for vessels in a series.



SECTION 2 Standards and Criteria

1 Overview of Standards and Criteria

An overview of standards and criteria is given in Section 2, Table 1. All the maneuvers, except stopping, are to be executed on both port and starboard and averaged values are to be used for rated and non-rated criteria.

TABLE 1
Overview of Standards and Criteria

Measure of Maneuverability	Criteria and Standard	Maneuver	IMO Standard	ABS Guide Requirement
	Required f	or Optional Class Not	ation	
Turning Ability	Tactical Diameter	Turning Circle	TD < 5L	Rated
				$Rtd \ge 1$
	Advance		Ad < 4.5L	Not rated
				Ad < 4.5L
Course Changing and	First Overshoot Angle	10/10 Zig-zag test	$\alpha 10_1 \le f_{101}(L/V)$	Rated
Yaw Checking Ability				$Rt\alpha_{10} \ge 1$
	Second Overshoot Angle		$\alpha 10_2 < f_{102}(L/V)$	Not rated
				$\alpha 10_2 < f_{102}(L/V)$
	First Overshoot Angle	20/20 Zig-zag test	$\alpha 20_1 \leq 25$	Rated
				$Rt\alpha_{20} \ge 1$
Initial Turning Ability	Distance traveled before	10/10 Zig-zag test	$\ell_{10} \le 2.5L$	Rated
	10-degrees course change			$Rti \ge 1$
Stopping Ability	Track Reach	Crash stop	$TR < 15L^{(1)}$	Not rated
				$TR < 15L^{(1)}$
	Head Reach		None	Rated
				$Rts \ge 1$
	Recommended, Not I	Required for Optional	Class Notation	
Straight-line Stability	Residual turning rate	Pull-out test	$r \neq 0$	Not rated
and Course Keeping				$r \neq 0$
Ability	Width of instability (2)	Simplified spiral	$\alpha_U \leq f_u(L/V)$	Not rated
	loop			$\alpha_U \leq f_u(L/V)$

Notes:

- 1 For large, low powered vessels, TR < 20L.
- 2 Applicable only for path-unstable vessels.

2 Turning Ability

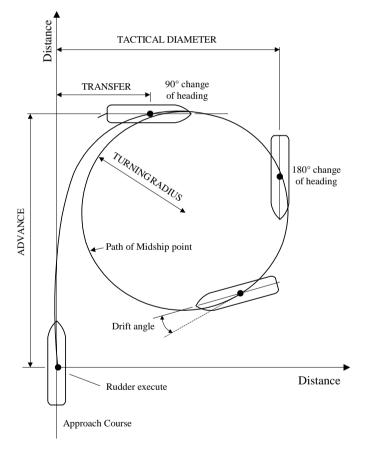
2.1 Definitions

Turning ability is the measure of the ability to turn the vessel using hard-over rudder (or other primary mean of directional control), the result being a minimum "advance at 90° change of heading" and "tactical diameter" defined by the "transfer at 180° change of heading".

A turning circle maneuver is to be performed to both starboard and port. The rudder angle must be the maximum design rudder angle permissible at the test speed, but is not required to be more than 35 degrees (applicable only to a vessel equipped with conventional rudder as a primary mean of directional control).

The rudder angle is executed following a steady approach with zero yaw rate. The essential information to be obtained from this maneuver is tactical diameter, advance and transfer (see Section 2, Figure 1). In addition, the speed lost in a turn and maximum roll angle, as well as the peak and final yaw rates, should be recorded.

FIGURE 1 Turning Circle Test



2.2 Advance and Tactical Diameter Criteria

Tactical diameter and advance are to be determined from the turning circle test, as defined in Section 2, Figure 1.

IMO requires that the tactical diameter is to be less than 5 ship lengths and the advance is to be less than 4.5 ship lengths [IMO 2002a]:

$$Ad \le 4.5L \tag{2.1}$$

Provided that condition (2.1) is satisfied, the rating of turning ability is found from the following formulae or from Section 2, Figure 2.

if
$$(4.26 - 1.62 \cdot 10^{-6} \Delta) \cdot L < TD \le 5 \cdot L$$
 then $Rtd = 1$ if $(3.63 - 1.62 \cdot 10^{-6} \Delta) \cdot L < TD \le (4.26 - 1.62 \cdot 10^{-6} \Delta) \cdot L$ then $Rtd = 2$ if $(2.79 - 1.62 \cdot 10^{-6} \Delta) \cdot L < TD \le (3.63 - 1.62 \cdot 10^{-6} \Delta) \cdot L$ then $Rtd = 3$ if $(2.16 - 1.62 \cdot 10^{-6} \Delta) \cdot L < TD \le (2.79 - 1.62 \cdot 10^{-6} \Delta) \cdot L$ then $Rtd = 4$

where

TD = tactical diameter, in meters

 $(2.16 - 1.62 \cdot 10^{-6} \Delta) \cdot L > TD$

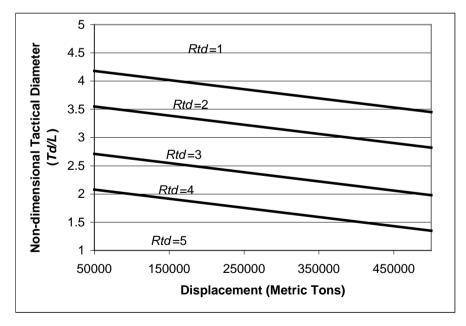
L = vessel length, in meters

 Δ = vessel displacement, in metric tons

Rtd = rating of turning ability

The above rating is based on statistics of sea trials [Barr, et al 1981].

FIGURE 2
Rating of Turning Ability



then Rtd = 5.....(2.2)

3 Initial Turning/Course Changing and Yaw Checking Ability

3.1 Definitions

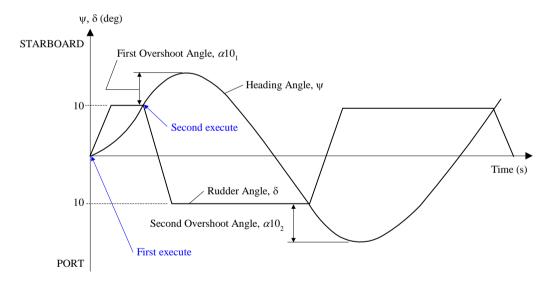
The initial turning ability is defined by the change-of-heading response to a moderate helm, in terms of heading deviation per unit distance sailed or in terms of the distance covered before realizing a certain heading deviation (such as the "time to second execute" demonstrated when entering the zig-zag maneuver).

The yaw-checking ability of the vessel is a measure of the response to counter-rudder applied in a certain state of turning, such as the heading overshoot reached before the yawing tendency has been cancelled by the counter-rudder in a standard zig-zag maneuver.

A zig-zag test should be initiated to both starboard and port and begins by applying a specified amount of rudder angle to an initially straight approach ("first execute"). The rudder angle is then alternately shifted to either side after a specified deviation from the vessel's original heading is reached ("second execute" and following) (see Section 2, Figure 3).

Two kinds of zig-zag tests are included in the Standards [IMO 2002a], the 10/10 and 20/20 zig-zag tests. The 10/10 zig-zag test uses rudder angles of 10 degrees to either side following a heading deviation of 10 degrees from the original course. The 20/20 zig-zag test uses 20-degree rudder angles coupled with a 20-degree change of heading from the original course. The essential information to be obtained from these tests is the overshoot angles, initial turning time to second execute and the time to check yaw.





3.2 The First Overshoot Angle in the 10/10 Zig-zag Maneuver Criteria

The first overshoot angle in the zig-zag test is a measure of the vessel's course checking ability. It is defined in Section 2, Figure 3. The requirements below are the result of harmonization of the IMO standards [IMO 2002a] rating system based on statistics of sea trials [Barr, et al 1981].

As measured in the 10/10 zig-zag test, the first overshoot angle, $\alpha 10_1$, is to be evaluated with the following auxiliary function:

$$f_{101}(L/V) = \begin{cases} 10.0 & \text{if} \quad L/V \le 10 \,\text{sec.} \\ 5 + 0.5 \cdot \left(L/V\right) & \text{if} \quad 10 \,\text{s} < L/V < 30 \,\text{sec.} \\ 20.0 & \text{if} \quad L/V \ge 30 \,\text{sec.} \end{cases}$$
 (2.3)

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where

L = vessel length, in meters

V = vessel speed, in m/s

The first overshoot angle in the 10/10 zig-zag test is to be rated as follows.

The rating for the first overshoot angle in the 10/10 zig-zag test, $Rt\alpha_{10} = 1$, can be assigned only if:

$$10.04 + 2.22C_b < f_{101}(L/V). \tag{2.4}$$

where

 C_b = block coefficient

L = vessel length, in meters

V = test speed, in m/s

Provided that condition (2.4) is satisfied:

if
$$10.04 + 2.22C_b < \alpha 10_1 \le f_{101}(L/V)$$
 then $Rt\alpha_{10} = 1$(2.5)

Provided that condition (2.4) is satisfied:

if
$$7.42 + 2.22C_b < \alpha 10_1 \le 10.04 + 2.22C_b$$
 then $Rt\alpha_{10} = 2$(2.6)

If condition (2.4) is not satisfied:

if
$$7.42 + 2.22C_b < \alpha 10_1 \le f_{101}(L/V)$$
 then $Rt\alpha_{10} = 2$(2.7)

Assignment of other ratings does not depend on condition (2.4) and is to be done according to the following formulae.

if
$$3.92 + 2.22C_h < \alpha 10_1 \le 7.42 + 2.22C_h$$
 then $Rt\alpha_{10} = 3$

if
$$1.29 + 2.22C_b < \alpha 10_1 \le 3.92 + 2.22C_b$$
 then $Rt\alpha_{10} = 4$

if
$$\alpha 10_1 \le 1.29 + 2.22C_b$$
 then $Rt\alpha_{10} = 5$(2.8)

where

 C_b = block coefficient

 $Rt\alpha_{10}$ = rating for the first overshoot angle in the 10/10 zig-zag test

3.3 The Second Overshoot Angle in the 10/10 Zig-zag Maneuver Criteria

The second overshoot angle in the 10/10 zig-zag test, $\alpha 10_2$, as required by IMO standards [IMO 2002a], is to be evaluated with the following function:

$$f_{102}(L/V) = \begin{cases} 25.0 & \text{if} \quad L/V \le 10 \, s \\ 17.5 + 0.75 \cdot (L/V) & \text{if} \quad 10 \, s < L/V < 30 \, s \\ 40.0 & \text{if} \quad L/V \ge 30 \, s \end{cases}$$
 (2.9)

Based on IMO standards [IMO 2002a], the second overshoot angle in the 10/10 zig-zag test should meet the following requirement:

$$\alpha 10_2 < f_{102}(L/V)$$
(2.10)

where

L = vessel length, in meters

V = test speed in m/s

3.4 The First Overshoot Angle in the 20/20 Zig-zag Maneuver Criteria

The requirements below are the result of harmonization of the IMO standards [IMO 2002a] rating system based on the statistics of the sea trials [Barr, et al 1981]. As measured in the 20/20 zig-zag test, the first overshoot angle, $\alpha 20_1$, is to be rated as follows.

$$\begin{array}{ll} \text{if} & 20.09 + 4.44C_b < \alpha 20_1 \leq 25 & \text{then } Rt\alpha_{20} = 1 \\ \\ \text{if} & 14.84 + 4.44C_b < \alpha 20_1 \leq 20.09 + 4.44C_b & \text{then } Rt\alpha_{20} = 2 \\ \\ \text{if} & 7.84 + 4.44C_b < \alpha 20_1 \leq 14.84 + 4.44C_b & \text{then } Rt\alpha_{20} = 3 \\ \\ \text{if} & 2.59 + 4.44C_b < \alpha 20_1 \leq 7.84 + 4.44C_b & \text{then } Rt\alpha_{20} = 4 \\ \end{array}$$

then $Rt\alpha_{20} = 5$(2.11)

where

L = vessel length, in meters

if $\alpha 20_1 \le 2.59 + 4.44C_h$

 $Rt\alpha_{20}$ = rating for the first overshoot angle in the 20/20 zig-zag test.

3.5 Resulting Overshoot Angle Rating

Provided both ratings $Rt\alpha_{10}$ and $Rt\alpha_{20}$ are one or more, the resulting overshoot angle rating is to be calculated as:

$$Rt\alpha = 0.5(Rt\alpha_{10} + Rt\alpha_{20}) \tag{2.12}$$

3.6 Initial Turning Ability Criterion

The initial part of the zig-zag test is used to judge the vessel's initial turning ability. Specifically, with the application of a 10-degree rudder angle, the vessel must not travel more than 2.5 ship lengths before the vessel's heading has changed 10 degrees.

The requirements below are the result of harmonization of the IMO standards [IMO 2002a] rating system based on statistics of the sea trials [Barr, et al 1981]. Assignment of the initial turning ability rating is to be done as follows:

$$\begin{array}{ll} \text{if} & 2.24L < \ell_{10} \leq 2.50L & \text{then } Rti = 1 \\ \\ \text{if} & 2.07L < \ell_{10} \leq 2.24L & \text{then } Rti = 2 \\ \\ \text{if} & 1.89L < \ell_{10} \leq 2.07L & \text{then } Rti = 3 \\ \\ \text{if} & 1.63L < \ell_{10} \leq 1.89L & \text{then } Rti = 4 \\ \end{array}$$

if
$$\ell_{10} \le 1.63L$$
 then $Rti = 5$(2.13)

where

L = vessel length between perpendiculars, in meters

 $\ell_{10}=$ distance that the vessel travels from the moment of the first execute until the course angle reaches 10 degrees in the 10/10 zig-zag maneuver

Rti = rating for the initial turning ability

4 Stopping Ability

4.1 Definitions

Stopping ability is measured by the "track reach" and "head reach" realized in a stop engine-full astern maneuver performed after a steady approach at the test speed until ahead speed in ship coordinates changes sign (i.e., vessel starts going backward).

- *Track Reach* is defined as a distance along the vessel's track that the vessel covers from the moment that the "full astern" command is given until ahead speed changes sign. See Section 2, Figure 4.
- *Head Reach* is defined as a distance along the direction of the course at the moment when the "full astern" command was given. The distance is measured from the moment when the "full astern" command is given until the vessel is stopped dead in the water. See Section 2, Figure 4.

4.2 Track Reach Criterion

The stopping ability of the vessel is judged using a full astern crash stop maneuver. Based on IMO requirements [IMO, 2002a], the track reach (see Section 2, Figure 4) should generally not exceed 15 ship lengths (measured along the path).

However, in the case of low-powered large displacement vessels, this value may be modified, but in no case should exceed 20 ship lengths, subject to special consideration and approval by ABS. Determination of whether a vessel falls into the category of "low powered large displacement vessels" also will be done by ABS.

4.3 Head Reach Criterion

where

Head reach criterion in a form of rating is based on statistics of sea trials [Barr, et al 1981].

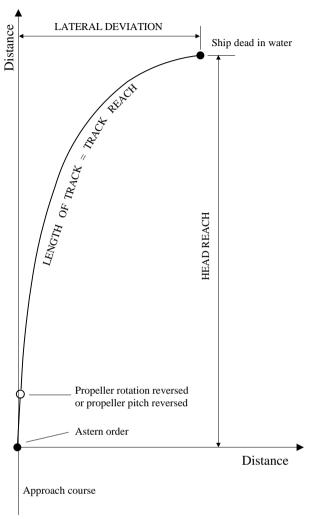
Stopping ability rating only if:

TR < 20L only for low-powered large displacement vessel TR < 15L in all other cases......(2.14)

TR = track reach, in meters, as defined in 2/4.1 (see Section 2, Figure 4)

L = vessel length, in meters

FIGURE 4 Stopping Ability Test



Defintions Used in Stopping Test

Provided that condition (2.14) is satisfied, the head reach is to be rated as follows:

if
$$Fn(69.4 + 0.000139 \cdot \Delta) < HR/L$$
 then $Rts = 1$(2.15)

where

HR = non-dimensional head reach, measured in ship lengths, as defined in 2/4.1 (see

Section 2, Figure 4)

L = vessel length, in meters

 Δ = displacement, in metric tons

 $Fn = \text{Froude Number, } \frac{V}{\sqrt{gL}}$ (2.16)

g = gravity acceleration (9.807 m/s²)

L = vessel length, in meters

V = test speed in, m/s

Provided that condition (2.14) is satisfied, assignment of other ratings is to be done according to the following formulae:

f $Fn(56.2 + 0.000139 \cdot \Delta) < HR/L \le Fn(64.9 + 0.000139 \cdot \Delta)$ then Rts = 2

if $Fn(29.8 + 0.000139 \cdot \Delta) < HR/L \le Fn(56.2 + 0.000139 \cdot \Delta)$ then Rts = 3

if $Fn(16.6 + 0.000139 \cdot \Delta) < HR/L \le Fn(29.8 + 0.000139 \cdot \Delta)$ then Rts = 4

if $HR/L \le Fn (16.6 + 0.000139 \cdot \Delta)$ then Rts = 5....(2.17)

where

HR = non-dimensional head reach, measured in ship lengths, (see Section 2, Figure 4)

 Δ = displacement, in metric tons

Fn = Froude Number

Rts = rating for stopping ability

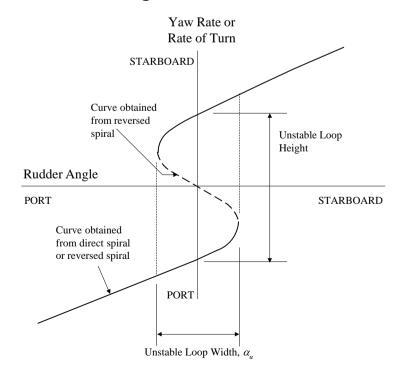
5 Straight-line Stability and Course Keeping Ability

5.1 Definitions

A vessel is straight-line stable on a straight course if, after a small disturbance, it will soon settle on a new straight course without any corrective rudder. The resultant deviation from the original heading will depend on the degree of inherent stability and on the magnitude and duration of the disturbance.

The course-keeping quality is a measure of the ability of the steered vessel to maintain a straight path in a predetermined course direction without excessive oscillations of rudder or heading. In most cases, reasonable course control is possible where there is small straight-line instability. The relationship between rudder angle and yaw rate is used to quantify a magnitude of straight-line instability, in particular with unstable loop, α_{II} , as recommended by IMO standards [IMO 2002a], see Section 2, Figure 5.

FIGURE 5
Relation between Rudder Angle and Yaw (Turn) Rate for Straight-line Unstable Vessel

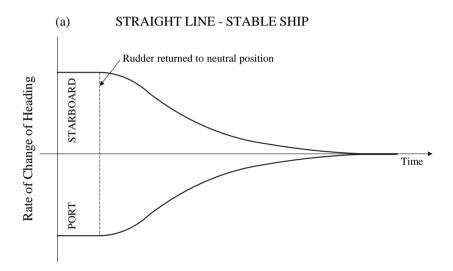


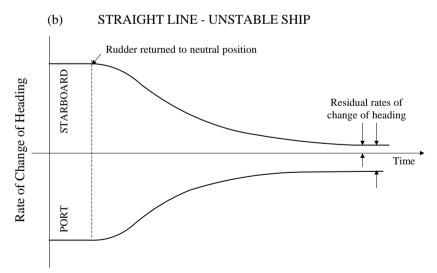
5.2 The Pull-out Test

The pull-out test allows for the determination of whether a vessel is dynamically stable and able to keep the course.

After the completion of the turning circle test, the rudder is returned to neutral position, (zero for twin screw vessels, may not equal to zero for single screw vessels) and kept there until a steady turning rate is obtained. This test gives a simple indication of a vessel's dynamic stability on a straight course. If the vessel is stable, the rate of turn will decay to zero (within accuracy of vessel equipment) for turns to both port and starboard (see Section 2, Figure 6a). If the vessel is unstable, then the rate of turn will reduce to some residual rate of turn (see Section 2, Figure 6b). The residual rates of turn to port and starboard indicate the magnitude of instability at the neutral rudder angle. Normally, pull-out maneuvers are performed in connection with the turning circle, zig-zag, or initial turning tests, but they may be carried out separately. (*Note:* This test should only be used to identify course instability.)

FIGURE 6 Pull-out Test





5.3 Tests for Straight-line Unstable Vessels

If the vessel is found to be straight-line unstable by the pull-out test, one of spiral tests may also be performed:

- The direct spiral maneuver (Dieudonné Spiral) is an orderly sequence of turning circle tests to obtain a steady turning rate versus rudder angle relation. The maneuver requires a very long time and therefore is not recommended for sea trial.
- The reverse spiral (Bech Spiral) test may provide a more rapid procedure than the direct spiral test in developing the spiral curve and enables obtaining the dashed or unstable portion of the yaw rate versus rudder angle relationship in Section 2, Figure 5 which is not obtainable from the Dieudonné test. In the reverse spiral test, the vessel is steered to obtain a constant yaw rate, the mean rudder angle required to produce this yaw rate is measured and the yaw rate versus rudder angle plot is created. Points on the curve of yaw rate versus rudder angle may be taken in any order. A more detailed description of the reverse spiral is given in 4/3.5.

5.4 Maximum Width of Unstable Loop

If the vessel is found to be straight-line unstable (there is a residual turn rate after the rudder was returned to the neutral position), the magnitude of the instability loop (see Appendix 3, Figure 11) should be assessed in order to determine if an average helmsman can control the vessel. In order to assure this capability, the following maximum value of the width of the instability loop, α_U , is specified by the following formulae (see also Section 2, Figure 7):

$$\alpha_{U} \le f_{U}(L/V) = \begin{cases} 0 & \text{if } L/V < 9s \\ \left(\frac{1}{3} \frac{L}{V} - 3\right) & \text{if } 9 \le L/V < 45s \end{cases}$$
 (2.18)

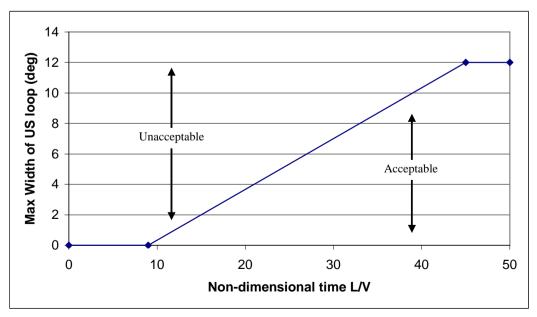
where

L = vessel length, in meters

V = test speed measured, in m/s

No rating is assigned in relation to the width of the instability loop.

FIGURE 7
Maximum Width of Unstable Loop





SECTION 3 Prediction of Maneuverability

1 General

An optional class notation may be requested on the basis of maneuvering prediction, which can be performed using one of three methods:

- Comparative prediction, see Subsection 3/2
- Numerical simulation, see Subsection 3/3
- Scaled model test, see Subsection 3/4

2 Comparative Prediction

Comparative prediction of maneuvering could be performed based on maneuvering information available from a similar vessel.

Choice of a similar vessel is to be justified with the previous positive experience. Considerations of similarity are to be included in the submittal.

3 Numerical Simulation

3.1 General

Numerical simulation based on model test data is a recommended prediction tool.

This numerical simulation is to be performed for a vessel in full load conditions (see definition in Subsection 1/4). The mathematical model (definition – see below) used for numerical simulation is to be validated with sea trial results for a vessel in trial load conditions.

Numerical simulation is the integration of differential equations describing ship motions in a horizontal plane. A sample of these equations (also called the "mathematical model") is considered in detail in Appendix 1, their brief theoretical background is reviewed in Appendix 3 and the derivation of these equations is given in Appendix 4. These Appendices are included for convenient reference.

The most important input data required by the mathematical model are hydrodynamic forces and moments acting on the submerged part of the hull and usually presented in non-dimensional form commonly known as "hydrodynamic coefficients", "maneuvering coefficients" or "maneuvering derivatives".

The only reliable source of hydrodynamic coefficients is a model test. These coefficients, however, are reusable for a vessel with similar underwater hull geometry. See 3/3.2.

3.2 Data Sources for Hydrodynamic Coefficients

Determining hydrodynamic forces and moments is the most difficult problem for maneuverability prediction. If data on hydrodynamic forces and moments is available for a similar existing vessel, it may be possible to use them for a vessel being designed.

If hydrodynamic coefficients from similar vessels are used, the justification of similarity is to be included in the submittal.

If enough information on hydrodynamic forces and moments (including nonlinear components) is available for the similar hull, these forces and moments can be used in a mathematical model to simulate maneuvers described in Section 2. Requirements for a mathematical model are listed in 3/3.4. The theoretical background of such a model is given in Appendix 3.

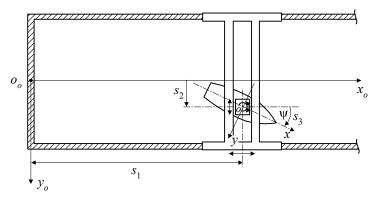
If data on hydrodynamic forces and moments are not available, the forces and moments should be acquired through a model test, as described in 3/3.3, and then used for numerical simulation with the mathematical model

3.3 Model Test as a Source of Data for Hydrodynamic Coefficients

If a model test is to be carried out to obtain hydrodynamic coefficients, the Planar Motion Mechanism (PMM) Technique is the preferred method. PMM testing is preferable to self-propelled model tests since the PMM tests can be run at the correct vessel (not model) self-propulsion point giving the correct flow over the rudder.

The PMM is a horizontal oscillator used in a traditional long towing tank. The PMM is capable of forcing a ship model to oscillate in pure sway (s_2) or yaw (s_3) or a combination of the two. Most PMM's consist of two rams that oscillate the model horizontally. See Section 3, Figure 1 below.

FIGURE 1
Scheme of Planar Motion Mechanism with Three Degrees of Freedom



The phasing of these rams determines if the model is oscillated in pure sway or pure yaw. The tests are used to determine the hydrodynamic forces and moments acting upon a model as a result of these motions. The results of the tests are used to derive a linear or nonlinear hydrodynamics model describing the vessel maneuvering capability. The model is towed at the design speed and the amplitude of the sway and yaw motion is varied.

The maneuvering forces and moments are assumed to be low frequency values and independent of frequency. The PMM model test must be conducted according to ITTC-Recommended Procedure 7.5-02-06-02 "Testing and Extrapolation Methods Maneuverability Captive Model Test Procedure".

Experimental results, obtained with Rotating Arm could also be used as a source of data for hydrodynamic coefficients. Self-propelled model test results are admissible only by special consideration by ABS on a case-by-case basis.

The following are the requirements for the model test:

- The model test is to be performed for both operational and trial loading conditions.
- The model test is to be carried out by an ABS-recognized facility, which is usually a member of the International Towing Tank Conference (ITTC).
- The model test must be conducted both for full load and ballast conditions
- A copy of the model test report and measured data in electronic format should be submitted to ABS. The report should contain a brief description of the method used (PMM, Rotating Arm, etc.), detailed description of the model, analysis of uncertainties and errors, detailed description of data processing and definitions and values for linear and nonlinear hydrodynamic derivatives and other coefficients. Electronic data are to be presented as an ASCII file with detailed description of the layout of the data.

All the model test data will be treated by ABS as confidential information.

3.4 Maneuvering Simulation

Once hydrodynamic derivatives and other coefficients have been determined either from similar vessels or through model test, the numerical simulation of maneuvers described in Section 2 has to be performed in order to predict maneuverability of a vessel being designed.

Numerical simulation of maneuvers is numerical integration of a system of ordinary differential equations describing ship motion in a horizontal plane. Such a system is also called a "mathematical model". Numerical integration of a system of ordinary differential equations is a standard mathematical procedure; relevant software is available and is a part of most engineering and mathematical packages.

The following are requirements for a mathematical model to be used for maneuvering simulation:

- The mathematical model is to be capable of simulating tight maneuvers for both straight-line stable and straight-line unstable vessels.
- The mathematical model is to account for forward speed loss during maneuvers.
- The mathematical model is to account for forces and moments caused by asymmetric flow around propeller.
- The mathematical model is to account for interaction of propeller and rudder.
- The mathematical model is to be capable of taking into account rudder deflection rate.
- The mathematical model is to be capable of simulating control actions such as deflection of the rudder and changing speed for "full ahead" to "crash astern" during a time interval typical for a vessel being designed.
- All the simulations are to be carried out for full load and ballast conditions at test speed.
- Slow speed (within the range 6-8 knots) maneuvering simulation is recommended in addition to the test speed maneuvering simulation. If such simulation is performed, it is recommended that the results are made available for the Owner/operator.
- It is recommended that for twin screw vessels, the model is to account for the interaction of the two systems.
- It is recommended to include roll into the mathematical model for fast vessels, especially for container carriers and vessels with small value of GM.

The following information is to be submitted to ABS:

- Formulation for system of differential equations
- Definition for each coefficient and its numerical value
- Straight-line stability analysis
- Time history of control inputs (commanded speed and rudder deflection angle) for each maneuver required in Section 2
- Time histories of vessel response: motions surge, sway, yaw and respective velocities obtained during simulation of maneuvers required in Section 2
- Value of maneuverability criteria as described in Section 2 with the rating figure for each of them

4 Scale Model Test

Self-propelled model test results are admissible only by special consideration by ABS on a case-by-case basis.

5 Validation of Prediction of Maneuverability

If an optional class notation is requested on the basis of maneuvering prediction, the prediction is to be validated with full scale sea trials.

Validation includes carrying out maneuvering prediction for a vessel in trial load condition exactly in the same way as it is done for the vessel in full load conditions. The results of this prediction are to include all criteria and standard values of the required maneuvers from Section 2, Table 1.

The results of this prediction are to be compared with the results of full scale sea trials. The allowable difference between prediction and trials is to be determined by ABS.

If numerical simulation is used as a method of maneuvering prediction, the validation may be done following the ITTC-recommended Procedure 7.5-02-06-03 "Testing and Extrapolation Methods Maneuverability Validation of Maneuvering Simulation Models"

If proprietary data are used, ABS will treat such data as confidential.



SECTION 4 Sea Trials

1 Required and Recommended Maneuvers

Sea trials are the final confirmation of a vessel's maneuvering qualities and its maneuverability prior to its delivery. The required maneuvers are:

- Turning test. For initial turning and steady turning ability
- 10/10 zig-zag test. For yaw checking ability, course-keeping ability and initial turning/course changing ability
- 20/20 zig-zag test. For yaw checking ability and course-keeping ability
- Stopping test (Crash Stop). For emergency stopping ability

The recommended maneuvers are:

- Pull-out test. For inherent straight-line stability
- *One of the spiral tests.* For inherent straight-line stability if the pull-out test indicated that the vessel is directionally unstable. (Recommended not mandatory)

Additional guidance on conducting of sea trials is also available from [SNAME 1987], [IMO 2002b] and [ISO 2005].

2 Conditions of Trials

Maneuverability of a vessel may be significantly influenced by hydrodynamic interaction with the sea bottom, banks and other vessels passing nearby. In addition, winds, waves, currents and tides also affect maneuverability. In order to get credible results, sea trials are to be carried out in the following conditions:

- Deep and unrestricted waters. Water depth at the trial site is to be more than four times of vessel draft at midship. The site should be free from other traffic and far enough from banks that any maneuver would not make any bank closer than two ship lengths.
- Winds and waves. The trials are not to be conducted if sea state is greater than 4. Wind is not to exceed Beaufort 5. It is not recommended to conduct the trials if the sea state is greater than 2 and wind greater than Beaufort 3.
- *Tides and currents*. It is recommended to avoid places with strong current and/or tidal influence when choosing a trial site. If current cannot be avoided, it should be uniform and the tests should be performed both for initial following and initial ahead current.

Vessel loading conditions also can affect measurements of maneuverability. The following are the requirements for loading conditions:

- If the decision is made to perform sea trials in full load conditions (definition, see Subsection 1/2), a vessel is to be loaded within 5% of full load draft, without trim and on even keel [IMO 2002b].
- If the decision is made to perform sea trials in trial load conditions (definition, see Subsection 1/2), there should be minimal possible trim and the propeller is to be immersed sufficiently

The following requirements are applied for the approach conditions:

- The approach speed is not to be less then 90% of the speed corresponding to 85% of MCR. It is recommended that some or all of the trials be repeated at slow speed (within the range of 6-8 knots). If such trials are performed, it is recommended that the results are made available to the Owner/Operator.
- Prior to beginning any test, a vessel should run in steady course for enough time for the relevant vessel machinery to be at steady state.

3 Guidelines on Test Procedures

3.1 Turning Circle Maneuver

A turning circle maneuver is to be performed to both starboard and port. The rudder angle is to be the maximum design rudder angle permissible at the test speed, but it is not required to be more than 35 degrees.

The rudder angle is executed following a steady approach with zero yaw rate. See Section 2, Figure 1. During trials, it is necessary to complete a turning circle of at least 720 degrees for both starboard and port turns.

The following information is to be obtained:

- Tactical diameter
- Advance
- Transfer
- Loss of speed in steady turn
- Time taken to change heading by 90 degrees
- Time taken to change heading by 180 degrees
- Final yaw rate

If possible, the entire trajectory is to be recorded.

3.2 Zig-zag Maneuver

A zig-zag test is to be performed by applying a specified rudder angle (10 degrees for 10/10 zig-zag test and 20 degrees for 20/20 zig-zag test) to an initially straight approach ("first execute"). Once change of heading has reached this specified value (10 degrees or 20 degrees, respectively), the rudder is then immediately deflected to the opposite side with the same angle. See Section 2, Figure 3 and Section 4, Figure 1.

If possible, the entire trajectory is to be recorded. The following information is to be recorded from the zig-zag test (see Section 4, Figure 1):

- *Initial turning time*. The time from the first execute until the vessel reaches the specified heading (10 degrees for 10/10 zig-zag maneuver and 20 degrees for 20/20 zig-zag maneuver, respectively).
- Reverse rudder heading angle. Actual heading angle at which the rudder has been reversed. "Second execute" is to be made when the vessel reaches the specified heading change (10 degrees for 10/10 zig-zag maneuver and 20 degrees for 20/20 zig-zag maneuver, respectively). However, as reversing of the rudder also takes finite time, the actual change in heading may be different from the specified value.
- Overshoot angle. The difference between the specified value (10 degrees for 10/10 zig-zag maneuver and 20 degrees for 20/20 zig-zag maneuver, respectively) and maximal heading angle reached before the course is reversed. Both first and second overshoot angles are to be recorded.
- *Time to check yaw*. Time elapsed from the moment of the first or second execute to when maximum change of heading is reached.
- *Reach*. The time between the first execute and the instant when vessel heading is zero.
- *Time for complete cycle*. The time between the first execute and the instant when the time is zero after the third execute.

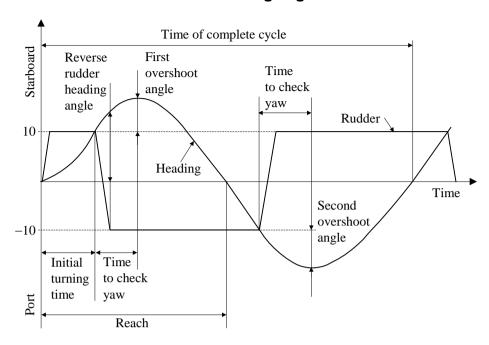


FIGURE 1
Elements of 10/10 Zig-zag Maneuver

3.3 Stopping Test (Crash Stop)

The stopping test must be performed starting from the test speed (not to be less than 90% of the speed corresponding to 85% of MCR). Once this speed is achieved and all the relevant machinery is operating in steady state, the "full astern" command is given from the engine control position on the bridge. The test is considered to be completed when the vessel speed is zero. See Section 2, Figure 4.

If possible, the entire trajectory is to be recorded. The following information must be recorded:

- *Head reach.* The distance traveled in the direction of initial heading
- Track reach. The total distance traveled along the vessel's path
- Lateral deviation. The distance traveled in the direction, perpendicular to the direction of initial heading

3.4 Pull-out Test

It is recommended to perform the pull-out tests right after the turning tests. As the test is performed to both starboard and port, a pull-out test in the corresponding direction follows. After the completion of the turning circle test, the rudder is returned to the midship position and kept there until a steady turning rate is obtained. This test gives a simple indication of a vessel's dynamic stability on a straight course. If the vessel is stable, the rate of turn will decay to zero for turns to both port and starboard. If the vessel is unstable, then the rate of turn will reduce to some residual rate of turn (see Section 2, Figure 6).

If possible, the entire trajectory is to be recorded. The following information is to be recorded during the pull-out test:

- Residual yaw rate after the rudder is centered in a starboard turn
- Residual yaw rate after the rudder is centered in a port turn

3.5 Spiral Maneuver

If a vessel is found to be directionally unstable during the pull-out test, performing one of the spiral tests is recommended. The spiral tests are described in 2/5.3. As a direct spiral (Spiral Dieudonné) maneuver takes a significant amount of time, the reverse spiral (Bech spiral) is also acceptable.

To perform the reverse spiral, the vessel is steered at a constant rate of turn. The rudder angle necessary to reach this constant rate of turn is recorded. In order to perform the maneuver with a constant rate of turn, a vessel must be equipped with a rate-gyro (swing indicator) or with any other means of differentiation of the course readings from a gyro-compass with an accurate (±1 degree) rudder angle indicator.

The maneuver is performed by several consecutive deflections of a rudder in order to reach the given turning rate. The following recommendation is cited after ITTC – Recommended Procedure on Full scale Maneuvering Sea Trials Procedure 7.5-04-02-01:

"The ship is made to approach the desired rate of turn, by applying a moderate rudder angle. As soon as the desired rate of turn is obtained, the rudder is actuated such as to maintain this rate of turn as precisely as possible. The helmsman should now aim to maintain the desired rate of turn using progressively decreasing rudder motions until steady values of speed and rate of turn have been obtained. Steady rate of turn will usually be obtained very rapidly, since rate steering is easier to perform than normal compass steering. However, adjustments to the rudder angle may be required until the ship achieves a steady speed; therefore, it is necessary to allow some time before the values of turning rate and rudder deflation are measured"

If possible, the entire trajectory should be recorded.

4 Data Acquisition Instrumentation

It is recommended to use the following data acquisition instrumentation for maneuverability sea trials:

- Global Positioning System (GPS) to record vessel speed and position. It is recommended to use two GPS devices at bow and stern as it allows determining vessel heading as well as course.
- Gyrocompass and Doppler log to record course.
- Rudder angle indicator to record rudder deflection.
- Turning rate (swing) indicator.
- Shaft RPM indicator.

If possible, time histories of all of the above values should be recorded in electronic form.

The procedure for environmental correction recommended by [IMO 2002b] is found in Appendix 6.

5 Information to Submit

The post-trials submittal is to be presented in the form shown in Appendix 5. In addition to this form, information on environmental conditions should be reported, including strength and mean direction of waves and wind. The shipyard will conduct the trials with the Surveyor witnessing. The shipyard will submit a report to ABS for review.

Information on time histories and trajectories should be presented in electronic form as an ASCII file with a detailed description of its layout. All trial data submitted to ABS will be treated as confidential information.



SECTION 5 Onboard Information

1 General

The wheelhouse poster is recommended by IMO Resolution A.601(15) [IMO 1987]. This Section describes the contents and source of maneuverability data needed for the wheelhouse poster. A sample of the wheelhouse poster is given in Appendix 7.

A pilot card is another form of information recommended to be available onboard a vessel by IMO Resolution A.601(15) [IMO 1987]. ABS recommends using an enhanced form of the card prepared on the basis of a significant number of interviews with pilots [Barr 1990], However the IMO-recommended pilot card will be accepted by ABS as well.

Both the wheelhouse poster and a pilot card are required for optional class notation.

2 Wheelhouse Poster

The wheelhouse poster is to be permanently displayed in the wheelhouse.

The following information for turning and emergency stopping maneuvers is required to be posted on the navigation bridge. The information on turning includes:

- Expected turn trajectory of the center of gravity and another point furthest from it.
- These trajectories must be shown in a rectangular coordinate system with the origin at the point where the rudder was deflected. Length units are cables.
- Time and speed when vessel turns 90, 180, 270 and 360 degrees from the original course.

The following information on emergency stopping maneuvers is to be available:

- Trajectories of full astern stopping maneuvers plotted along with turning (maximum rudder).
- Diagram of the stopping characteristics, including information on track reach (distance traveled), speed and time to stop for approach speeds corresponding to slow ahead, half ahead, full ahead and full sea ahead.
- Full astern maneuvering is to be presented with the envelope (a line connecting all the possible vessel-dead-in-the-water positions).

All the above information is to be presented for full load conditions.

If an optional class notation is requested on the basis of sea trials in full load conditions, the above information is to be based on full scale sea trials.

If an optional class notation is requested on the basis of maneuvering prediction, the above information is to be based on maneuvering prediction.

In addition to the above, it is recommended to include the following information for the wheelhouse poster if such information is available:

- Turning information for the ballast conditions in deep unrestricted waters.
- It is recommended to include the turning information in shallow water (with a clearance of 20% to 50% of the draft) in full load conditions.

A sample of the poster can be found in Appendix 7.

3 Pilot Card

The pilot card, to be filled by the Master, is intended to provide information to the pilot upon boarding the ship.

Availability of the empty card is required as a condition of optional class notations.

The pilot card is to contain information on the current condition of the ship with regards to its loading, propulsion and maneuvering equipment as well as other relevant equipment.

A form of a pilot card is available in Appendix 7. Use of a pilot card from [IMO 1997] is also permissible.



APPENDIX 1 Example

1 General Ship Data

A typical VLCC was chosen for the sample analysis. General ship data including rudder information is available in Appendix 1, Table 1. The sample demonstrates the prediction of maneuverability performance in early and detailed design stages including numerical simulation and model test data from [Roseman 1987]. The sample also includes the evaluation of maneuverability performance based on the result of numerical simulation.

TABLE 1
General Data for Sample Ship

Length between perpendiculars, L, m	349.8
Breadth molded, B, m	58.3
Draft in full load, T, m	19.4
Block coefficient	0.875
Displacement, metric tons	355600
Longitudinal position of center of gravity, x_{cg} , m	0
Speed, corresponding to 85% of engine output, V_s , knots	15
Movable rudder area, A_R , m ²	164.8
Rudder chord, Ch, m	10.8
Rudder span, Sp, m	15.2
Rudder deflection rate, r_{δ} , rad/s	0.04
Submerged area of bow profile, A_B , m ²	8.0
Stern type (1 – closed, 2 – open), ST	1
Number or propellers	1
Engine type:	Diesel
Time to stop engine, min	7
Time to achieve full reverse power from 0 RPM, min	2
Time to move camshaft to reverse position, sec	10
Maxim continuous rating of engine MCR, RPM	93
Nominal continuous rating of engine NCR, RPM	87
Reverse rating (70% of MCR), RPM	65

2 Prediction of Maneuverability in Early Design Stage

2.1 Prediction and Evaluation of Elements of Turning Circle

Prediction of elements of the turning circle is done using Equations (A4.1) through (A4.5) in Appendix 4. Results are given in Appendix 1, Table 2.

TABLE 2
Prediction of Turning Circle in Early Design Stage

Steady turning diameter (in ship lengths) STD/L	2.41
Tactical diameter (in ship lengths) TD/L	3.21
Advance (in ship lengths) Ad/L	2.99
Transfer (in ship lengths) Tr/L	1.53
Velocity of steady turn, V_T knots	5.79

According to the criterion for advance Equation (2.1) ($Ad \le 4.5L$) in Section 2, this vessel would pass this criterion as described in 2/3.1.

Equation (2.2) in Section 2 shows that the vessel has passed the requirements for tactical diameter $(3.68L < 3.21L \le 3.06L)$ with rating Rtd = 1.

2.2 Prediction and Evaluation of Stopping Ability

Prediction of track reach is done using Equations (A4.6) through (A4.9) and Appendix 4, Tables 2 and 3.

TABLE 3
Prediction of Stopping Ability in Early Design Stage

Value	Low Boundary	High Boundary
Coefficient A for VLCC	14	16
Coefficient <i>B</i> for diesel engine 0.6		1.0
Coefficient C for $V_s = 15$ knots	7.2	
Coefficient C _L	0	.8
Track reach (in ship lengths)	13.7	18.3

According to the criteria described in 2/3.4, the vessel is in compliance with the track reach requirement at the low boundary 13.7L < 15L but does not pass on high boundary 18.3L < 15L. However, taking into account size of the vessel, the stopping ability still may be considered acceptable as 18.3L < 20L, subject to special consideration and approval by ABS.

3 Maneuvering Simulation

3.1 Hydrodynamic Coefficients

Hydrodynamic coefficients from the model test [Roseman 1987] are given in Appendix 1, Table 4.

TABLE 4
Hydrodynamic Coefficients

Symbol	Value	Symbol	Value	Symbol	Value	Symbol	Value
$X'_{\dot{u}}$	-0.00086	$Y'_{r \delta }$	0.0	$N_{v v }'$	0.00754	a_1	-0.001331
X'_{vr}	0.01095	$Y'_{v r }$	-0.0152	N_r'	-0.00294	b_1	-0.001011
$X'_{\nu\nu}$	0.00287	$Y'_{\dot{r}}$	-0.00025	$N_{r r }^{\prime}$	0.0	c_1	0.001827
$X'_{\delta\delta}$	-0.001	Y_{δ}'	0.00416	$N'_{r\eta}$	0.0	a_2	-0.000894
X' _{rr}	0.0	$Y'_{\delta\eta}$	0.00416	$N'_{ v r}$	-0.00495	b_2	-0.000649
$X'_{vv\eta}$	0.0	$Y'_{r\eta}$	0.00138	$N'_{\dot{v}}$	-0.00005	c_2	0.001543
$X'_{\delta\delta\eta\eta}$	-0.00135	$Y'_{v\eta}$	-0.00266	N_δ'	-0.00216	a_3	-0.000894
$Y'_{\dot{v}}$	-0.0146	$Y'_{v v \eta}$	0.0	$N'_{\delta\eta}$	-0.00216	b_3	0.001016
Y _* '	0.000063	$Y'_{*\eta}$	0.000063	$N'_{v\eta}$	0.00138	c_3	0.0000004
Y_{v}'	-0.011	$N'_{\dot{r}}$	-0.000964	$N'_{v v \eta}$	0.0	a_4	-0.001722
$Y'_{v v }$	-0.0398	N_*'	-0.000033	$N'_{r\eta}$	-0.00072	b_4	-0.000619
Y'_r	0.00394	N_{v}^{\prime}	-0.00798	$N'_{*\eta}$	-0.000033	c_4	-0.000813

These coefficients are intended for use with the sample mathematical model considered in Subsection A3/4.

3.2 Mathematical Model

A mathematical model is referred to as "linear" if the differential equations are linear. It means that all the terms of the equations are linear functions or constants with respect to state variables: linear velocities and yaw rate.

The mathematical model that contains nonlinear differential equations is commonly referred to as "nonlinear". It means that among the terms of the system of equations, any types of functions of state variables could be found: constant, linear, square, third power, sine function, etc.

Methods of numerical integration work equally well on linear or nonlinear differential equations. However, systems of linear differential equations have an analytical solution, so application of numerical integration is redundant for a linear mathematical model.

On the other hand, use of linear mathematical models is limited. They cannot be used for simulation of tight maneuvers and they cannot be used at all for straight-line unstable vessels. However, linear models are essential in determining if a vessel is straight-line stable or straight-line unstable. A linear model can be obtained from the nonlinear model by ignoring all of the nonlinear terms in the equations.

The sample mathematical model described in Equations (A3.59), (A3.68) and (A3.75) in Appendix 3, was used for maneuverability simulation. These formulae are rewritten below with all of the acceleration-related terms placed in the left-hand side and all the other terms placed in right-hand side as a necessary step to present the system of ordinary differential equations in standard (Cauchy) form.

$$\left(m - \frac{\rho}{2} L^3 X_u' \right) \cdot \dot{u} = m \left[vr + x_{vg} r^2 \right] + \\ + \frac{\rho}{2} L^2 u^2 \left[a_i + b_i \eta + c_i \eta^2 \right] + \\ + \frac{\rho}{2} L^4 X_{rr}' r^2 + \frac{\rho}{2} L^2 X_{vv}' v^2 + \frac{\rho}{2} L^3 X_{vr}' vr + \\ + \frac{\rho}{2} L^2 X_{vvg}' v^2 (\eta - 1) + \\ + \frac{\rho}{2} L^2 u^2 \left(X_{ob}' \delta_R^2 + X_{ob\eta\eta} \delta_R^2 \eta^2 \right) \right)$$
 Resistance due to turn at self-propulsion point
$$\left(m - \frac{\rho}{2} L^3 Y_v' \right) \cdot \dot{v} + \left(m x_{cg} - \frac{\rho}{2} L^4 Y_r' \right) \dot{r} = m u r + \\ + \frac{\rho}{2} L^2 u Y_{vv}' v + \frac{\rho}{2} L^3 u Y_{vr}' r + \frac{\rho}{2} L^2 Y_{v\phi | v}' v | v | + \frac{\rho}{2} L^3 Y_{v | v}' v | r | + \\ + \left(\frac{\rho}{2} L^2 u Y_{vv}' v + \frac{\rho}{2} L^3 u Y_{v\eta}' r + \frac{\rho}{2} L^2 Y_{v\phi | v}' v | v | + \frac{\rho}{2} L^3 Y_{v | v}' v | r | + \\ + \left(\frac{\rho}{2} L^2 u Y_{v\eta}' v + \frac{\rho}{2} L^3 u Y_{r\eta}' r + \frac{\rho}{2} L^2 Y_{v\phi | v}' v | v | + \frac{\rho}{2} L^3 Y_{v | v}' v | r | + \\ + \frac{\rho}{2} L^2 u^2 Y_{v\phi}' \delta_R + \frac{\rho}{2} L^3 u Y_{v\eta}' r + \frac{\rho}{2} L^2 Y_{v\phi | v}' v | v | + \frac{\rho}{2} L^3 u^2 Y_{\phi | v}' \delta_R (\eta - 1) + \\ + \frac{\rho}{2} L^2 u^2 Y_{v'}' \delta_R + \frac{\rho}{2} L^3 u Y_{v\eta}' v + \frac{\rho}{2} L^3 u Y_{v\eta}' v | v | + \frac{\rho}{2} L^2 u^2 Y_{\phi | v}' \delta_R (\eta - 1) + \\ + \frac{\rho}{2} L^4 u V_{v'}' r + \frac{\rho}{2} L^3 u V_{v'}' v + \\ + \frac{\rho}{2} L^4 u V_{v'}' r + \frac{\rho}{2} L^3 u V_{vy}' v | v | v | + \frac{\rho}{2} L^4 V_{v|\eta}' v | r | + \\ + \frac{\rho}{2} L^5 v_{v|\eta'}' r | r | + \frac{\rho}{2} L^3 N_{v|\eta'}' v | v | + \frac{\rho}{2} L^4 N_{v|\eta'}' v | r | + \\ + \frac{\rho}{2} L^5 u V_{v'}' r + \frac{\rho}{2} L^3 u V_{vy}' v + \frac{\rho}{2} L^3 N_{v|\eta'}' v | v | \cdot (\eta - 1) + \\ + \frac{\rho}{2} L^3 u^2 N_{v'}' \delta_R + \frac{\rho}{2} L^3 u N_{v\eta} v + \frac{\rho}{2} L^3 N_{v|\eta'}' v | v | \cdot (\eta - 1) + \\ + \frac{\rho}{2} L^3 u^2 N_{v'}' \delta_R + \frac{\rho}{2} L^4 u N_{ob}' \rho \delta_R | r | + \frac{\rho}{2} L^3 u^2 N_{ob}' \delta_R (\eta - 1) + \\ + \frac{\rho}{2} L^3 u^2 N_{v'}' + \frac{\rho}{2} L^3 u^2 N_{v\eta}' \eta (\eta - 1)$$
 Rudder moment
$$+ \frac{\rho}{2} L^3 u^2 N_{v'}' + \frac{\rho}{2} L^3 u^2 N_{v\eta}' \eta (\eta - 1)$$
 Rudder moment
$$+ \frac{\rho}{2} L^3 u^2 N_{v'}' + \frac{\rho}{2} L^3 u^2 N_{v\eta}' \eta (\eta - 1)$$
 Rudder moment
$$+ \frac{\rho}{2} L^3 u^2 N_{v'}' + \frac{\rho}{2} L^3 u^2 N_{v'}' \eta (\eta - 1)$$

To avoid handling bulky equations (A1.1-A1.3), the left-hand side is presented in a generic form:

$$\begin{cases} \left(m - \frac{\rho}{2}L^{3}X'_{\dot{u}}\right) \cdot \dot{u} = f_{X}\left(u, v, r, \delta_{R}, u_{C}\right) \\ \left(m - \frac{\rho}{2}L^{3}Y'_{\dot{v}}\right) \cdot \dot{v} + \left(mx_{cg} - \frac{\rho}{2}L^{4}Y'_{\dot{r}}\right) \cdot \dot{r} = f_{Y}\left(u, v, r, \delta_{R}, u_{C}\right) \\ \left(I_{z} - \frac{\rho}{2}L^{5}N'_{\dot{r}}\right) \cdot \dot{r} + \left(mx_{cg} - \frac{\rho}{2}L^{4}N'_{\dot{v}}\right) \cdot \dot{v} = f_{N}\left(u, v, r, \delta_{R}, u_{C}\right) \end{cases}$$

$$(A1.4)$$

Here, the left-hand side of the system is presented with generic functions f_X , f_Y and f_N with state variable u, v, r and control parameters δ_R and u_c . To present the system (A1.4) in standard (Cauchy) form, accelerations have to be expressed explicitly similar to the procedure described in Subsection A1/2. Acceleration in the first equation of (A1.4) is expressed in a trivial way, making the derivation easier for the remaining two equations.

$$\begin{cases} \left(m - \frac{\rho}{2}L^{3}Y_{\dot{v}}'\right) \cdot \dot{v} + \left(mx_{cg} - \frac{\rho}{2}L^{4}Y_{\dot{r}}'\right) \cdot \dot{r} = f_{Y}(u, v, r, \delta_{R}, u_{C}) \\ \left(I_{z} - \frac{\rho}{2}L^{5}N_{\dot{r}}'\right) \cdot \dot{r} + \left(mx_{cg} - \frac{\rho}{2}L^{4}N_{\dot{v}}'\right) \cdot \dot{v} = f_{N}(u, v, r, \delta_{R}, u_{C}) \end{cases}$$
(A1.5)

The same system in the matrix format:

$$\begin{pmatrix} m - \frac{\rho}{2} L^{3} Y'_{\dot{v}} & m x_{cg} - \frac{\rho}{2} L^{4} Y'_{\dot{r}} \\ m x_{cg} - \frac{\rho}{2} L^{4} N'_{\dot{v}} & I_{z} - \frac{\rho}{2} L^{5} N'_{\dot{r}} \end{pmatrix} \cdot \begin{pmatrix} \dot{v} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} f_{Y}(u, v, r, \delta_{R}, u_{C}) \\ f_{N}(u, v, r, \delta_{R}, u_{C}) \end{pmatrix} ...$$
(A1.6)

Equation (A1.6) allows explicit expression of the acceleration:

$$\begin{pmatrix} \dot{v} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} m - \frac{\rho}{2} L^3 Y'_{\dot{v}} & m x_{cg} - \frac{\rho}{2} L^4 Y'_{\dot{r}} \\ m x_{cg} - \frac{\rho}{2} L^4 N'_{\dot{v}} & I_z - \frac{\rho}{2} L^5 N'_{\dot{r}} \end{pmatrix}^{-1} \begin{pmatrix} f_Y(u, v, r, \delta_R, u_C) \\ f_N(u, v, r, \delta_R, u_C) \end{pmatrix}$$
 (A1.7)

To simplify further consideration, the following notation is used:

$$\mathbf{B} = \begin{pmatrix} m - \frac{\rho}{2} L^3 Y'_{\dot{v}} & m x_{cg} - \frac{\rho}{2} L^4 Y'_{\dot{r}} \\ m x_{cg} - \frac{\rho}{2} L^4 N'_{\dot{v}} & I_z - \frac{\rho}{2} L^5 N'_{\dot{r}} \end{pmatrix} \dots$$
(A1.8)

Substitution of (A1.8) in (A1.7) with account of Equations (A3.22-A3.23) of Appendix 3 yields:

$$\begin{pmatrix} \dot{v} \\ \dot{r} \end{pmatrix} = \frac{1}{\det(\mathbf{B})} \begin{pmatrix} B_{22} & -B_{12} \\ -B_{21} & B_{11} \end{pmatrix} \begin{pmatrix} f_{Y}(u, v, r, \delta_{R}, u_{C}) \\ f_{N}(u, v, r, \delta_{R}, u_{C}) \end{pmatrix} =$$

$$= \frac{1}{\det(\mathbf{B})} \begin{pmatrix} B_{22} \cdot f_{Y}(u, v, r, \delta_{R}, u_{C}) - B_{12} \cdot f_{N}(u, v, r, \delta_{R}, u_{C}) \\ B_{11} \cdot f_{N}(u, v, r, \delta_{R}, u_{C}) - B_{21} \cdot f_{Y}(u, v, r, \delta_{R}, u_{C}) \end{pmatrix}$$
.....(A1.9)

which leads to the final Cauchy form of the system of equations:

$$\begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \frac{f_X(u, v, r, \delta_R, u_C)}{m - 0.5 \rho L^3 X'_u} \\ \frac{B_{22} \cdot f_Y(u, v, r, \delta_R, u_C) - B_{12} \cdot f_N(u, v, r, \delta_R, u_C)}{\det(\mathbf{B})} \\ \frac{B_{11} \cdot f_N(u, v, r, \delta_R, u_C) - B_{21} \cdot f_Y(u, v, r, \delta_R, u_C)}{\det(\mathbf{B})} \end{pmatrix}$$
(A1.10)

The system of differential equations (A1.10) is ready for numerical integration.

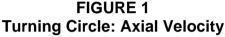
3.3 Simulation of Turning Circle

To simulate the turning circle maneuver, the rudder deflection, δ_R , was set to 35 degrees and commanded speed, u_c , was set constant and equal to approach speed of 15 knots, as defined in Subsection 1/4. The system of ordinary differential equations (A1.10) was numerically integrated with initial conditions corresponding to straight ahead sailing, $u = u_C$, v = 0, r = 0, time step for integration was four seconds and total duration of simulation was one hour. The results of integration are shown in Appendix 1, Figures 1, 2 and 3, respectively. To simulate the approach, the rudder was executed after 60 seconds of simulation.

In order to restore trajectory, velocities and yaw rate have to be integrated. In particular, yaw angle is recovered by the integration of yaw rate:

$$\psi(t) = \int_{0}^{t} r(t)dt \qquad (A1.11)$$

The restored yaw angle is shown in Appendix 1, Figure 4.



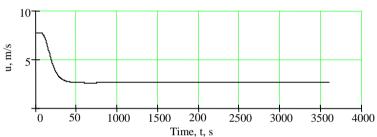


FIGURE 2
Turning Circle: Sway Velocity



Time t, s

FIGURE 3
Turning Circle: Yaw Rate

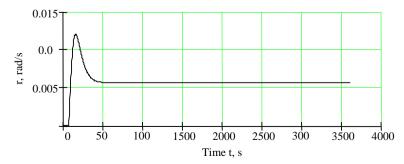
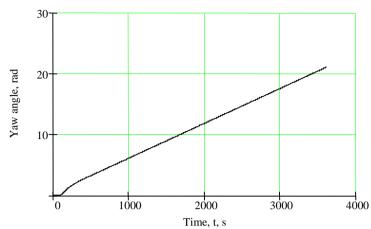


FIGURE 4
Turning Circle: Yaw Angle



Axial (surge) and sway velocities are defined in a ship-fixed system of coordinates. Derivatives of ship coordinates in a global coordinate system are obtained through the following coordinate transformation:

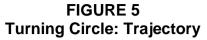
$$\dot{x}(t) = u(t)\cos(\psi(t)) - v(t)\sin(\psi(t))$$

$$\dot{y}(t) = u(t)\sin(\psi(t)) + v(t)\cos(\psi(t)) \qquad (A1.12)$$

Then, the coordinates as a function of time, which allow the plotting of the trajectory, are calculated as:

$$x(t) = \int_{0}^{t} \dot{x}(t)dt; \quad y(t) = \int_{0}^{t} \dot{y}(t)dt$$
 (A1.13)

The resulting trajectory is shown in Appendix 1, Figure 5. Parameters of the maneuver are summarized in Appendix 1, Table 5.



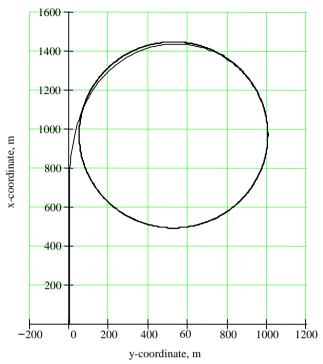


TABLE 5
Parameters of Turning Circle

Parameter	Value
Time to change course 90 degrees, seconds	173
Time to change course 180 degrees, seconds	409
Tactical diameter in ship lengths	2.79
Advance in ship lengths	2.77
Velocity of steady turn, knots	5.33
Diameter of steady turn, ship lengths	2.73

3.4 Simulation of 10/10 Zig-zag Test

To simulate the 10/10 Zig-zag test, the rudder was deflected 10 degrees and once the vessel achieved 10 degrees of course change, the rudder was deflected to the other side. The commanded speed was kept the same during the maneuver. The system of ordinary differential equations (A1.10) was numerically integrated with initial conditions corresponding to straight ahead sailing $u = u_C$, v = 0, r = 0, time step for integration was one second and total duration of the simulation was 15 minutes. To simulate the approach, the rudder was executed after 60 seconds of simulation. The results of integration are shown in Appendix 1, Figures 6, 7 and 8, respectively. The yaw angle was obtained by integration of the yaw rate, as defined by Equation (A1.11), and is shown in Appendix 1, Figure 9 along with time history of rudder deflation. Trajectory was restored with Equations (A1.12-A1.13) and is shown in Appendix 1, Figure 10. Parameters resulting from the simulation of the 10/10 Zig-zag maneuver are summarized in Appendix 1, Table 6.

The distance traveled along the path is calculated as:

$$S = \sum_{i=2}^{N} \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}$$
 (A1.14)

FIGURE 6
10/10 Zig-zag Maneuver: Axial Velocity

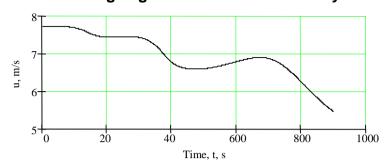
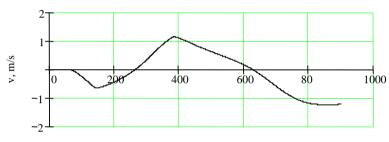


FIGURE 7
10/10 Zig-zag Maneuver: Sway Velocity



Time t, s

FIGURE 8 10/10 Zig-zag Maneuver: Yaw Rate

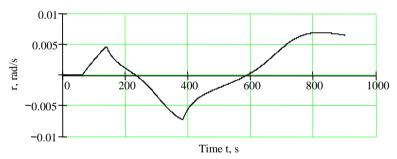


FIGURE 9
10/10 Zig-zag Maneuver: Yaw and Rudder Angles

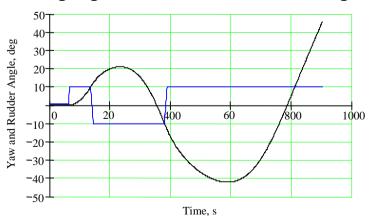


FIGURE 10 10/10 Zig-zag Maneuver: Trajectory

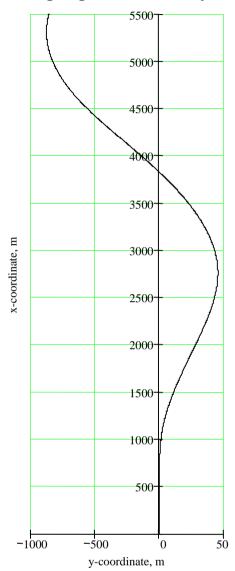


TABLE 6
Parameters of 10/10 Zig-zag Maneuver

Parameter	Value
Time of the first execute, seconds	74.1
Time of the second execute, seconds	318.7
First overshoot angle, degrees	11.1
Second overshoot angle, degrees	32.2
Distance traveled until the course reached 10 degrees, ship lengths	1.62

3.5 Simulation of the 20/20 Zig-zag Test

To simulate the 20/20 Zig-zag test, the rudder was deflected 20 degrees and once the vessel achieved 20 degrees of course change, the rudder was deflected to the other side. The commanded speed was kept the same during the maneuver. The system of ordinary differential equations (A1.10) was numerically integrated with initial conditions corresponding to straight ahead sailing $u = u_C$, v = 0, r = 0, the time step for integration was one second and the total duration of the simulation was 15 minutes. To simulate the approach, the rudder was executed after 60 seconds of simulation. The results of integration are shown in Appendix 1, Figures 11, 12 and 13, respectively. The yaw angle was obtained by integration of the yaw rate, as defined by Equation (A1.11), and is shown in Appendix 1, Figure 14 along with the time history of rudder deflation. The trajectory was restored with Equations (A1.12-A1.13) and is shown in Appendix 1, Figure 15. Parameters resulting from the simulation of the 20/20 Zig-zag maneuver are summarized in Appendix 1, Table 7.

FIGURE 11 20/20 Zig-zag Maneuver: Axial Velocity

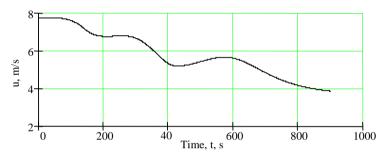


FIGURE 12 20/20 Zig-zag Maneuver: Sway Velocity

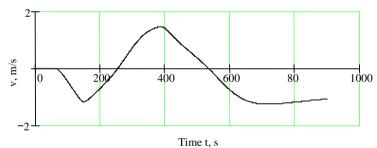


FIGURE 13 20/20 Zig-zag Maneuver: Yaw Rate

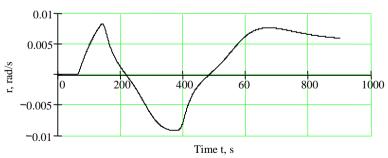


FIGURE 14 20/20 Zig-zag Maneuver: Yaw and Rudder Angles

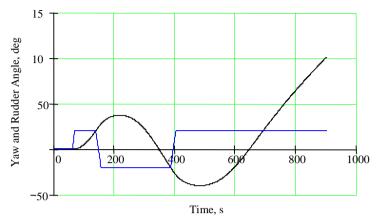


FIGURE 15 20/20 Zig-zag Maneuver: Trajectory

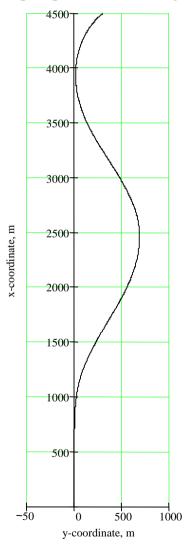


TABLE 7
Parameters of 20/20 Zig-zag Maneuver

Parameter	Value
Time of the first execute, seconds	77.3
Time of the second execute, seconds	325.1
First overshoot angle, degrees	17.38
Second overshoot angle, degrees	20.0
Distance traveled until the course reached 20 degrees, ship lengths	1.67

3.6 Simulation of the Pull-out Test

To simulate the pull-out test, the rudder was deflected 35 degrees and once the vessel achieved steady turning, the rudder was returned to the neutral position. The commanded speed was kept the same during the maneuver. The system of ordinary differential equations (A1.10) was numerically integrated with initial conditions corresponding to straight ahead sailing $u = u_C$, v = 0, r = 0, the time step for integration was two seconds and the total duration of the simulation was two hours. To simulate the approach, the rudder was executed after 60 seconds of simulation. The results of integration are shown in Appendix 1, Figures 16, 17 and 18, respectively. The yaw angle was obtained by integration of the yaw rate, as defined by Equation (A1.11), and is shown in Appendix 1, Figure 19 along with time history of rudder deflation. The trajectory was restored with Equations (A1.12-A1.13) and is shown in Appendix 1, Figure 20. Parameters resulting from the simulation of the pull-out maneuver are summarized in Appendix 1, Table 8.

FIGURE 16
Pull-out Maneuver: Axial Velocity

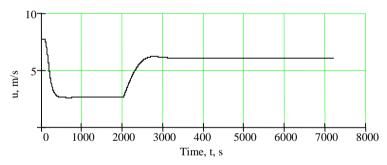


FIGURE 17
Pull-out Maneuver: Sway Velocity

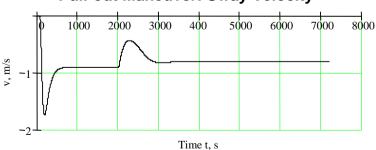


FIGURE 18
Pull-out Maneuver: Yaw Rate

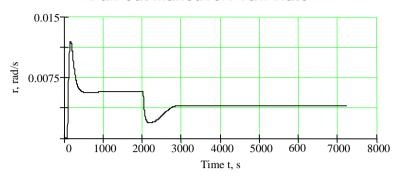


FIGURE 19
Pull-out Maneuver: Yaw and Rudder Angles

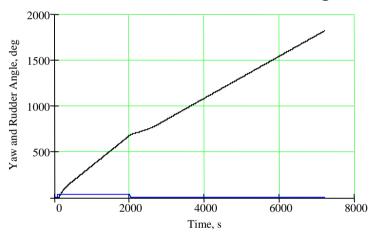


FIGURE 20 Pull-out Maneuver: Trajectory

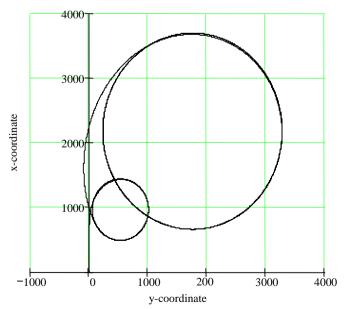


TABLE 8
Parameters of Pull-out Maneuver

Parameter	Value
Time of the first execute, seconds	60
Time of the second execute, seconds	2000
Residual yaw rate value, rad/s	0.0038
Straight-line-stability	Unstable

3.7 Simulation of Direct Spiral Maneuver

To simulate the direct spiral maneuver, the rudder was deflected 35 degrees and once the vessel achieved steady turning, the rudder deflection was decreased until the vessel changed direction of turn. The constant yaw rate has to be achieved each time before the rudder deflection is changed. The commanded speed was kept the same during the maneuver. The system of ordinary differential equations (A1.10) was numerically integrated with initial conditions corresponding to straight ahead sailing $u = u_C$, v = 0, r = 0, the time step for integration was two seconds and the total duration of the simulation was four hours in each side. The results of integration for starboard are shown in Appendix 1, Figures 21, 22 and 23, respectively. Similar results for the port side are shown in Appendix 1, Figures 24, 25 and 26.

The yaw angle was obtained by integration of the yaw rate, as defined by Equation (A1.11), and is shown in Appendix 1, Figure 27 for starboard and in Appendix 1, Figure 28 for port along with the time history of rudder deflation. The trajectory was restored with Equations (A1.12-A1.13) and is shown in Appendix 1, Figure 29 for starboard and Appendix 1, Figure 30 for port. The stability loop is shown in Appendix 1, Figure 31. Parameters resulting from the simulation of the direct spiral maneuver are summarized in Appendix 1, Table 9.

FIGURE 21
Direct Spiral Maneuver, Starboard: Axial Velocity

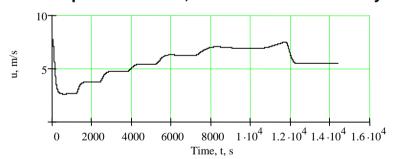


FIGURE 22
Direct Spiral Maneuver, Starboard: Sway Velocity

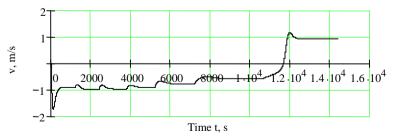


FIGURE 23
Direct Spiral Maneuver, Starboard: Yaw Rate

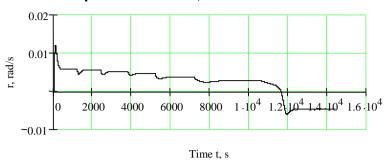


FIGURE 24
Direct Spiral Maneuver, Port: Axial Velocity

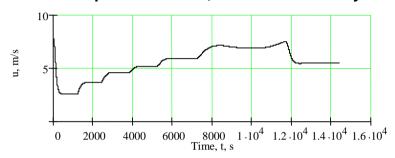


FIGURE 25
Direct Spiral Maneuver, Port: Sway Velocity

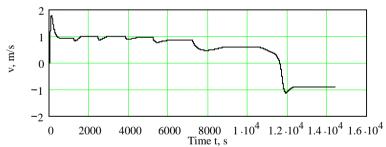


FIGURE 26
Direct Spiral Maneuver, Port: Yaw Rate

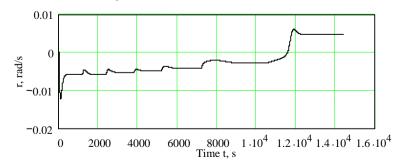


FIGURE 27
Direct Spiral Maneuver, Starboard: Yaw and Rudder Angles

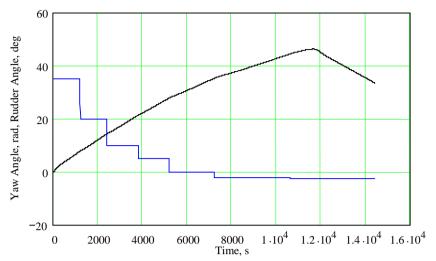


FIGURE 28
Direct Spiral Maneuver, Port: Yaw and Rudder Angles

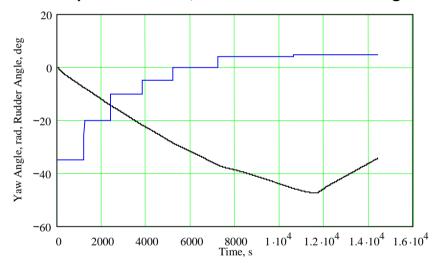


FIGURE 29
Direct Spiral Maneuver, Starboard: Trajectory

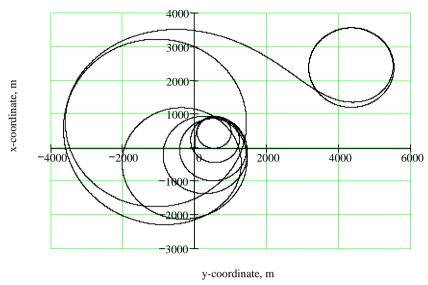


FIGURE 30 Direct Spiral Maneuver, Port: Trajectory

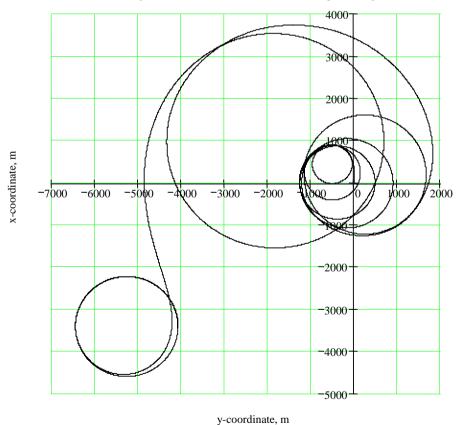
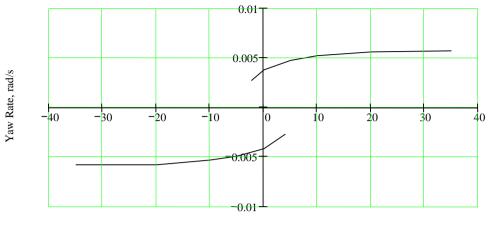


FIGURE 31
Relation between Rudder Angle and Yaw (Turn) Rate



Rudder Angle, deg

TABLE 9
Parameters of Direct Spiral Maneuver

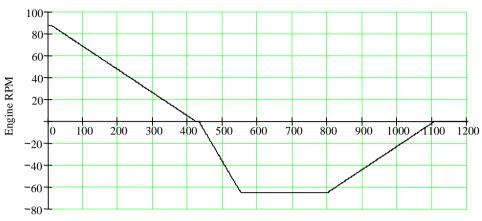
Parameter	Value			
	Sto	ırboard	Port	
	Time, s	Angle, deg	Time, s	Angle, deg
Execute 1	0	35	0	35
Execute 2	1200	20	1200	20
Execute 3	2400	10	2400	10
Execute 4	3800	5	3800	5
Execute 5	5200	0	5200	0
Execute 6	7200	-2.25	7200	-4.0
Execute 7	10600	-2.75	10600	-4.5
Height of instability loop, rad/s	0.00545			
Width of instability loop, deg			4.5	

3.8 Simulation of Stopping

To simulate the stopping maneuver, the rudder was kept neutral. The commanded speed was changed following an assumption of time necessary to reverse the engine, available in Appendix 1, Table 1. A time history of the engine's RPM is assumed based on these numbers. It is shown in Appendix 1, Figure 32. It is also assumed, once the vessel is dead in the water, that the engine is stopped. It is also assumed that 10 seconds is enough time to comprehend the "full astern" command and to start the reverse sequence. The commanded speed is calculated based of the engine's RPM, assuming the approach speed of 15 knots corresponding to nominal continuous rating as defined in Subsection 1/4. The resulting time history of commanded speed is shown in Appendix 1, Figure 33.

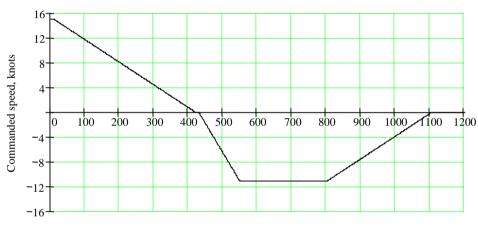
The system of ordinary differential equations (A1.10) was numerically integrated with initial conditions corresponding to straight ahead sailing $u=u_C$, v=0, r=0, the time step for integration was one second and the total duration of the simulation was 13 minutes 20 seconds. The results of integration are shown in Appendix 1, Figures 34, 35 and 36, respectively. The yaw angle was obtained by integration of the yaw rate, as defined by Equation (A1.11), and is shown in Appendix 1, Figure 37. The trajectory was restored with Equations (A1.12-A1.13) and is shown in Appendix 1, Figure 38. Track reach was calculated with Equation (A1.14). Parameters resulting from the simulation of the stopping maneuver are summarized in Appendix 1, Table 10.

FIGURE 32
Time History of Engine RPM during Stopping Maneuver



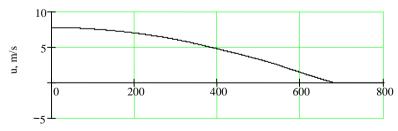
Time, s

FIGURE 33
Time History of Commanded Speed during Stopping Maneuver



Time, s

FIGURE 34
Stopping Maneuver: Axial Velocity



Time, t, s

FIGURE 35
Stopping Maneuver: Sway Velocity

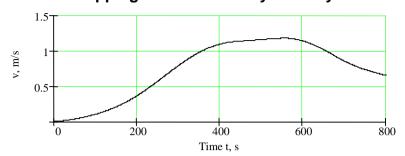


FIGURE 36 Stopping Maneuver: Yaw Rate

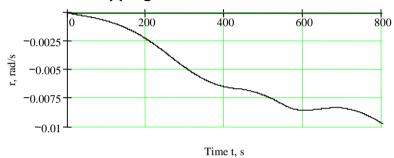
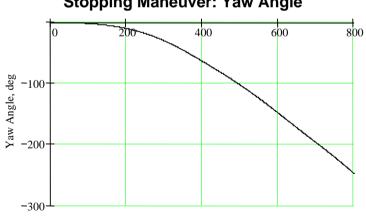


FIGURE 37
Stopping Maneuver: Yaw Angle



Time, s

FIGURE 38
Stopping Maneuver: Trajectory

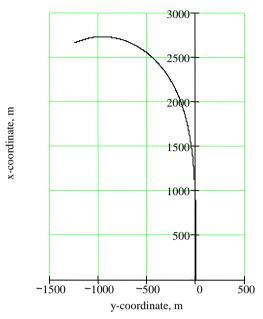


TABLE 10 Parameters of Stopping Maneuver

Parameter	Value
Time till vessel is dead in water, seconds	678
Head reach, ship lengths	7.42
Lateral deviation, ship lengths	3.77
Track reach, ship lengths	9.97

3.9 Evaluation of Maneuverability Based on Simulation

Evaluation of the maneuverability based on the simulation results, available from Appendix 1, Tables 5 through 10, is summarized in Appendix 1, Table 11.

TABLE 11 Evaluation of Maneuverability

Criteria	Simulated	Required	Result or Rating
Advance, ship lengths	2.4	4.5	Passed
Tactical diameter	2.79	$3.05 < Td \le 2.21$	Rtd = 3
10/10 Zig-zag 1st overshoot angle, deg	11.07	$9.36 < \alpha 10_1 \le 11.98$	$Rt\alpha_{10}=2$
10/10 Zig-zag 2 nd overshoot angle, deg	32.4	$\alpha 10_2 < f_{101}(L/V) = 40$	Passed
20/20 Zig-zag 1st overshoot angle, deg	17.32	$11.725 < \alpha 20_1 \le 18.725$	$Rt\alpha_{20}=3$
Resulting overshoot angle rating		$Rt\alpha = 2.5$	
Distance traveled until course change reaches 10 degrees during 10/10 Zig-zag maneuver, ship lengths	1.59	$\ell_{10} \le 1.633$	Rti = 5
Pull-out test	Vessel is straight-line unstable		
Width of instability loop, degrees	4.5	$\alpha_U < 12$	Passed
Track reach, ship lengths	9.92	TR < 15	Passed
Head reach, ship lengths	7.64	<i>HR</i> ≤ 8.7	Rts = 5
Resultant rating	3.875		
Expected optional class notation	MAN-A		



APPENDIX 2 Definitions and Nomenclature for Maneuvering Simulation

The following symbols are used for non-dimensional mass characteristics of the vessel:

$$m' = \frac{\rho m}{0.5 \rho L^3} = 2C_B \frac{B}{L} \frac{T}{L}$$
 non-dimensional vessel mass

$$I'_z = \frac{\rho mk^2}{0.5\rho L^5} = 2C_B \frac{B}{L} \frac{T}{L} \frac{k}{L}$$
 non-dimensional vessel inertia

$$x'_G = x_G/L$$
 non-dimensional distance of vessel's center of gravity forward of amidships

The following symbols are used for velocities and accelerations in relation to ship motions:

u = axial (surge) velocity

 \dot{u} = axial (surge) acceleration

v = swaying velocity

 \dot{v} = swaying acceleration

r = yaw rate or yaw angular velocity

 \dot{r} = yawing acceleration

The following symbols are used for non-dimensional velocities and accelerations in relation to ship motions:

v' = v/V non-dimensional sway velocity

r' = rL/V non-dimensional angular velocity – yaw rate

 $\dot{v}' = \dot{v} L/V^2$ non-dimensional sway acceleration

 $\dot{r}' = \dot{r} L^2/V^2$ non-dimensional angular acceleration – vaw

The following symbols are used for mass characteristics of the vessel:

m =mass of the vessel, in metric tons

 $x_{c\sigma}$ = distance of vessel's center of gravity forward of amidships

 I_z = mass moment of inertia of a vessel relative to vertical axis

The following symbols are used for hydrodynamic forces on the vessel's hull and rudder:

X = sum of all forces acting on the hull in ship-fixed abscissa axis or axial (surge) forces

Y =sum of all forces acting on the hull in ship-fixed ordinate axis or swaying forces

N = sum of all moments acting on the hull in horizontal plane or yawing moments

 X_{Rd} = force acting on the rudder in ship-fixed abscissa axis

 Y_{rd} = force acting on the rudder in ship-fixed ordinate axis

 N_{Rd} = moment acting on the rudder ship-fixed coordinate system

The following symbols are used for hydrodynamic derivatives and other coefficients of equation of motions (for more information on linear formulation, see A3/2.2 and A3/4 for nonlinear formulation). Two values are given for each of these figures: dimensional and non-dimensional

$$X_{\dot{u}} = \frac{\partial X}{\partial \dot{u}}$$

$$X'_{\dot{u}} = \frac{X_{\dot{u}}}{0.5 \rho L^3}$$

Axial (surge) hydrodynamic derivative by axial acceleration or axial force coefficient per axial acceleration (equals surge-surge added mass taken with the opposite sign and at zero frequency: $-A_{11}$)

$$X_{\dot{v}} = \frac{\partial X}{\partial \dot{v}}$$

$$X'_{\dot{v}} = \frac{X_{\dot{v}}}{0.5\rho L^3}$$

Axial (surge) hydrodynamic derivative by axial acceleration or axial force coefficient per sway acceleration (equals surge-sway added mass taken with the opposite sign and at zero frequency: $-A_{12}$; it equals zero due to symmetry of vessel hull)

$$X_{\dot{r}} = \frac{\partial X}{\partial \dot{v}}$$

$$X'_{\dot{r}} = \frac{X_{\dot{r}}}{0.5\rho L^4}$$

Axial (surge) hydrodynamic derivative by yaw acceleration component or axial force coefficient per yaw acceleration (equals surge-yaw added mass taken with the opposite sign and at zero frequency: $-A_{16}$; it equals zero due to symmetry of vessel hull)

$$X_u = \frac{\partial X}{\partial u}$$

$$X_u' = \frac{X_u}{0.5\rho L^2 V}$$

Axial (surge) hydrodynamic derivative by axial speed component – axial force coefficient per axial velocity (equals zero, as a resistance is proportional to the second power of speed)

$$X_{v} = \frac{\partial X}{\partial v}$$

$$X_{v}' = \frac{X_{v}}{0.5\rho L^{2}V}$$

Axial (surge) hydrodynamic derivative by sway velocity – axial (surge) force coefficient per sway velocity (equals zero due to symmetry of vessel hull)

$$X_{vv} = \frac{\partial^2 X}{\partial v^2}$$

$$X'_{vv} = \frac{X_{vv}}{0.5\rho L^2}$$

The second axial (surge) hydrodynamic derivative by sway velocity

$$X_{vr} = \frac{\partial^2 X}{\partial v \partial r}$$

$$X'_{vr} = \frac{X_{vr}}{0.5 \rho L^3}$$

The second axial (surge) hydrodynamic derivative by swaying velocity and yaw rate

 $X_{rr} = \frac{\partial^2 X}{\partial r^2}$

$$X'_{rr} = \frac{X_{rr}}{0.5\rho L^4}$$

The second axial (surge) hydrodynamic derivative by yaw rate

 $X_{\nu\nu\eta}$

$$X'_{vv\eta} = \frac{X_{vv\eta}}{0.5 \, \rho L^2}$$

First order coefficient used to present X_{vv} as a function of propulsion ratio η .

 X_{δ}

$$X_{\delta}' = \frac{X_{\delta}}{0.5\rho L^2 V^2}$$

First order coefficient used to present rudder drag as a function of rudder deflection angle δ_R (equals zero, as drag is proportional to the second power of speed)

 $X_{\delta\delta}$

$$X'_{\delta\delta} = \frac{X_{\delta\delta\eta}}{0.5\rho L^2 V^2}$$

Second order coefficient used to present rudder drag a function of rudder deflection angle δ_R (does not include any influence of a propeller, so propulsion ratio $\eta = 0$)

 $X_{\delta\delta\eta\eta}$

$$X'_{\delta\delta\eta\eta} = \frac{X_{\delta\delta\eta\eta}}{0.5\rho L^2 V^2}$$

Second order coefficient used to present the rudder drag coefficient $X_{\delta\delta}$ as a function of propulsion ratio η . Takes into account influence of a propeller on rudder.

 $Y_{ii} = \frac{\partial Y}{\partial \dot{u}}$

$$Y_{ii}' = \frac{Y_{ii}}{0.5\rho L^3}$$

Sway hydrodynamic derivative – sway force coefficient per axial (surge) acceleration (equals to sway-surge added mass taken with the opposite sign and at zero frequency: $-A_{21}$; it equals zero due to symmetry of vessel hull)

$Y_{\dot{v}} = \frac{\partial Y}{\partial \dot{v}}$	$Y_{\dot{v}}' = \frac{Y_{\dot{v}}}{0.5\rho L^3}$	Sway hydrodynamic derivative – sway force coefficient per sway acceleration (equals sway-sway added mass taken with the opposite sign and at zero frequency: $-A_{22}$)
$Y_{\dot{r}} = \frac{\partial Y}{\partial \dot{r}}$	$Y_{\dot{r}}' = \frac{Y_{\dot{r}}}{0.5\rho L^4}$	Sway hydrodynamic derivative – sway force coefficient per yaw acceleration (equals to sway-yaw added mass taken with the opposite sign and at zero frequency: $-A_{26}$)
$Y_u = \frac{\partial Y}{\partial u}$	$Y_u' = \frac{Y_u}{0.5\rho L^2 V}$	Sway hydrodynamic derivative – sway force coefficient per axial (surge) velocity (equals zero due to symmetry of vessel hull)
$Y_{v} = \frac{\partial Y}{\partial v}$	$Y_{v}' = \frac{Y_{v}}{0.5\rho L^{2}V}$	Sway hydrodynamic derivative – 1 st order sway force coefficient per sway velocity
$Y_r = \frac{\partial Y}{\partial r}$	$Y_r' = \frac{Y_r}{0.5\rho L^3 V}$	Sway hydrodynamic derivative -1^{st} order sway force coefficient per yaw rate
$Y_{ u u }$	$Y'_{\nu \nu } = \frac{Y_{\nu \nu }}{0.5\rho L^2}$	The second order coefficient of sway force, presenting Y as a function of product of sway velocity by absolute value of the sway velocity $v/v/$
$Y_{v \setminus r }$	$Y'_{\nu r } = \frac{Y_{\nu r }}{0.5\rho L^3}$	The second order coefficient of sway force, presenting Y as a function of product of sway velocity by absolute value of yaw rate $v/r/$
$Y_{ u\eta}$	$Y'_{\nu\eta} = \frac{Y_{\nu\eta}}{0.5\rho L^2 V}$	The first order coefficient used to present the sway hydrodynamic derivative Y_{ν} as a function of propulsion ratio minus unity $(\eta - 1)$
$Y_{r\eta}$	$Y'_{r\eta} = \frac{Y_{r\eta}}{0.5\rho L^3 V}$	The first order coefficient used to present the sway hydrodynamic derivative Y_r as a function of propulsion ratio minus unity $(\eta - 1)$
$Y_{ u u \eta}$	$Y'_{\nu \nu \eta} = \frac{Y_{\nu \nu \eta}}{0.5\rho L^2}$	The first order coefficient used to present the second order sway coefficient $Y_{\nu \nu }$ as a function of propulsion ratio minus unity $(\eta - 1)$
Y_*	$Y_*' = \frac{Y_*}{0.5\rho L^2 V^2}$	The lateral force caused by asymmetry of propeller flow at straight course – drift angle and rudder deflection angle are zero
$Y_{*_{m{\eta}}}$	$Y'_{*\eta} = \frac{Y_{*\eta}}{0.5\rho L^2 V^2}$	The first order coefficient to present the lateral propeller force Y_* as a function of propulsion ratio minus unity $(\eta - 1)$
Y_{δ}	$Y_{\delta}' = \frac{Y_{\delta}}{0.5\rho L^2 V^2}$	The first order coefficient for presenting of a lift force developed by rudder as a function of rudder deflection angle δ_R
$Y_{\delta r }$	$Y_{\delta r }' = \frac{Y_{\delta r }}{0.5\rho L^3 V}$	The second order coefficient of rudder lift force, presenting it as a function of product of sway velocity by absolute value of yaw rate $v/r/$.
$Y_{\delta\eta}$	$Y'_{\delta\eta} = \frac{Y_{\delta\eta}}{0.5\rho L^2 V^2}$	The first order coefficient to present the coefficient of the rudder lifting force as a function of propulsion ration minus unity $(\eta - 1)$
$N_{\dot{u}} = \frac{\partial N}{\partial \dot{u}}$	$N_{\dot{u}} = \frac{N_{\dot{u}}}{0.5\rho L^4}$	Yaw hydrodynamic derivative – yaw force coefficient per axial (surge) acceleration (equals yaw-surge added mass taken with the opposite sign and at zero frequency: $-A_{61}$; it

equals zero due to symmetry of vessel hull)

lix 2	Definitions and Nomenclature for Maneuvering Simulation		
$N_{\dot{v}} =$	$\frac{\partial N}{\partial \dot{v}}$	$N_{\dot{v}}' = \frac{N_{v}}{0.5\rho L^4}$	Yaw hydrodynamic derivative – yaw force coefficient per sway acceleration (equals yaw-sway added mass taken with the opposite sign and at zero frequency: $-A_{62}$)
$N_{\dot{r}} =$	$\frac{\partial N}{\partial \dot{r}}$	$N_{\dot{r}}' = \frac{N_{\dot{r}}}{0.5\rho L^5}$	Yaw hydrodynamic derivative – yaw force coefficient per yaw acceleration (equals to yaw-yaw added mass taken with the opposite sign and at zero frequency: $-A_{66}$)
$N_u =$	$\frac{\partial N}{\partial u}$	$N_u' = \frac{N_u}{0.5\rho L^2 V}$	Yaw hydrodynamic derivative – yaw moment coefficient per axial (surge) velocity (equals zero due to symmetry of vessel hull)
$N_v =$	$\frac{\partial N}{\partial v}$	$N_v' = \frac{N_v}{0.5\rho L^3 V}$	Yaw hydrodynamic derivative – 1 st order yaw moment coefficient per sway velocity
$N_r =$	$\frac{\partial N}{\partial r}$	$N_r' = \frac{N_r}{0.5\rho L^4 V}$	Yaw hydrodynamic derivative – 1 st order yaw moment coefficient per yaw rate
$N_{v v }$		$N'_{\nu \nu } = \frac{N_{\nu \nu }}{0.5\rho L^3}$	The second order coefficient of sway force, presenting N as a function of product of sway velocity by absolute value of the sway velocity $v/v/v$
$N_{r r }$		$N'_{r r } = \frac{N_{r r }}{0.5\rho L^5}$	The second order coefficient of sway force, presenting N as a function of product of yaw rate by absolute value of yaw rate $r/r/$
$N_{v r }$		$N'_{v r } = \frac{N_{v r }}{0.5\rho L^4}$	The second order coefficient of sway force, presenting N as a function of product of sway velocity by absolute value of yaw rate $v/r/$
$N_{v\eta}$		$N'_{\nu\eta} = \frac{N_{\nu\eta}}{0.5\rho L^3 V}$	The first order coefficient used to present the yaw hydrodynamic derivative N_v as a function of propulsion ratio minus unity $(\eta - 1)$
$N_{r\eta}$		$N'_{r\eta} = \frac{N_{r\eta}}{0.5\rho L^4 V}$	The first order coefficient used to present the yaw hydrodynamic derivative N_r as a function of propulsion ratio minus unity $(\eta - 1)$
$N_{v v \eta}$		$N'_{\nu \nu \eta} = \frac{N_{\nu \nu \eta}}{0.5\rho L^2}$	The first order coefficient used to present the second order yaw coefficient $N_{\nu/\nu/}$ as a function of propulsion ratio minus unity $(\eta-1)$
N_*		$N_*' = \frac{N_*}{0.5 \rho L^3 V^2}$	The moment of lateral force caused by asymmetry of propeller flow at straight course – drift angle and rudder

metry of e and rudder deflection angle are zeros

The first order coefficient to present the moment of lateral propeller force N_* as a function of propulsion ratio minus unity $(\eta - 1)$

The first order coefficient for presenting of a moment of lift force developed by rudder as a function of rudder deflection angle δ_R

The second order coefficient of a moment of rudder lift force, presenting it as a function of absolute value of yaw rate |r|.

The first order coefficient to present the coefficient of the moment of rudder lifting force as a function of propulsion ration minus unity $(\eta - 1)$

 $N_{\delta}' = \frac{N_{\delta}}{0.5\rho L^3 V^2}$

 $N_{\delta|r|} = \frac{N_{\delta|r|}}{0.5\rho L^4 V}$

 $N'_{\delta\eta} = \frac{N_{\delta\eta}}{0.5\rho L^3 V^2}$

 $N_{*_{\eta}}$

 N_{δ}

 $N_{\delta \mid r \mid}$

 $N_{\delta\eta}$

The following symbols are used for other non-dimensional values:

$$t' = \frac{Vt}{L}$$
 non-dimensional time

$$\eta = \frac{V}{u}$$
 propulsion ratio



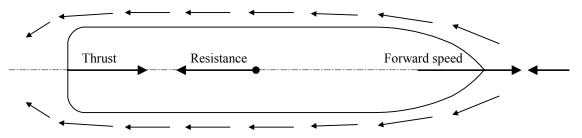
APPENDIX 3 Theoretical Background

1 Forces Acting on Vessel in Horizontal Plane

1.1 Straight Forward Sailing with Constant Speed

When a vessel is moving straight ahead with constant speed in calm water, all the forces acting on it are in perfect balance and thrust is counter-balanced by resistance. See Appendix 3, Figure 1. Thrust is generated by a propulsive device, most common of which are propellers. Resistance is due to the combined action of ship wave, viscous friction and eddy-making resistance.

FIGURE 1
Straight Ahead Sailing with Constant Forward Speed

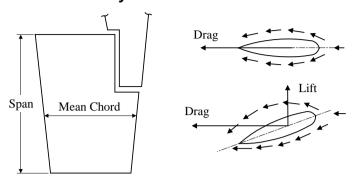


Straight sailing for a long period of time without helmsman or autopilot intervention is not always possible for all vessels. The capacity of the vessel to stay in straight ahead motion without intervention depends on the directional stability qualities of the hull. Physics of directional stability are reviewed in Subsection A3/3 and are addressed in Subsection 2/5 and 4/3.2 and 4/3.5.

1.2 Rudder Forces

A rudder is a lifting surface with a rudder force consisting of two components: drag and lift. Drag is directed along the chord of the rudder while lift is perpendicular to it. Drag on the rudder is similar to the drag on the entire hull with the exception of the wave resistance, as the rudder is usually completely submerged, so only viscous friction and eddy-making resistance remain. Lift force is directed perpendicular to the longitudinal axis. Lift force is caused by the difference in hydrodynamic pressures, which result from the different fluid velocities on the two sides of the rudder when it is deflected. Therefore, lift is only present on the deflected rudder. See Appendix 3, Figure 2 (for definition of rudder geometry, see Section 1, Figure 2).

FIGURE 2
Geometry and Forces on Rudder



1.3 Hydrodynamic Forces on Hull

Once the rudder is deflected, a moment is created by the rudder lift force. See Appendix 3, Figure 3. This moment deviates the vessel from its original course. Once there is an angle between the course and the centerline of the vessel, lift is developed on the hull. See Appendix 3, Figure 4.

The force on the rudder is usually small in comparison to forces on the hull, so the rudder is only the initiator of a turn, while the hull lift force actually makes the vessel turn. The hull works as a lifting surface with a very small ratio between chord and span. See Appendix 3, Figure 5.

Bank(s) and bottom may have a significant effect on hydrodynamic forces in shallow and/or restricted waters.

FIGURE 3
On the Moment of Rudder Lift Force

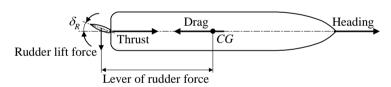


FIGURE 4
Hydrodynamic Forces on the Hull

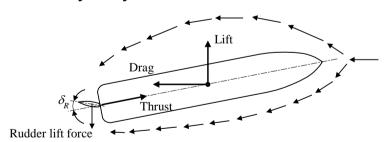
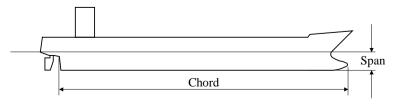


FIGURE 5
Vessel Hull as Lifting Surface



1.4 Other Forces Acting on Vessel During a Maneuver

Speed is a vector characterized not only by its magnitude but by its direction as well. Even if speed does not change its magnitude during a maneuver, it changes in direction. As a result, a vessel is subjected to acceleration and inertial reaction forces.

Water and air in open sea are rarely completely calm. There are environmental forces caused by wind, waves and current.

Forces acting on a vessel during maneuvering may include external forces from tugs and thrusters.

2 Equations of Motions in Horizontal Plane

2.1 Accepted System of Coordinates and General Equations of Motions

There are two coordinate systems accepted in this Guide: one is fixed to the Earth $(X_0O_0Y_0)$ and another is attached to the vessel (XOY). Both systems of coordinates are shown in Appendix 3, Figure 6 [IMO, 2002b].

Despite the fact that a vessel, like any other body, has six degrees of freedom, consideration of only three of them is generally enough for solving most maneuverability problems. All the motions are assumed to be in the horizontal plane, so that only surge, sway and yaw are further considered:

$$\begin{split} m \Big[\dot{u} - vr - x_{cg} r^2 \Big] &= X + X_{Rd} \\ m \Big[\dot{v} + ur + x_{cg} \dot{r} \Big] &= Y + Y_{Rd} \\ I_z \dot{r} + m x_{cg} \Big[\dot{v} + ur \Big] &= N + N_{Rd} \end{split} \tag{A3.1}$$

where (dots above the symbol denote derivatives)

 $X = \sup$ sum of all forces acting on the hull in ship-fixed abscissa axis or surge or axial forces

Y = sum of all forces acting on the hull in ship-fixed ordinate axis or sway forces

 $N = \sup$ sum of all moments acting on the hull in horizontal plane or vaw moments

 X_{Rd} , Y_{Rd} and N_{Rd} are corresponding rudder forces and moment.

u = surge or axial component of instantaneous speed

 \dot{u} = surge or axial acceleration

v = sway velocity

 \dot{v} = sway acceleration

r = yaw rate or yaw angular velocity

 \dot{r} = yaw acceleration

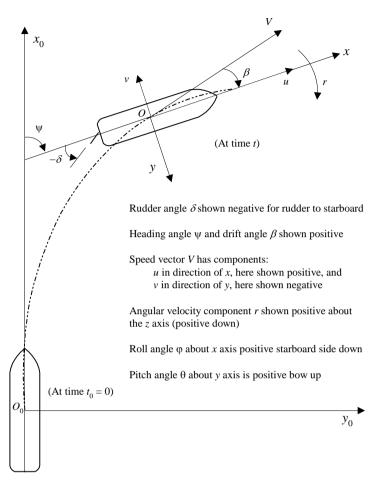
m = vessel mass

 I_z = mass moment of inertia of a vessel relative to vertical axis

 X_{cg} = abscissa of the center of gravity

These equations are obtained by application of Newton's second law in a ship-fixed coordinate system. The symbols X, Y and N on the right-hand side are the sum of all forces (or moments) acting in the corresponding direction, while the left-hand part of the equations express the inertia, being essentially mass (or moment of inertia in case of yaw) times acceleration terms.

FIGURE 6
Earth and Ship-fixed Coordinate Systems (IMO, 2002b)



Surface ship with body axes O(xyz) maneuvering within space-fixed inertial frame with axes $O_0(x_0y_0z_0)$

2.2 Expressions for the Hull Forces in Linear Formulation

Generation of hydrodynamic lift is a major physical phenomenon governing vessel maneuvering. As the hydrodynamic lift forces depend on velocity and accelerations, they are presented in the following form:

$$X = F_{x}(u, v, \dot{u}, \dot{v}, r, \dot{r})$$

$$Y = F_{y}(u, v, \dot{u}, \dot{v}, r, \dot{r})$$

$$N = F_{y}(u, v, \dot{u}, \dot{v}, r, \dot{r})$$
(A3.2)

where

 F_x , F_y = components of the hydrodynamic force

 F_{yy} = hydrodynamic moment in the horizontal plane

Analytical representations for these functions are not known. Approximations using Taylor series are used instead.

2.3 Taylor Series Expansion of Hull Forces

Expansion into Taylor series is a technique that uses partial derivatives to present the value of a function of multiple variables. Here is an example of a generic function F of three variables x, y and z, presented up to the second order of approximation:

$$F(x_{0}, y_{0}, z_{0}) + \frac{\partial F}{\partial x}(x - x_{0}) + \frac{\partial F}{\partial y}(y - y_{0}) + \frac{\partial F}{\partial z}(z - z_{0}) + \frac{1}{2} \frac{\partial^{2} F}{\partial x^{2}}(x - x_{0})^{2} + \frac{1}{2} \frac{\partial^{2} F}{\partial y^{2}}(y - y_{0})^{2} + \frac{1}{2} \frac{\partial^{2} F}{\partial z^{2}}(z - z_{0})^{2} + \frac{1}{2} \frac{\partial^{2} F}{\partial x \partial y}(x - x_{0})(y - y_{0}) + \frac{1}{2} \frac{\partial^{2} F}{\partial x \partial z}(x - x_{0})(z - z_{0}) + \frac{1}{2} \frac{\partial^{2} F}{\partial y \partial z}(y - y_{0})(z - z_{0}) + \dots$$

$$(A3.3)$$

The first four terms of Equation (A3.3) represent linear approximation because the variables are just multiplied by corresponding coefficients. The initial point (defined above with coordinates x_0 , y_0 , z_0) corresponds to the initial moment at Appendix 3, Figure 6, so the velocities and accelerations caused by the maneuver are all equal or close to zero.

$$u_0 = V, v_0 = 0, \ \dot{u}_0 = 0, \ \dot{v}_0 = 0, \ \dot{r}_0 = 0, \ \dot{r}_0 = 0 \dots$$
 (A3.4)

Respectively, all the hydrodynamic forces on the hull caused by the maneuver are also equal to zero at the initial moment:

$$X(u_0, v_0, \dot{u}_0, \dot{v}_0, r_0, \dot{r}_0) = 0$$

$$Y(u_0, v_0, \dot{u}_0, \dot{v}_0, r_0, \dot{r}_0) = 0$$

$$N(u_0, v_0, \dot{u}_0, \dot{v}_0, r_0, \dot{r}_0) = 0.$$
(A3.5)

As a result, the following formulae could be written for hydrodynamic maneuvering forces on the hull:

$$X = \frac{\partial X}{\partial u}(u - V) + \frac{\partial X}{\partial v}v + \frac{\partial X}{\partial \dot{u}}\dot{u} + \frac{\partial X}{\partial \dot{v}}\dot{v} + \frac{\partial X}{\partial r}r + \frac{\partial X}{\partial \dot{r}}\dot{r}$$

$$Y = \frac{\partial Y}{\partial u}(u - V) + \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial \dot{u}}\dot{u} + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{r}}\dot{r}$$

$$N = \frac{\partial N}{\partial u}(u - V) + \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial \dot{u}}\dot{u} + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{r}}\dot{r} \qquad (A3.6)$$

The formulae are simplified due to the following circumstances:

- Most vessels are symmetric about the centerline and change of surging force does not change the swaying force, $\frac{\partial Y}{\partial u} = \frac{\partial Y}{\partial \dot{u}} = 0$. The same could be stated about the moment, $\frac{\partial N}{\partial u} = \frac{\partial N}{\partial \dot{u}} = 0$.
- Similarly to the above, the change of swaying velocity and yaw rate do not affect the surging force, $\frac{\partial X}{\partial y} = \frac{\partial X}{\partial \dot{y}} = 0$ and $\frac{\partial X}{\partial r} = \frac{\partial X}{\partial \dot{r}} = 0$.

Final linear formulae for the hull forces are:

$$X = \frac{\partial X}{\partial u}(u - V) + \frac{\partial X}{\partial v}v$$

$$Y = \frac{\partial Y}{\partial v}v + \frac{\partial Y}{\partial \dot{v}}\dot{v} + \frac{\partial Y}{\partial r}r + \frac{\partial Y}{\partial \dot{r}}\dot{r}$$

$$N = \frac{\partial N}{\partial v}v + \frac{\partial N}{\partial \dot{v}}\dot{v} + \frac{\partial N}{\partial r}r + \frac{\partial N}{\partial \dot{r}}\dot{r} \qquad (A3.7)$$

2.4 Concept of Hydrodynamic Derivatives

Equations (A3.7) present the forces and moment as a linear combination (multiplication by a constant and summation) of velocities and accelerations. Coefficients in these formulae customarily are called "Hydrodynamic Derivatives" and defined as follows:

$$X_u = \frac{\partial X}{\partial u}$$
; $X_v = \frac{\partial X}{\partial v}$ surge hydrodynamic derivatives, dimension: [kN-s/m]

$$Y_{\nu} = \frac{\partial Y}{\partial \nu}$$
; $Y_{\dot{\nu}} = \frac{\partial Y}{\partial \dot{\nu}}$ sway hydrodynamic derivative by transversal component of velocity and accelerations, dimensions: [kN-s/m] and [kN-s²/m], respectively

$$Y_r = \frac{\partial Y}{\partial r}$$
; $Y_{\dot{r}} = \frac{\partial Y}{\partial \dot{r}}$ sway hydrodynamic derivative by yaw rate and yaw acceleration, dimensions: [kN-s/rad] and [kN-s²/rad], respectively

$$N_{\nu} = \frac{\partial N}{\partial \nu}$$
; $N_{\dot{\nu}} = \frac{\partial N}{\partial \dot{\nu}}$ yaw hydrodynamic derivative by transversal component of velocity and accelerations, dimensions: [kN-s] and [kN-s²], respectively.

$$N_r = \frac{\partial N}{\partial r}$$
; $N_{\dot{r}} = \frac{\partial N}{\partial \dot{r}}$ yaw hydrodynamic derivative by yaw rate and yaw acceleration, dimensions: [kN-m-s/rad] and [kN-m-s²/rad]

It is more convenient, however, to work with non-dimensional forces. Therefore, the following non-dimensional forms are defined for the hydrodynamic derivatives:

$$X'_{u} = \frac{X_{u}}{0.5\rho L^{2}V}; \quad X'_{v} = \frac{X_{v}}{0.5\rho L^{2}V}$$
non-dimensional surge hydrodynamic derivatives
$$Y'_{v} = \frac{Y_{v}}{0.5\rho L^{2}V}; \quad Y'_{\dot{v}} = \frac{Y_{v}}{0.5\rho L^{3}V}; \quad Y'_{r} = \frac{Y_{r}}{0.5\rho L^{3}V}; \quad Y'_{\dot{r}} = \frac{Y_{\dot{r}}}{0.5\rho L^{4}}$$
non-dimensional sway hydrodynamic derivatives

$$N'_{\nu} = \frac{N_{\nu}}{0.5 \rho L^3 V}; \ N'_{\dot{\nu}} = \frac{N_{\nu}}{0.5 \rho L^4}; \ N'_{r} = \frac{N_{r}}{0.5 \rho L^4 V}; \ N'_{\dot{r}} = \frac{N_{\dot{r}}}{0.5 \rho L^5}$$
 non-dimensional yaw moment hydrodynamic derivatives

where

L = length between perpendiculars, in m

 ρ = density of water, in kg/m³

V = straight-ahead speed before the maneuver, in m/s

These non-dimensional derivatives are to be used with non-dimensional variables:

$$u' = \frac{u}{V}; \quad v' = \frac{u}{V}; \quad \dot{u}' = \frac{\dot{u}L}{V^2}; \quad \dot{v}' = \frac{\dot{v}L}{V^2}; \quad r' = \frac{rL}{V}; \quad \dot{r}' = \frac{\dot{r}L^2}{V^2}; \quad V' = 1 \dots (A3.8)$$

Non-dimensional linear expressions for the hydrodynamic forces X' and Y' and the moment N' are:

$$X' = X'_{u}(u'-1) + X'_{v}v'\dot{r}'$$

$$Y' = Y'_{v}v + Y'_{\dot{v}}\dot{v} + Y'_{r}r' + Y'_{\dot{r}}\dot{r}'$$

$$N' = N'_{v}v + N'_{v}\dot{v}' + N'_{r}r' + N'_{\dot{r}}\dot{r}' \qquad (A3.9)$$

2.5 Expressions for Rudder Forces

As linear expressions were used for hydrodynamic forces on the hull, similar linear expressions are used for rudder forces:

$$X_{Rd} = 0$$

$$Y_{Rd} = Y_{\delta} \delta_{R}$$

$$N_{Rd} = N_{\delta} \delta_{R}$$
.....(A3.10)

The X-component of rudder force is small relative to surge (axial) hull forces and may be neglected. Y_{δ} and N_{δ} are to be defined in the same way as the hull hydrodynamic derivative, but the expansion into the Taylor series is done with respect to the rudder deflection angle.

The remaining components of the rudder force and its moment are customarily presented in non-dimensional form:

$$Y'_{\delta} = \frac{Y_{\delta}}{0.5\rho L^2 V^2}; \quad N'_{\delta} = \frac{N_{\delta}}{0.5\rho L^3 V^2}$$
 (A3.11)

The non-dimensional expressions for the rudder-induced forces are:

$$X'_{Rd} = 0$$

$$Y'_{Rd} = Y'_{\delta} \delta_{R}$$

$$N'_{Rd} = N'_{\delta} \delta_{R}...$$
(A3.12)

2.6 Linear Equations of Motion

As the hull and rudder forces formulations are based on linear approximation, the same assumption is to be used for the inertial forces in the equations of motion (A3.1). The axial (surge) component of velocity is presented as:

$$u = V + u_t$$
 (A3.13)

where

 u_t = difference between the axial component of instantaneous velocity and the speed before the turn

Substitution of the above formula into Equation (A3.1) produces a nonlinear term $u_t r$ that has to be excluded from linear equations:

$$m\dot{u} = X + X_{Rd}$$

$$m[\dot{v} + Vr + x_{cg}\dot{r}] = Y + Y_{Rd}$$

$$I_z\dot{r} + mx_{cg}(Vr + \dot{v}) = N + N_{Rd} \qquad (A3.14)$$

The non-dimensional form of these equations is as follows:

$$m'\dot{u}' = X' + X'_{Rd}$$

$$m'[\dot{v}' + V'r' + x'_{cg}\dot{r}'] = Y' + Y'_{Rd}$$

$$I'_{z}\dot{r}' + m'x'_{cg}(V'r' + \dot{v}') = N' + N'_{Rd}$$
(A3.15)

where:

$$m' = \frac{m}{0.5\rho L^3}$$
 non-dimensional mass
$$I'_z = \frac{I_z}{0.5\rho L^5}$$
 non-dimensional moment of inertia

$$x'_{cg} = \frac{x_{cg}}{L}$$
 non-dimensional longitudinal position of center of gravity

The non-dimensional form of the equations of motion in linear formulation is:

$$m'\dot{u}' = X'_{u}u' + X'_{v}v'$$

$$m'[\dot{v}' + V'r' + x'_{cg}\dot{r}'] = Y'_{v}v' + Y'_{\dot{v}}\dot{v}' + Y'_{r}r' + Y'_{\dot{r}}\dot{r}' + Y'_{\delta}\delta_{R}$$

$$I'_{\dot{z}}\dot{r}' + m'x'_{cg}(V'r' + \dot{v}') = N'_{v}v' + N'_{\dot{v}}\dot{v}' + N'_{r}r' + N'_{\dot{r}}\dot{r}' + N'_{\delta}\delta_{R} \qquad (A3.16)$$

In these equations, the angle of rudder deflection represents control action. It is customary to gather all terms expressing the control action on the right side of the equations while placing all the terms describing reactions of the controlled object on the left:

$$m'\dot{u}' - X'_{u}u' - X'_{v}v' = 0$$

$$(m' - Y'_{v})\dot{v}' - Y'_{v}v' - (Y'_{r} - m'x'_{cg})\dot{r}' - (Y'_{r} - mV')r' = Y'_{\delta}\delta_{R}$$

$$(I'_{z} - N'_{\dot{r}})\dot{r}' - (N'_{r} - V'm'x'_{cg})r' - (N'_{\dot{v}} - m'x'_{cg})\dot{v}' - N'_{v}v' = N'_{\delta}\delta_{R} \qquad (A3.17)$$

The above linear equations are useful to analyze the vessel's straight-line stability, approximately assess the steady turning of a straight-line stable vessel and to evaluate the initial turning ability of a vessel. The most common use of these equations is to evaluate the straight-line stability, which for a straight-line stable vessel is usually balanced with the steady turning ability. The straight-line stability will be discussed in A3/3.3, but the steady turning ability and the initial turning ability as predicted by the linear equations will be discussed below and in A3/3.1. The system of equations (A3.17) represents the final form of the vessel maneuverability equations within the framework of linear approximations.

2.7 Equations of Motions in Standard (Cauchy) Form

There are three unknowns in the equations of motion Equation (A3.17): surge velocity, sway velocity and yaw rate. These are functions as they change with time. Surge, sway and yaw accelerations are also part of the equations. As these are derivatives of the unknowns, the equations of motion (A3.17) are differential equations.

Among the equations (A3.17), sway and yaw are coupled and are to be solved together, while the surge equation could be solved separately once the swaying velocity is known. As a result, sway and yaw are to be considered first:

$$\begin{cases} (m'-Y'_{\dot{v}})\dot{v}'-Y'_{v}v'-(Y'_{\dot{r}}-m'x'_{cg})\dot{r}'-(Y'_{r}-mV')r'=Y'_{\delta}\delta_{R}\\ (I'_{z}-N'_{\dot{r}})\dot{r}'-(N'_{r}-V'm'x'_{cg})r'-(N'_{\dot{v}}-m'x'_{cg})\dot{v}'-N'_{v}v'=N'_{\delta}\delta_{R} \end{cases}(A3.18)$$

The solution of the differential equation is a function that where substituted into the equation converts it into true equality. The solution of the system of two differential equations is a system of two related functions, respectively. It is convenient to present them in the form of a vector using the non-dimensional sway velocity as the first coordinate of the vector and the non-dimensional yaw rate as the second coordinate. Then, sway acceleration and yaw angular acceleration comprise the vector of derivatives. Finally, rudder forces and moment are also presented in vector form:

$$\vec{X} = \begin{pmatrix} v' \\ r' \end{pmatrix}; \quad \dot{\vec{X}} = \begin{pmatrix} \dot{v}' \\ \dot{r}' \end{pmatrix}; \quad \vec{F} = \begin{pmatrix} Y_{\delta}' \delta_R \\ N_{\delta}' \delta_R \end{pmatrix} \dots (A3.19)$$

In order to apply standard methods of solution to (A3.19), it has to be presented in matrix-vector form with an explicit expression of derivative vector on the left-hand side. First, the system (A4.6) could be presented as an equality of two vectors:

$$\begin{pmatrix} (m'-Y'_{\dot{v}})\dot{v}'-Y'_{\dot{v}}v'-(Y'_{\dot{r}}-m'x'_{cg})\dot{r}'-(Y'_{r}-mV')r'\\ (I'_{z}-N'_{\dot{r}})\dot{r}'-(N'_{r}-V'm'x'_{cg})r'-(N'_{\dot{v}}-m'x'_{cg})\dot{v}'-N'_{\dot{v}}v' \end{pmatrix} = \begin{pmatrix} Y'_{\delta}\delta_{R}\\ N'_{\delta}\delta_{R} \end{pmatrix} \dots (A3.20)$$

The left-hand side further is presented as a sum of two vectors in which the first contains only sway and yaw accelerations, while the second contains sway and yaw velocities:

$$\begin{pmatrix} (m' - Y'_{\dot{v}})\dot{v}' - (Y'_{\dot{r}} - m'x'_{cg})\dot{r}' \\ (I'_{z} - N'_{\dot{r}})\dot{r}' - (N'_{\dot{v}} - m'x'_{cg})\dot{v}' \end{pmatrix} + \begin{pmatrix} -Y'_{v}v' - (Y'_{r} - m'V')r' \\ -(N'_{r} - V'm'x'_{cg})r' - N'_{v}v' \end{pmatrix} = \begin{pmatrix} Y'_{\delta}\delta_{R} \\ N'_{\delta}\delta_{R} \end{pmatrix} \dots (A3.21)$$

Each of these vectors also can be presented as the result of multiplication of a matrix by the velocity or acceleration vector, respectively:

$$\begin{pmatrix} (m'-Y'_{\dot{v}}) & -(Y'_{\dot{r}}-m'x'_{cg}) \\ -(N'_{\dot{v}}-m'x'_{cg}) & (I'_{z}-N'_{\dot{r}}) \end{pmatrix} \begin{pmatrix} \dot{v}' \\ \dot{r}' \end{pmatrix} + \begin{pmatrix} -Y'_{v} & -(Y'_{r}-m'V') \\ -N'_{v} & -(N'_{r}-V'm'x'_{cg}) \end{pmatrix} \begin{pmatrix} v' \\ r' \end{pmatrix} = \begin{pmatrix} Y'_{\delta}\delta_{R} \\ N'_{\delta}\delta_{R} \end{pmatrix} \dots (A3.22)$$

To simplify further derivation, the following vector notations are used:

$$\mathbf{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} = \begin{pmatrix} (m' - Y'_{\dot{v}}) & -(Y'_{\dot{r}} - m'x'_{cg}) \\ -(N'_{\dot{v}} - m'x'_{cg}) & (I'_{z} - N'_{\dot{r}}) \end{pmatrix} \dots$$
(A3.23)

$$\mathbf{C} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} -Y_{\nu}' & -(Y_{r}' - m'V') \\ -N_{\nu}' & -(N_{r}' - V'm'x_{cg}') \end{pmatrix}$$
(A3.24)

The system of equations now is re-written as:

$$\begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \begin{pmatrix} \dot{v}' \\ \dot{r}' \end{pmatrix} + \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} v' \\ r' \end{pmatrix} = \begin{pmatrix} Y'_{\delta} \delta_R \\ N'_{\delta} \delta_R \end{pmatrix} ...$$
(A3.25)

Alternatively, a more compact format can be used:

$$\mathbf{B} \cdot \dot{\vec{X}} + \mathbf{C} \cdot \vec{X} = \vec{F} \tag{A3.26}$$

In order to use standard methods for solution of a system of differential equations, the acceleration vector must be expressed explicitly, so all other terms must be transferred to the right-hand side of the equation:

$$\mathbf{B} \cdot \dot{\vec{X}} = -\mathbf{C} \cdot \vec{X} + \vec{F} \tag{A3.27}$$

Then both sides have to be multiplied by the inverse matrix \mathbf{B}^{-1} :

$$\mathbf{B}^{-1}\mathbf{B}\cdot\dot{\vec{X}} = -\mathbf{B}^{-1}\mathbf{C}\cdot\vec{X} + \mathbf{B}^{-1}\vec{F} \qquad (A3.28)$$

Multiplication of the inverse matrix by the original matrix results in the identity matrix, which does not change the vector being multiplied by it:

$$\mathbf{B}^{-1}\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I} ; \qquad \mathbf{I} \cdot \dot{\vec{X}} = \dot{\vec{X}}$$
 (A3.29)

As a result, the system of differential equations in standard or Cauchy form:

$$\dot{\vec{X}} = \mathbf{A} \cdot \vec{X} + \vec{H} \tag{A3.30}$$

where matrix A is usually called the "coefficient matrix":

$$\mathbf{A} = -\mathbf{B}^{-1}\mathbf{C}; \qquad \vec{H} = \mathbf{B}^{-1}\vec{F}$$
 (A3.31)

The inverse matrix is to be expressed as:

$$\mathbf{B}^{-1} = \frac{1}{\det(\mathbf{B})} \begin{pmatrix} B_{22} & -B_{12} \\ -B_{21} & B_{11} \end{pmatrix} \dots (A3.32)$$

where $det(\mathbf{B})$ is the determinant of the matrix \mathbf{B} . It is expressed as:

$$\det(\mathbf{B}) = B_{11}B_{22} - B_{12}B_{21} \dots (A3.33)$$

As a result, the coefficient matrix **A** is expressed as follows:

$$\mathbf{A} = -\frac{1}{B_{11}B_{22} - B_{12}B_{21}} \begin{pmatrix} -B_{22} & B_{12} \\ B_{21} & -B_{11} \end{pmatrix} \cdot \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} =$$

$$= \frac{1}{B_{11}B_{22} - B_{12}B_{21}} \begin{pmatrix} -(C_{11}B_{22} + B_{12}C_{21}) & -(C_{12}B_{22} + B_{12}C_{22}) \\ -C_{11}B_{21} + B_{11}C_{21} & -C_{12}B_{21} + B_{11}C_{22} \end{pmatrix}$$
.....(A3.34)

The vector \vec{H} , expressing the rudder action, becomes:

$$\vec{H} = \frac{\delta_R}{B_{11}B_{22} - B_{12}B_{21}} \begin{pmatrix} B_{22}Y_{\delta} - B_{12}N_{\delta} \\ -B_{21}Y_{\delta} + B_{11}N_{\delta} \end{pmatrix} \dots (A3.35)$$

2.8 Solution of Linear Equations of Motions

The system of differential equations (A3.30) is linear but non-homogeneous, as it includes the term \vec{H} , expressing external action, namely, the rudder control. In order to solve such a system, the homogeneous equations have to be considered first:

$$\dot{\vec{X}} = \mathbf{A} \cdot \vec{X} \tag{A3.36}$$

As this system is linear, its solution is known and can be formulated as:

$$\vec{X} = e^{\mathbf{A}t} \cdot \vec{X}_0 \tag{A3.37}$$

Here, $e^{\mathbf{A}t}$ is an exponential matrix and \vec{X}_0 is a vector of initial conditions for non-dimensional sway velocity and yaw rate at the moment when a rudder is deflected. As straight sailing is assumed before the steering action took place, these must be zero:

$$\vec{X}_0 = \begin{pmatrix} v'(t=0) \\ r'(t=0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \dots (A3.38)$$

The exponential matrix has a very important property. Its derivative is similar to itself:

$$\frac{d}{dt}(e^{\mathbf{A}t}) = \mathbf{A} \cdot e^{\mathbf{A}t} \tag{A3.39}$$

When substituted back into Equation (A3.36), the exponential matrix converts it to the true equality. As a result, in order to solve Equation (A3.36), the exponential matrix has to be computed. For the 2-by-2 coefficient matrix, the following formula is applicable:

$$e^{\mathbf{A}t} = \frac{\mathbf{A} - \lambda_{2}\mathbf{I}}{\lambda_{1} - \lambda_{2}} \exp(\lambda_{1}t) + \frac{\mathbf{A} - \lambda_{1}\mathbf{I}}{\lambda_{1} - \lambda_{2}} \exp(\lambda_{2}t) =$$

$$= \frac{\exp(\lambda_{1}t)}{\lambda_{1} - \lambda_{2}} \begin{pmatrix} A_{11} - \lambda_{2} & A_{12} \\ A_{21} & A_{22} - \lambda_{2} \end{pmatrix} + \frac{\exp(\lambda_{2}t)}{\lambda_{1} - \lambda_{2}} \begin{pmatrix} A_{11} - \lambda_{1} & A_{12} \\ A_{21} & A_{22} - \lambda_{1} \end{pmatrix} \dots (A3.40)$$

Parameters λ_1 and λ_2 are eigenvalues of coefficient matrix **A**. They are to be calculated from the following matrix equation:

$$\det(\mathbf{I} \cdot \lambda - \mathbf{A}) = 0 \tag{A3.41}$$

This equation is equivalent to:

$$\det\begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \lambda - \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = 0$$
 (A3.42)

Expanding:

$$\det \begin{pmatrix} \lambda + \frac{C_{11}B_{22} + B_{12}C_{21}}{B_{11}B_{22} - B_{12}B_{21}} & \frac{C_{12}B_{22} + B_{12}C_{22}}{B_{11}B_{22} - B_{12}B_{21}} \\ \frac{C_{11}B_{21} - B_{11}C_{21}}{B_{11}B_{22} - B_{12}B_{21}} & \lambda + \frac{C_{12}B_{21} - B_{11}C_{22}}{B_{11}B_{22} - B_{12}B_{21}} \end{pmatrix} = 0 ...$$
(A3.43)

The equation above is equivalent to the following system:

$$\begin{cases} (B_{11}B_{22} - B_{12}B_{21})\lambda^2 + (B_{11}C_{22} - B_{21}C_{12} - B_{12}C_{21} + B_{22}C_{11})\lambda + \\ + C_{11}C_{22} - C_{12}C_{21} = 0 \end{cases}$$

$$(A3.44)$$

$$B_{11}B_{22} - B_{12}B_{21} \neq 0$$

The first condition of the above system is a quadratic equation:

$$a\lambda^2 + b\lambda + c = 0. (A3.45)$$

The coefficients a, b and c are expressed in the terms of the hydrodynamic derivatives:

$$a = (m' - Y'_{\psi})(I_Z - N'_r) - (Y_r - m'x'_g)(N_{\psi} - m'x'_g)$$

$$b = -(m' - Y'_{\psi})(N'_r - V'm'x'_{cg}) - (N'_{\psi} - m'x'_{cg})(Y'_r - m'V') - (Y'_r - m'x'_{cg})N'_{\psi} - (I'_z - N'_r)Y'_{\psi}$$

$$c = Y'_{\psi}(N'_r - V'm'x'_{cg}) - (Y'_r - m'V')N'_{\psi} \qquad (A3.46)$$

Equation (A4.33), also known as the characteristic equation, has two solutions:

$$\lambda_{1,2} = \frac{-b \pm \sqrt{D}}{2a}; \quad D = b^2 - 4ac \quad ...$$
 (A3.47)

Eigenvalues define the type of behavior of a vessel in the horizontal plane. Non-dimensional sway velocity and yaw rate finally are expressed in the following form:

$$v' = V_1' \exp(\lambda_1 t') + V_2' \exp(\lambda_2 t') + V_3'$$

$$r' = R_1' \exp(\lambda_1 t') + R_2' \exp(\lambda_2 t') + R_3'$$
(A3.48)

where V_1' , V_2' , R_1' and R_2' are dependent on initial conditions (sway velocity and yaw rate at the moment the rudder was deflected), while V_3' and R_3' are defined by rudder and hull forces.

2.9 Steady Turning

If both eigenvalues are negative in Equation (A3.48), the exponential terms die out and become negligibly small after a certain period of time. The sway velocity and yaw rate then become constant, as the vessel enters the steady turning mode. A sample trajectory is shown in Section 2, Figure 1. When the rudder is deflected, the vessel does not turn immediately. It travels some distance both in the forward direction (Advance) and transversely (Transfer). This is an initial part of the trajectory, while terms with exponents in Equation (A3.48) still have significant magnitude. Once the transition is over, the vessel is involved in the steady turn. Exponential terms in Equation (A3.48) are very small at this stage and the vessel behavior is described by the constant part of Equation (A3.48).

Vessel turning consists of three phases of turning. These are first phase, second phase and third phase. The third phase can be approximated using linear results for steady turning. Vessel turning is assumed to begin while the vessel is traveling on a straight path and when the rudder execute command is given and the rudder turns to respond. The vessel begins to accelerate prior to changing course. This is the first phase of turning and is a transient phase and consists of all acceleration. The next phase is the second phase, which is the transition phase and is the most difficult and nonlinear to analyze. The second phase can only be analyzed by using numerical simulations. The final phase of vessel turning is the steady turning phase and consists of no acceleration and only steady linear and angular motion. This can be examined approximately, by looking at the linearized equations of motion. For a straight-line stable vessel, the linearized turning radius and corresponding drift angle is:

$$R = L \left[\frac{1}{\delta} \frac{C'}{N'_{v} Y'_{\delta} - Y'_{v} N'_{\delta}} \right]$$

$$\beta = -\delta \left[\frac{N'_{\delta} (Y'_r - m') - Y'_{\delta} N'_r}{C'} \right]$$

where

$$C' = Y'_{v}(N'_{r} - V'm'x'_{cg}) - (Y'_{r} - m'V')N'_{v}$$

Cautionary note: It should be noted herein that the linearized equations of motion are not valid under all circumstances, which include a straight-line unstable vessel and a particularly tight turning radius.

3 Directional Stability

3.1 Vessel Behavior in Horizontal Plane and Steady Turning

An important aspect of maneuverability is the ability of a vessel to stay on course. No rudder deflection is needed to analyze the course-keeping ability, so rudder forces are set to zero and the solution contains only exponential terms:

$$v' = V_1' \exp(\lambda_1 t') + V_2' \exp(\lambda_2 t')$$

$$r' = R_1' \exp(\lambda_1 t') + R_2' \exp(\lambda_2 t')$$
(A3.49)

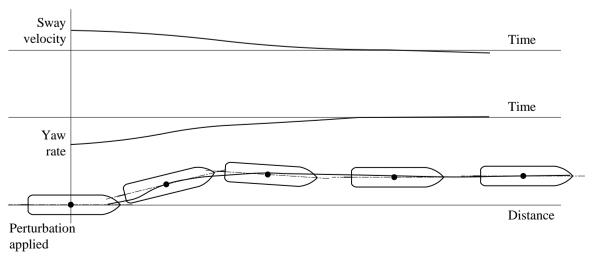
Contrary to the previous case, initial conditions are no longer zero; they are assumed to be of small magnitudes, resulting from perturbations (wind, current, waves or other factors) always existing in the real sea.

$$\vec{X}_0 = \begin{pmatrix} v'(t=0) \\ r'(t=0) \end{pmatrix} = \begin{pmatrix} \varepsilon_v \\ \varepsilon_r \end{pmatrix} \dots (A3.50)$$

Once these perturbations are applied, the vessel deviates from its initial course, but if the eigenvalues are negative, it will return to a straight-line path as the exponential terms die out. A sample trajectory is illustrated in Appendix 3, Figure 7, along with sway velocity and yaw rate time histories.

If at least one eignevalue is positive, then both sway velocity and yaw rate tend to infinity, which is physically impossible. It means that the mathematical model of ship motions expressed in the form of a system of linear differential equations (A3.48) is not adequate and cannot be applied. So, both negative eigenvalues of coefficient matrix $\bf A$ is the condition of applicability of the model (A3.48).

FIGURE 7
Reaction on Small Perturbation if Eigenvalues are Negative



3.2 Stability of Motion

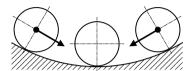
The physical reason why model (A3.48) cannot be used in the calculation of the trajectory can be attributed to the phenomenon known as stability of motion or motion stability. This phenomenon is similar to the stability of equilibrium, illustrated in Appendix 3, Figure 8.

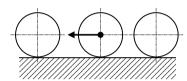
FIGURE 8 Stability of Equilibrium

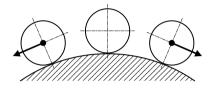




c) Unstable Equilibrium







The three states shown in Appendix 3, Figure 8 are related to the equilibrium of a heavy sphere on a smooth surface. In the first case (Appendix 3, Figure 8a), the surface is curved and the sphere is in the trough. The sphere is in a perfect equilibrium as the force of gravity is balanced with a reaction of the surface. If the equilibrium is perturbed and the sphere is shifted from its original position, a resultant force appeared from the disturbed state, pulls the sphere back to its original position. Such equilibrium is stable as any small perturbation finally leads to the restoration of the original state.

If the surface is flat (Appendix 3, Figure 8b), there is no reaction on any perturbation and the sphere stays in its new position for an indefinite time, as the new position is also an equilibrium position. Such a type of equilibrium is defined as neutral.

If the surface is curved but the sphere is located on the crest (Appendix 3, Figure 8c), any small perturbation takes it away from the equilibrium and it will never return. Such equilibrium is defined as unstable and it cannot last for a long time, as small perturbations always exist in the real world.

A complete analogy exists between stability of equilibrium and the stability of motion. If a vessel has stable motions in the horizontal plane, small perturbations do not alter its straight-line course. The combination of the forces on its hull tends to restore the vessel to another straight-line path after some transition. Such a vessel is considered as "straight-line stable".

If a vessel does not have stable motions in the horizontal plane, any small perturbation results in a combination of hull forces that take a vessel off course. Such a vessel is considered as "straight-line unstable".

Hull form has a decisive influence on straight-line stability. However, absence of straight-line stability does not constitute design failure. In many cases, other design requirements and considerations make a straight-line unstable hull form the only option available. In many cases, straight-line instability is compensated for with proper tuning of an autopilot and/or the actions of a helmsman.

Linear equations (A3.17) cannot be used for prediction of maneuverability of a straight-line unstable vessel – equations of motion have to be nonlinear and more terms should be kept in the approximation (A3.3). Nonlinear equations of motion are addressed in Subsection A3/4.

3.3 Evaluation of Straight-line Stability

Presence or absence of the straight-line stability is determined by the signs of the eigenvalues of the coefficient matrix (See A3/2.7 for definition). The eigenvalues are determined from the solution of the characteristic equation (A4.33) that is expressed by Equation (A3.45). The sign of both eigenvalues depend on the coefficients a, b and c, defined by formulae (A3.46).

Hydrodynamic derivatives $Y_{\dot{v}}'$ and $N_{\dot{r}}'$ are non-dimensional added mass in sway (added mass 22) and added moment of inertia (added mass 66) in yaw taken with the opposite sign. As a result, value $(m'-Y_{\dot{v}}')(I_Z-N_{\dot{r}}')$ is positive. Hydrodynamic derivatives $Y_{\dot{r}}'$ and $N_{\dot{v}}'$ are non-dimensional coupled added mass sway-yaw (added mass 26) and yaw-sway (added mass 62), while $m'x_g'$ is the non-dimensional correction moment due to center of gravity and does not coincide with the origin of the coordinate system. All these quantities are small in comparison with non-dimensional mass and added mass in sway, and vertical moment of inertia with added moment of inertia in yaw. As a result, the coefficient a is substantially positive.

Coefficient b is also a positive quantity. Hydrodynamic derivative Y'_{ν} is negative, and it describes influence of sway velocity on the lift force on the hull. This force increases with increasing of angle of attack/drift angle β that is defined in Appendix 3, Figure 6. As seen from Appendix 3, Figure 6, where sway velocity is shown negative and the drift angle positive, with the accepted coordinate system, positive lift force Y is developed with negative drift angle β . Negative drift angle is a result of negative sway velocity ν . So, the coefficient describing the influence of sway velocity on lift force – hydrodynamic derivative Y'_{ν} must be negative and the term $-(I'_z - N'_{\dot{\nu}})Y'_{\dot{\nu}}$ is positive.

Hydrodynamic derivative N'_{ν} describes influences of sway velocity on the moment due to lift force. As the center of hydrodynamic pressure is located forward of midship for most conventional hull configurations, the lever of the moment is positive while N'_{ν} is negative due to negative Y'_{ν} . As a result, the term $-(Y'_{r}-m'x'_{cg})N'_{\nu}$ is positive.

Hydrodynamic derivative Y'_r expresses the influence of yaw rate on the lifting force. When the vessel is turning with positive yaw rate (see Appendix 3, Figure 6), the forward part of the vessel has positive transverse velocity and the aft part has negative transverse velocity. This means that the contribution of the forward part to the lift force is negative, (analogous to the previously considered case with Y'_v) while the contribution of the aft part is positive. For the usual hull configuration, the absolute value of the contribution from the forward part to the lift force is larger than that of the aft part (it is the reason why the hydrodynamic pressure center is located forward of midship). Hence, Y'_r is negative and the term $-(N'_v - m'x'_{cg})(Y'_r - m'V')$ is positive.

Hydrodynamic derivative N'_r expresses the influence of yaw rate on the moment due to lifting force. As the contribution to the lift force from the forward part is negative, its moment is also negative since all the distances forward are positive, and therefore, the lever also is positive. The contribution of the aft part is positive, but all distances measured in the aft direction are negative, so the lever is negative and the moment of the aft contribution is negative. As both fore and aft parts produce negative moments, the hydrodynamic derivative N'_r is negative and the term $-(m'-Y'_{\dot{\nu}})(N'_r-V'm'x'_{cg})$ is positive.

The above reasoning concludes that, for conventional hull configuration, coefficient b is positive. As it follows from the formulae (A3.47), in order for both eigenvalues to be negative, the square root of the discriminant, \sqrt{D} , has to be smaller than b, which is possible only for positive values of coefficient c.

For motions in the horizontal plane, the discriminant, D, is always positive and the eigenvalues are always real numbers. If the discriminant, D, happens to be negative, the eigenvalues become complex; which means that a vessel would experience oscillatory sway and yaw motions in calm waters, which usually does not occur in practice.

Finally, the straight-line stability condition is formulated as:

$$c = Y'_v(N'_r - V'm'x'_{cg}) - (Y'_r - m'V')N'_v > 0$$
 (A3.51)

Re-writing the condition (A3.51) reveals its physical meaning:

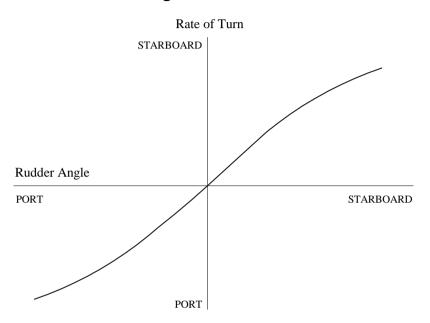
$$\frac{N_r' - V'm'x_{cg}'}{Y_r' - m'V'} > \frac{N_v'}{Y_v'} \tag{A3.52}$$

The condition of straight-line stability could be interpreted as the requirement for the lever of hull hydrodynamic moment generated by yaw to be greater than the lever of the hydrodynamic moment generated by sway.

3.4 Measure of Straight-line Instability

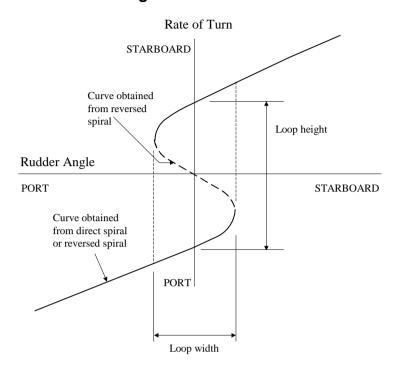
If a vessel is path-stable, each angle of deflection of the rudder leads to a certain yaw rate or rate of turn. Dependence of these two quantities for the path-stable vessel is illustrated in Appendix 3, Figure 9. The curve crosses the yaw rate axis only once at zero. This means that if the rudder is not defected, yaw rate is zero and the vessel sails straight.

FIGURE 9
Relation between Rudder Angle and Yaw (Turn) Rate for Straight-line Stable Vessel



If a vessel is straight-line unstable, the character of this relationship is more complex. See Appendix 3, Figure 10. The curve crosses the yaw rate axis three times: at zero, at some negative and positive values of yaw rate. Despite the fact that the curve still crosses the yaw rate axis at zero, this motion is unstable. This is exactly the picture described in A3/3.3, when any small perturbation takes the straight-line unstable vessel out of its expected straight trajectory. The other two crossings represent stable motion modes. It means that the vessel is turning with negative or positive yaw rate value. This is possible even if the rudder is not deflected and a vessel cannot stay on course unless there is intervention from the helmsman or autopilot. Location of these points gives important information on how strong the instability is. The larger the absolute value of the yaw rate at the other two crossing points, the faster the vessel will be deviated from the desired course.

FIGURE 10
Relation between Rudder Angle and Yaw (Turn) Rate for Straight-line Unstable Vessel



4 Nonlinear Equations of Motion

4.1 General

The linear equations of motion (A3.17) have only limited use. If a vessel is straight-line stable, they can be used, in principle, for maneuvering prediction, if the considered maneuvers are not too tight. If they are tight, the result will not be accurate enough, as contributions of nonlinear terms become significant and they could no longer be ignored. If a vessel is path-unstable, the linear system of equations cannot be applied at all, as the solution will have a tendency of unlimited increase and only nonlinear terms could stop its growth.

A nonlinear system is derived by nonlinear terms in the Taylor expansion (A3.3). Usually, the expansion is made up to the third power, as the terms of higher order are small in most cases. In general, which terms will be kept is determined by both theoretical consideration and practical experience. Numerical values of hydrodynamic derivatives come from model tests with planar motion mechanism (PMM), rotating arm or with a free running model. Therefore, the particular expressions for the terms for practical use may be dependent on how the experimental data was processed. As a result, all further consideration should be seen as an example only, as other forms of mathematical models may be used. Physical meaning stays the same. However, it could be expressed in a slightly different way. There are numerous formulations of the nonlinear equations, but the most common are the cubic and quadratic nonlinearity. The quadratic nonlinearity will be described below because of the availability of a complete set sample data. However, the cubic nonlinearity may be more common and easy to use.

4.2 Surging Equation

Consider the surging equation of motion from the system (A3.1):

$$m[\dot{u} - vr - x_{cg}r^2] = X + X_{Rd}$$
 (A3.53)

Expansion of the hull force *X* into Taylor series now includes nonlinear terms:

$$X = \frac{\rho}{2} L^3 X'_{\dot{u}} \dot{u} + \frac{\rho}{2} L^4 X'_{rr} r^2 + \frac{\rho}{2} L^2 X'_{vv} v^2 + \frac{\rho}{2} L^3 X'_{vr} vr \qquad (A3.54)$$

The term with $X_{\dot{u}}\dot{u}$ has an inertial meaning: it reflects changes of inertial properties of a body moving in a fluid in comparison with a body moving in vacuum. Therefore, $X'_{\dot{u}}$ actually is a non-dimensional surging added mass of a vessel taken with the opposite sign ($-A_{11}$ in seakeeping notation, but calculated for zero frequency).

The terms with $X'_{rr}r^2$, $X'_{vv}v^2$ and $X'_{vr}vr$ express additional resistance caused by the turn. In a sense, this can be seen as a drag component of lift force, if a hull is considered as a lifting surface. See Appendix 3, Figure 4.

All the above hull-force-related terms were written with the implicit assumption that vessel speed does not change much and the thrust created by the propeller is perfectly balanced with resistance – vessel in self-propulsion point. However, this is not necessarily true since tight maneuvers may involve significant change of vessel speed. Also, the model should be capable of simulating acceleration and stopping, so the term expressing a balance between thrust and resistance is necessary.

The balance between thrust and resistance in straight course can be expressed as follows:

$$RT(u) = R(u) - T(u) = \frac{\rho}{2} L^2 u^2 \left[a_i + b_i \eta + c_i \eta^2 \right].$$
 (A3.55)

where:

R(u) = full value of resistance in straight course

T(u) = propeller thrust in straight sailing

 $a_i, b_i, c_i =$ coefficients of the fitted polynomial

Usually, it is presented in several intervals for different values of vessel propulsion ratio, η , which is expressed as:

$$\eta = \frac{u_c}{u} \tag{A3.56}$$

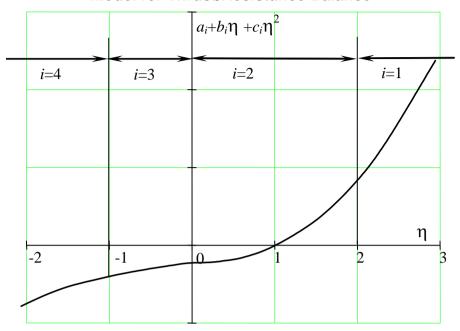
where

 u_c = "commanded speed" – the speed in straight course in calm water corresponding to the requested number of revolutions of the propeller

Equation (A3.55) possesses the following properties.

If $\eta = 1$ or $u = u_c$, the resistance and thrust are in perfect balance, so $RT(u_c) = 0$. This is the self-propulsion point. If $\eta = 1$ or $u_c = 0$, the propeller does not produce any thrust, so RT(u) = R(u) and the function (A3.55) represents straight course resistance only. The appearance of the fitted polynomial is shown in Appendix 3, Figure 11.

FIGURE 11
Model for Thrust/Resistance Balance



The formula (A3.55) takes into account the changing axial component of speed during a maneuver on straight course thrust/resistance balance. However, change of axial velocity u may have a significant influence on additional resistance caused by turn (drag component of lift force). It should be accounted for with:

$$X_{\eta} = \frac{\rho}{2} L^2 X'_{\nu\nu\eta} \nu^2 (\eta - 1) \dots (A3.57)$$

where

 $X'_{\nu\nu\eta}$ = hydrodynamic derivative produced when the model was not in the self-propulsion point

The influence of speed changes of other components in Equation (A3.54) may also be included if this influence is found to be significant for the modeled vessel.

The rudder force X_{Rd} (included in Equation (A3.53) is actually additional resistance (or drag) created by the deflected rudder. As flow around the rudder depends on the propeller, it may be different, depending on whether a vessel is in self-propulsion point or not. The formula below accounts for that:

$$X_{Rd} = \frac{\rho}{2} L^2 u^2 \left(X_{\delta\delta}' \delta_R^2 + X_{\delta\delta\eta\eta} \delta_R^2 \eta^2 \right) \dots \tag{A3.58}$$

where

 $X'_{\delta\delta}$ = rudder drag coefficient measured at the self-propulsion point

 $X'_{\delta\delta nn}$ = correction in case a vessel is not in the self-propulsion point

Finally, the entire surge equation has the following appearance (with brief comments for fast reference):

$$m\left[\dot{u}-vr-x_{cg}r^{2}\right] = \frac{\rho}{2}L^{3}X'_{\dot{u}}\dot{u} +$$
 Inertial forces
$$+ \frac{\rho}{2}L^{2}u^{2}\left[a_{i}+b_{i}\eta+c_{i}\eta^{2}\right] +$$
 Balance of thrust and resistance
$$+ \frac{\rho}{2}L^{4}X'_{rr}r^{2} + \frac{\rho}{2}L^{2}X'_{vv}v^{2} + \frac{\rho}{2}L^{3}X'_{vr}vr +$$
 (A3.59) Resistance due to turn at self-propulsion point
$$+ \frac{\rho}{2}L^{2}X'_{vv\eta}v^{2}(\eta-1) +$$
 Correction for non self-propulsion point
$$+ \frac{\rho}{2}L^{2}u^{2}\left(X'_{\delta\delta}\delta_{R}^{2} + X_{\delta\delta\eta\eta}\delta_{R}^{2}\eta^{2}\right)$$
 Rudder force

4.3 Swaying Equation

Consider the swaying equation from the system (A3.1):

$$m[\dot{v} + ur + x_{cg}\dot{r}] = Y + Y_{Rd}$$
 (A3.60)

Expansion of the hull force *Y* into Taylor series now includes nonlinear terms:

The first four terms of Equation (A3.61) were present in the linear system (A3.17).

The first two terms express inertial forces in the fluid; hydrodynamic coefficients Y'_{i} and Y'_{i} are actually non-dimensional added masses: sway-sway and sway-yaw taken with the opposite sign and $(-A_{22}$ and $-A_{26}$ in seakeeping notation – calculated for zero frequency).

The terms $0.5\rho L^2 u Y_v' v$ and $0.5\rho L^3 u Y_r' r$ in their non-dimensional form are present in the linear system (A3.17). However, in Equation (A3.61), they are no longer linear as the transversal component of speed v is one of the variables.

The terms $0.5\rho L^2 Y'_{\nu|\nu|}\nu|\nu|$ and $0.5\rho L^3 Y'_{\nu|r|}\nu|r|$ are a nonlinear part of the lift force on the hull. Here, it is expressed with the second order of Taylor expansion, the absolute values are used to keep correct sign; the vessel hull is usually symmetrical relative to the XZ plane, so the lifting force has to be an odd function.

These terms, however, are not the only way to include the nonlinearity. It is also possible to use terms of the third order of Taylor expansion, for example, $0.5\rho L^3 Y'_{vvr}v^2 r$, while assuming that the terms of the second order are zero based on the above consideration of symmetry.

Presentation of hydrodynamic forces with Taylor series is just a technique and does not bear any physical meaning by itself, so the choice of the form of nonlinear terms is a matter of accuracy and convenience. In practice, the choice is also dependent on in which form the hydrodynamic derivatives were obtained in PMM test.

It is important, however, to distinguish inertial hydrodynamic forces Y_I from lift (vortex-induced) forces Y_H in Equation (A3.61).

$$Y = Y_I + Y_H \tag{A3.62}$$

$$Y_{I} = \frac{\rho}{2} L^{3} Y_{\dot{v}}' \dot{v} + \frac{\rho}{2} L^{4} Y_{\dot{r}}' \dot{r}$$
 (A3.63)

$$Y_{H} = \frac{\rho}{2} L^{2} u Y_{v}' v + \frac{\rho}{2} L^{3} u Y_{r}' r + \frac{\rho}{2} L^{2} Y_{v|v|}' v \mid v \mid + \frac{\rho}{2} L^{3} Y_{v|r|}' v \mid r \mid \dots$$
(A3.64)

Lift forces in Equation (A3.64) are assumed to be determined when the vessel is in the self-propulsion point. The following formula gives the correction for cases when speed lost in turns takes a vessel out of the self-propulsion point:

$$Y_{\eta} = \left(\frac{\rho}{2} L^{2} u Y'_{\nu \eta} v + \frac{\rho}{2} L^{3} u Y'_{r \eta} r + \frac{\rho}{2} L^{2} Y'_{\nu | \nu | \eta} v | v |\right) \cdot (\eta - 1) \dots (A3.65)$$

The flow around a propeller is not symmetrical and, as a result, a lateral force is created by any propeller. This effect is especially strong on a single-screw vessel. This force is modeled as:

$$Y_* = \frac{\rho}{2} L^2 u^2 Y_*' + \frac{\rho}{2} L^2 u^2 Y_{*\eta}' (\eta - 1)$$
 (A3.66)

The first term of Equation (A3.66) is the lateral force of propeller for the self-propulsion point and the second term implements a correction if a vessel is not in the self-propulsion point. Coefficients Y'_* and $Y'_{*\eta}$ are non-dimensional measures of this effect, which is considered here on the straight course only.

The rudder force includes a nonlinear term and a correction for a vessel not being in the self-propulsion point:

$$Y_{Rd} = \frac{\rho}{2} L^2 u^2 Y_{\delta}' \delta_R + \frac{\rho}{2} L^3 u Y_{\delta|r|}' \delta_R \mid r \mid + \frac{\rho}{2} L^2 u^2 Y_{\delta\eta}' \delta_R (\eta - 1)$$
 (A3.67)

where

 Y'_{δ} = non-dimensional rudder force coefficient

 $Y'_{\delta|r|}$ = non-dimensional rudder force coefficient accounting for yaw rate influence on rudder

 $Y'_{\delta n}$ = non-dimensional confident for correction if a vessel is not at the self-propulsion point.

Finally, the entire sway equation has the following appearance (with brief comments for fast reference):

$$m[\dot{v} + ur + x_{cg}\dot{r}] = \frac{\rho}{2}L^3Y'_{v}\dot{v} + \frac{\rho}{2}L^4Y'_{\dot{r}}\dot{r} +$$

$$+ \frac{\rho}{2}L^2uY'_{v}v + \frac{\rho}{2}L^3uY'_{r}r + \frac{\rho}{2}L^2Y'_{v|v|}v \mid v \mid + \frac{\rho}{2}L^3Y'_{v|r|}v \mid r \mid +$$

$$+ \left(\frac{\rho}{2}L^2uY'_{v\eta}v + \frac{\rho}{2}L^3uY'_{r\eta}r + \frac{\rho}{2}L^2Y'_{v|v|\eta}v \mid v \mid\right) \cdot (\eta - 1) +$$

$$+ \frac{\rho}{2}L^2u^2Y'_{\delta}\delta_R + \frac{\rho}{2}L^3uY'_{\delta|r|}\delta_R \mid r \mid + \frac{\rho}{2}L^2u^2Y'_{\delta\eta}\delta_R(\eta - 1) +$$

$$+ \frac{\rho}{2}L^2u^2Y'_{s} + \frac{\rho}{2}L^2u^2Y'_{s\eta}(\eta - 1)$$
Rudder force
$$+ \frac{\rho}{2}L^2u^2Y'_{s} + \frac{\rho}{2}L^2u^2Y'_{s\eta}(\eta - 1)$$
Lateral propeller force

4.4 Yawing Equation

Consider the yawing equation from the system (A3.1):

$$I_z \dot{r} + m x_{cg} [\dot{v} + ur] = N + N_{Rd}$$
 (A3.69)

Expansion of moment of the hull force, N, into Taylor series now should include nonlinear terms.

Following the pattern for consideration of sway, this moment could be separated into inertial and lift forces.

$$N = N_I + N_H \dots (A3.70)$$

$$N_I = \frac{\rho}{2} L^5 N_{\dot{r}}' \dot{r} + \frac{\rho}{2} L^4 N_{\dot{v}}' \dot{v} \qquad (A3.71)$$

These terms are present in the linear system (A3.17). These terms express inertial forces in the fluid; hydrodynamic coefficients $N'_{\dot{r}}$ and $N'_{\dot{v}}$ are actually non-dimensional added masses: yaw-yaw and yaw-sway taken with the opposite sign and ($-A_{66}$ and $-A_{62}$ in seakeeping notation – calculated for zero frequency).

The terms $0.5\rho L^4 u N_r' r$ and $0.5\rho L^3 u N_v' v$ in their non-dimensional form are present in the linear system (A3.17). However, in Equation (A3.72), they are no longer linear, as an axial component of speed u is one of the variables. The rest of the terms represent nonlinear contribution. These terms are of the second order and absolute value is used to keep the odd character of the functions, as it is necessary due to the symmetry of the hull relative to the XZ plane. Again, the above form is not the only way to present the nonlinear contribution.

As all the coefficients in these terms are obtained for the self-propulsion point, a correction is needed to take into account speed loss in a turn and the departure from the self-propulsion point:

$$N_{\eta} = \left(\frac{\rho}{2} L^{4} u N'_{r\eta} r + \frac{\rho}{2} L^{3} u N_{\nu\eta} v + \frac{\rho}{2} L^{3} N'_{\nu|\nu|\eta} v \mid v \mid\right) \cdot (\eta - 1) \dots (A3.73)$$

The lateral force created by the asymmetric flow around the propeller and expressed in the form (A3.66) makes a moment:

$$N_* = \frac{\rho}{2} L^3 u^2 N_*' + \frac{\rho}{2} L^3 u^2 N_{*\eta}' (\eta - 1) \qquad (A3.74)$$

The expression for the moment of rudder force is similar to Equation (A3.73):

$$N_{Rd} = \frac{\rho}{2} L^3 u^2 N_{\delta}' \delta_R + \frac{\rho}{2} L^4 u N_{\delta|r|}' \delta_R \mid r \mid + \frac{\rho}{2} L^3 u^2 N_{\delta\eta}' \delta_R (\eta - 1) \dots (A3.75)$$

$$\begin{split} I_z\dot{r} + mx_{cg} \left[\dot{v} + ur\right] &= \frac{\rho}{2} L^5 N_r'\dot{r} + \frac{\rho}{2} L^4 N_v'\dot{v} + \\ &+ \frac{\rho}{2} L^4 u N_r' r + \frac{\rho}{2} L^3 u N_v' v + \\ &+ \frac{\rho}{2} L^5 N_{r|r|}' r \mid r \mid + \frac{\rho}{2} L^3 N_{v|v|}' v \mid v \mid + \frac{\rho}{2} L^4 N_{v|r|}' v \mid r \mid + \\ &+ \left(\frac{\rho}{2} L^4 u N_{r\eta}' r + \frac{\rho}{2} L^3 u N_{v\eta} v + \frac{\rho}{2} L^3 N_{v|v|\eta}' v \mid v \mid\right) \cdot (\eta - 1) + \\ &+ \left(\frac{\rho}{2} L^3 u^2 N_\delta' \delta_R + \frac{\rho}{2} L^4 u N_{\delta|r|}' \delta_R \mid r \mid + \frac{\rho}{2} L^3 u^2 N_{\delta\eta}' \delta_R (\eta - 1) + \\ &+ \frac{\rho}{2} L^3 u^2 N_s' + \frac{\rho}{2} L^3 u^2 N_{s\eta}' (\eta - 1) \end{split} \qquad \text{Rudder moment}$$

A numerical example of using the nonlinear equations (A3.59), (A3.68) and (A3.76) is considered in Appendix 1.



APPENDIX 4 Empirical Methods of Prediction of Maneuverability

1 Empirical Prediction

1.1 Advantages and Applicability of Empirical Prediction Methods

Empirical prediction methods are capable of providing elements of turning and stopping performance using a very limited amount of input data, mostly basic ship particulars. These are especially useful in an early design stage, such as when only principal dimensions, hull form coefficients, speed, power, type of engine and rudder parameters have been determined.

The advantage of maneuverability prediction in the preliminary design stage is that it provides early indications of potential problems with maneuverability, when it is relatively easy to correct.

These methods are invariably based either on a series of model tests or statistics of full scale trial data. Therefore, applicability of the empirical methods is generally limited to vessels with characteristics within the range of parameters covered by the respective sample, either model- or full scale.

Due to the complexity of hydrodynamics involved in vessel maneuvering, accuracy of the empirical method is limited. Therefore, the results should be considered rather as guidance, and more accurate model test-based simulations should be used during the design as early as practical (see Subsection 3/1 for more information on these methods).

1.2 Prediction of Elements of Turning Circle

The method described below is taken from [Lyster and Knights 1979]. Use of this method is limited to single-screw monohulls, with shape and rudder parameters within the range specified in Appendix 4, Table 1. If the parameters of a vessel are not within the limits of Appendix 4, Table 1, results of the calculation may be misleading, and therefore, its use is not recommended. In such a case, prediction of maneuverability has to be performed later during the design process, as described in Subsection 3/1.

TABLE 1
Limitations of Empirical Technique for Prediction of Turning Circle

Parameter	Minimum	Maximum
<i>L</i> , m	55	350
C_B	0.56	0.88
δ_R (deg)	10	45
L/B	5.56	9.1
Trim/L	0	0.05
$Sp \cdot Ch/L \cdot T$	0.01	0.04
A_B/LT	-0.11	+0.04
V/\sqrt{L} (knot-m ^{-0.5})	0.20	1.0
T_L/T	0.5	1.0

Nomenclature for the symbols used in Appendix 4, Table 1 is given in Section 1 and repeated below.

First, steady turning diameter (in ship lengths) has to be calculated as:

$$\frac{STD}{L} = 4.19 - 203 \frac{C_B}{\delta_R} + 47.4 \frac{Trim}{L} - \frac{13.0B}{L} + \frac{194}{\delta_R} - 35.8 \frac{Sp \cdot Ch}{L \cdot T} (ST - 1) + \dots (A4.1) + 3.82 \frac{Sp \cdot Ch}{L \cdot T} (ST - 2) + 7.79 \frac{A_B}{L \cdot T} + 0.7 \left(\frac{T}{T_L} - 1 \right) \left(\frac{\delta_R}{|\delta_R|} \right) (ST - 1)$$

where

STD = steady turning diameter, in m

 C_R = block coefficient

 δ_R = rudder angle, in degrees (positive to starboard)

Trim = static trim, in m

L = length of the vessel, in m, measured between perpendiculars

B = molded breadth, in m

Sp = span of rudder, in m

Ch = mean chord of rudder, in m, as defined in Section 1, Figure 2

T = design draft at full load, in m

ST = stern type (1 = Closed 2 = Open, see Section 1, Figure 3)

 T_L = draft, in m, at which turning circle is estimated

 A_R = submerged bow profile area, in m²

Tactical diameter and advance (both in terms of ship length) are to be estimated as:

$$\frac{TD}{L} = 0.910 \frac{STD}{L} + 0.424 \frac{V_S}{\sqrt{L}} + 0.675 \dots$$
 (A4.2)

$$\frac{Ad}{L} = 0.519 \frac{TD}{L} + 1.33 \dots$$
 (A4.3)

where

TD = tactical diameter, in m

 $V_{\rm s}$ = test speed, in knots

Ad = advance, in m

The empirical method [Lyster and Knight 1979] also proposes formulae for other figures, which are not covered by the maneuverability criteria in Section 2, but may be useful to estimate the maneuverability of the design. These figures are transfer (in ship lengths) and speed in steady turn (as a fraction of speed):

$$\frac{Tr}{L} = 0.497 \frac{TD}{L} - 0.065 \dots (A4.4)$$

$$\frac{V_T}{V_S} = 0.074 \frac{TD}{L} + 0.149 \tag{A4.5}$$

where

Tr = transfer, in m

 V_T = velocity in a turn, in knots

1.3 Empirical Formulae for the Stopping Track Reach

The empirical method considered below is based on the IMO Maneuvering Explanatory Notes [IMO, 2002b]. Early stage prediction may be particularly important for large displacement low-powered vessels. The method is applicable for monohull, propeller-driven vessels powered by diesel or steam turbine.

The behavior of a vessel during a stopping maneuver is extremely complicated. However, a fairly simple mathematical model can be used to demonstrate the important aspects which affect the stopping ability of a vessel. For any vessel, the longest stopping distance can be assumed to result when the vessel travels in a straight line along the original course, after the astern order is given. In reality, the vessel will either veer off to port or starboard and travel along a curved track, resulting in a shorter track reach due to increased hull drag.

To calculate the stopping distance on a straight path, the following assumptions should be made:

- The resistance of the hull is proportional to the square of the vessel speed.
- The astern thrust is constant throughout the stopping maneuver and equal to the astern thrust generated by the propeller when the vessel eventually stops dead in the water; and
- The propeller is reversed as rapidly as possible after the astern order is given.

The track reach or the stopping distance along a straight track, in ship lengths, is to be estimated as lying between the following boundaries:

$$S_{low} = A_{low} \log_e (1 + B_{low}) + C$$

$$S_{high} = A_{high} \log_e (1 + B_{high}) + C$$
(A4.6)

where:

C

 S_{low} , S_{high} = low and high boundaries of the stopping distance, in ship lengths

 A_{low} , A_{high} = low and high boundaries for a coefficient dependent upon the mass of the vessel divided by its resistance coefficient (see Appendix 4, Table 2 for the numerical values)

 B_{low} , B_{high} = low and high boundaries for a coefficient dependent on the ratio of the vessel resistance immediately before the stopping maneuver, to the astern thrust when the vessel is dead in the water (see Appendix 4, Table 3 for the numerical values)

= coefficient dependent upon the product of the time taken to achieve the astern thrust and the initial speed of the vessel, the method of calculation of C is given below.

TABLE 2
Numerical Values for Boundaries of the Coefficient A

Vega el Turo	Coefficient A				
Vessel Type	Low Boundary, A _{low}	High Boundary, A_{high}			
Cargo ship	5	8			
Passenger/car ferry	8	9			
Gas carrier	10	11			
Product tanker	12	13			
VLCC	14	16			

The value of the coefficient B is controlled by the amount of astern power which is available from the power plant. With diesel machinery, the astern power available is usually about 85% of the ahead power, whereas with steam turbine machinery, this figure could be as low as 40%.

TABLE 3

Numerical Values for Boundaries of the Coefficient B

T. CM 1:	Coefficient B				
Type of Machinery	Low Boundary, Blow	High Boundary, Bhigh			
Diesel	0.6	1.0			
Steam turbine	1.0	1.5			

The value of the coefficient C is half the distance traveled, in ship lengths, by the vessel while the engine is reversed and full astern thrust is developed. The value of C will be larger for smaller vessels and defined by the following formulae:

$$C = \begin{cases} C_L & \text{if } V_S < 15 \text{ kn or } T_{Rv} < 60 \text{ s} \\ C_L \frac{V_S}{15} & \text{if } V_S > 0.25 T_{Rv} \\ C_L \frac{T_{Rv}}{60} & \text{if } V_S \le 0.25 T_{Rv} \end{cases}$$
 (A4.8)

where

 V_S = vessel speed, in knots. Speed for all maneuvering tests should be within 90-100% of this speed

 T_{Rv} = time, in seconds, necessary to achieve steady reverse thrust

 C_L = coefficient that depends on length and is to be calculated as:

$$= \begin{cases} 2.3 & \text{if} \quad L \le 100 \text{ m} \\ -0.012 \cdot L + 3.5 & \text{if} \quad 100 \text{ m} < L \le 200 \text{ m} \\ -0.003 \cdot L + 1.7 & \text{if} \quad 200 \text{ m} < L \le 300 \text{ m} \end{cases}$$
(A4.9)

L = length of the vessel, in m, measured between perpendiculars

Although all the values given for the coefficients A, B and C may only be considered as typical values for illustrative purposes, they indicate that large vessels may have difficulty satisfying the adopted stopping ability criterion of 15 ship lengths.



APPENDIX 5 Forms

Forms for reporting results of sea trials are given below. One form is to be filled out by the Surveyor witnessing the trials and another one by shipyard.

1 Form for Repo	rtin	ng Maneuv	ering	Data by S	urveyor		
Surveyor:							
SHIP DATA							
Ship Type						L/V	
L/B			В/Т			СВ	
Rudder Type		l .			Number	of Rudders	
Total Rudder Area/LT			•			Trim	
Propeller Type			-		Number o	f Propellers	
Engine Type			-		Appr	oach Speed	
			1			l	
TURNING CIRCLE							
TURNING CIRCLE		PORT	7	STB	en.	Pull-out	Toct
Tactical Diamete	or/I	TOKI	-	7		Tun-out	Test
Advanc						G _t 1	L /TT + 11
10 deg/10 deg Zig-zag	C/L					Stab	le/Unstable
1st Overshoot Ar	, ₀₁₀						
2 nd Overshoot Ar	_						
	•						
Distance traveled until cou change 10 de							
20 deg/20 deg Zig-zag							
1st Overshoot Ar	ngle						
STOPPING TEST		Head Reach/L		Track Reach	/L		
SPIRAL TEST*							
Width of Instability Loop, o	leg.						
Type of Spiral 7	Γest						
		Inverse/Simplific	ed/Direct				
* To be conducted only if pull-out t	est in						
Cianatana Tina 1D 4 CT	1						
Signature, Time and Date of T	rıals:						

2 Form for Reporti	ng Maneuve	ering L	Data to AB	S by Shi	pyard	
Shipyard:			Shipyard Repre	sentative:		
SHIP DATA						
Ship Type					L/V	
L/B		В/Т			СВ	
Rudder Type				Number o	of Rudders	
Total Rudder Area/LT					Trim	
Propeller Type				Number of	Propellers	
Engine Type				Appro	oach Speed	
TURNING CIRCLE						
	PORT		STBD	1	Pull-out	Гest
Tactical Diameter/L						
Advance/L					Stable	e/Unstable
10 deg/10 deg Zig-zag						
1st Overshoot Angle						
2 nd Overshoot Angle						
Distance traveled until course						
change 10 deg/L						
20 deg/20 deg Zig-zag			Т			
1 st Overshoot Angle						
STOPPING TEST	Head Reach/L		Track Reach/L	,		
SPIRAL TEST*						
Width of Instability Loop, deg.						
Type of Spiral Test						
	Inverse/Simplifie	d/Direct	1			
* To be conducted only if pull-out test i	ndicated path-instabil	ity.				
Signature, Time and Date of Trial	e.					



APPENDIX 6 Environmental Correction of Results of Sea Trials

The following procedure for environmental correction closely follows that from [IMO 2002b]

Vessel maneuverability can be significantly affected by the immediate environment, such as wind, waves and current. Environmental forces can cause reduced course-keeping stability or complete loss of the ability to maintain a desired course. They can also cause increased resistance to a vessel's forward motion, with consequent demand for additional power to achieve a given speed or reduce the stopping distance.

When the ratio of wind velocity to vessel speed is large, wind has an appreciable effect on vessel control. The vessel may be unstable in wind from many directions. Waves can also have a significant effect on course-keeping and maneuvering. It has been shown that for large wave heights a vessel may behave quite erratically and, in certain situations, can lose course stability.

Ocean current affects maneuverability in a manner somewhat differently from that of wind. The effect of current is usually treated by using the relative velocity between the vessel and the water. Local surface current velocities in the open ocean are generally modest and close to constant in the horizontal plane.

Therefore, trials shall be performed in the calmest weather conditions possible. If the minimum weather conditions for the criteria requirements are not applied, the trial results should be corrected.

Generally, it is easy to account for the effect of constant current. The turning circle test results may be used to measure the magnitude and direction of current. The vessel's track, heading and the elapsed time should be recorded until at least a 720° change of heading has been completed. The data obtained after the vessel's heading changes 180° are used to estimate magnitude and direction of the current. Position (x_{1i}, y_{1i}, t_{1i}) and (x_{2i}, y_{2i}, t_{2i}) in Appendix 6, Figure 1 are the positions of the vessel measured after a heading rotation of 360°. By defining the local current velocity, \vec{V}_i , for any two corresponding positions:

$$\vec{V}_i = \frac{1}{(t_{2i} - t_{1i})} \cdot \{ (x_{2i} - x_{1i}), (y_{2i} - y_{1i}) \}$$
 (A6.1)

The estimated current velocity can be obtained from the following equation:

$$\vec{V}_c = \frac{1}{n} \sum_{i=1}^n \vec{V}_i = \frac{1}{n} \sum_{i=1}^n \frac{1}{(t_{2i} - t_{1i})} \cdot \{ (x_{2i} - x_{1i}), (y_{2i} - y_{1i}) \}$$
 (A6.2)

If the constant time interval, $\Delta t = (t_{2i} - t_{1i})$, is used, this equation can be simplified and written:

$$\vec{V}_c = \frac{1}{n \cdot \Delta t} \left\{ \left(\sum_{i=1}^n x_{2i} - \sum_{i=1}^n x_{1i} \right), \left(\sum_{i=1}^n y_{2i} - \sum_{i=1}^n y_{1i}, \right) \right\} \dots$$
(A6.3)

The above vector, \vec{V}_c , obtained from a 720° turning test, will also include the effect of wind and waves.

The magnitude of the current velocity and the root mean square of the current velocities can be obtained from the equations:

$$V_c = \left| \vec{V}_c \right| \dots \tag{A6.4}$$

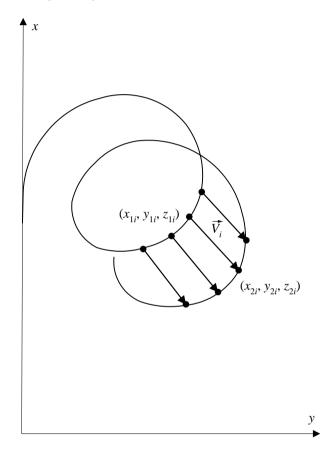
$$V_c(RMS) = \left[\frac{1}{n} \sum_{i=1}^{n} \left| \vec{V}_i - \vec{V}_c \right|^2 \right]^{\frac{1}{2}} ...$$
(A6.5)

 $V_c(RMS)$ represents the non-uniformity of $\vec{V_i}$, which may be induced from wind, waves and non-uniform current. All trajectories obtained from the sea trials should be corrected as follows:

$$\vec{Q}_1(t) = \vec{Q}(t) - \vec{V}_c t$$
 (A6.6)

where $\vec{Q}(t)$ is the measured position radius-vector and $\vec{Q}_1(t)$ is the corrected one of the vessel and $\vec{Q}_1(t) = \vec{Q}(t)$ at t = 0.

FIGURE 1
Correction of Trajectory for Current, Wind and Waves (IMO, 2002b)



Turning trajectory in wind, waves and current



APPENDIX 7 Sample Wheelhouse Poster and Pilot Card

This Appendix contains sample information to be included in the wheelhouse poster. For reading convenience, it is broken into several parts. As this information was produced from a real vessel, numerical values in some of the fields were erased to protect confidentiality. The layout of the wheelhouse poster is shown in Appendix 7, Figure 1.

FIGURE 1
Layout of the Wheelhouse Poster

Title						
Part 1	Part 2		Part 4 Anchor Chain			
Drafts and Propulsion	Ship's Particulars, Steering and Loading Conditions		Dort 2			
Part 5 Turning Circles at Maximum Rudder Angle			Part 3 verboard Rescue Maneuver			
Part 6 Emergency Stopping in Loaded Conditions	Part 8 Stopping Characteristics		Part 7 Emergency Stopping in Ballast Conditions			

Sample information included in each part of the Wheelhouse Poster is given below, except for the Title. Two versions of the pilot card sample are placed after the wheelhouse poster. One is one-sided and the other is double-sided. Both are taken from [Barr, 1990].

PART 1 Drafts and Propulsion

Ship's Name:					
-					
		 			
Call Sign:					

WARNING: The response of the above named Vessel may be different from those listed here if any of the following conditions, upon which the maneuvering information is based, are varied:

- 1) Calm weather-wind 10 Knots or less, calm sea;
- 2) No current;
- 3) Water depth twice the vessel's draft or greater;
- 4) Clean hull; and
- 5) Intermediate drafts or unusual trim

DRAFTS AT WHICH THE MANEUVERING DATA WERE OBTAINED

	Loaded Condition	Ballast Condition
Forward	m	m
Aft	m	m

PROPULSION PARTICULARS

Type of engine kW (hp)
T	pe of propeller FI	KED PITC	H-PROP	ELLER				
Ŀ	Engine order	RPM setting	- I			RPM setting		
	Full sea speed	108	25.00	26.40				
٦	Full	55	13.60	13.80	_	Full		75
ahead	Half	45	11.09	11.20	stern	Half		45
, a	Slow	35	8.44	8.70	B	Slow		35
	Dead slow	23	5.15	5.70		Dead slow	•	23
Cr	itical revolutions				-		NON	IE rpm
Mi	nimum revolution		23 r _l	om 🔷			5.	15 kts
Tit	me limit astern						NON	IE min
Tiı	me limit at minimum re	evolutions					NON	IE min
En	nergency full ahead to	full aster	'n				9 mi	n 20 s
St	Stop to full astern 15 min 0 s							n 0s
As	Astern power 75 % ahead							6 ahead
Ma	aximum number of co	nsecutive	starts				7+	· 7 = 14

PART 2 Ship's Particulars, Steering and Loading Conditions

SHIP'S PARTICULARS

Gross tonnage	
Net tonnage	
Displacement, maximum	t
Deadweight, maximum	t
Block coefficient at summer full load draft	

STEERING PARTICULARS

Type of rudder	SEMI-BALANCE		
Maximum rudder angle	35	0	
Time hard-over to hard-over with one power unit	27	s	
Time hard-over to hard-over with two power units	15	S	
Minimum speed to maintain course, propeller stopped	5	kts	
Rudder angle for neutral effect in loaded conditiond	0.8° sta	rboard	

THRUSTER EFFECT

Thruster	kW (hp)	Time delay for full thrust		Turning rate at zero speed	Time delay to reverse full thrust		Not effective above speed		
Bow	2000		9	s	10°/min	-min	18 s	5 k	ts
Stern				s	°/min	min	s	k	cts
Combined			W.D. II	s	°/min	min	s	k	cts

in Ballast

12 °/min

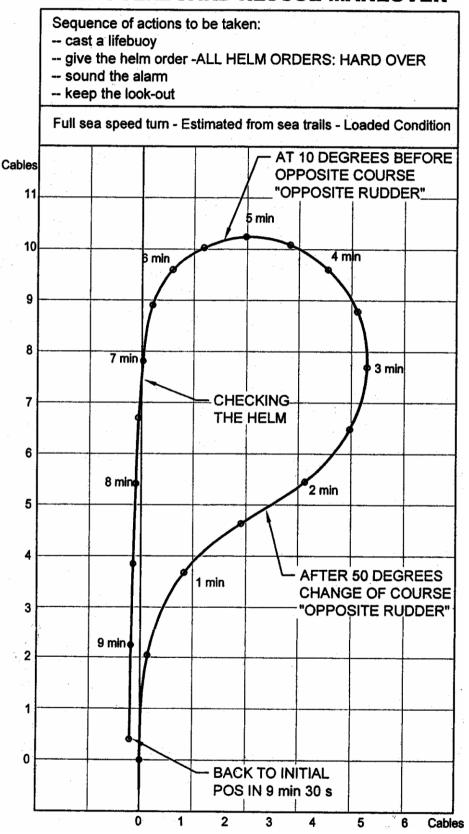
DRAFT INCREASE (Loaded Condition)

Estimated squat effect at 14m draft				
Under keel clearance	Ship's speed	Max. squat at bow / stem		
	13.0 kts	0.680 m		
7.0 m	14.0 kts	0.789 m		
	15.0 kts	0.850 m		
2.8 m	13.0 kts	0.905 m		
	14.0 kts	0.986 m		
	15.0 kts	1.132 m		

Heel effect			
Heel angle	Draft Increase		
2°	0.49	m	
4°	0.97	m	
8°	1.89	m.	
12°	2.73	m	
16°	3.50	m	

PART 3
Man Overboard Rescue Maneuver*

MAN OVERBOARD RESCUE MANEUVER



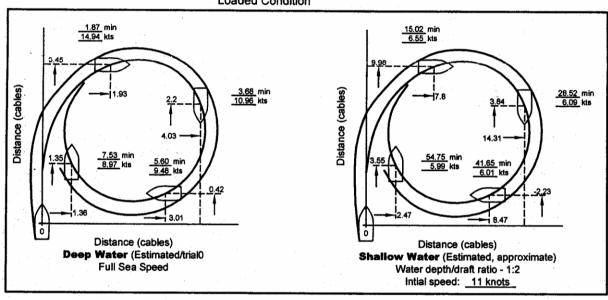
PART 4 Anchor Chain

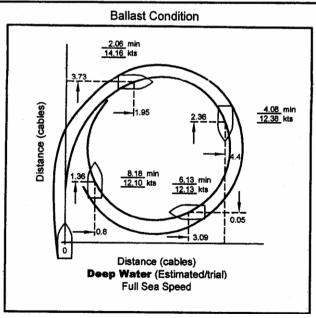
ANCHOR CHAIN

Number of shackles		Max. rate of heaving (min/shackle)	
Port	13	2 min 50 s	
Starboard	14	2 min 52 s	
Stem	-	-	

PART 5
Turning Circles at Maximum Rudder Angle



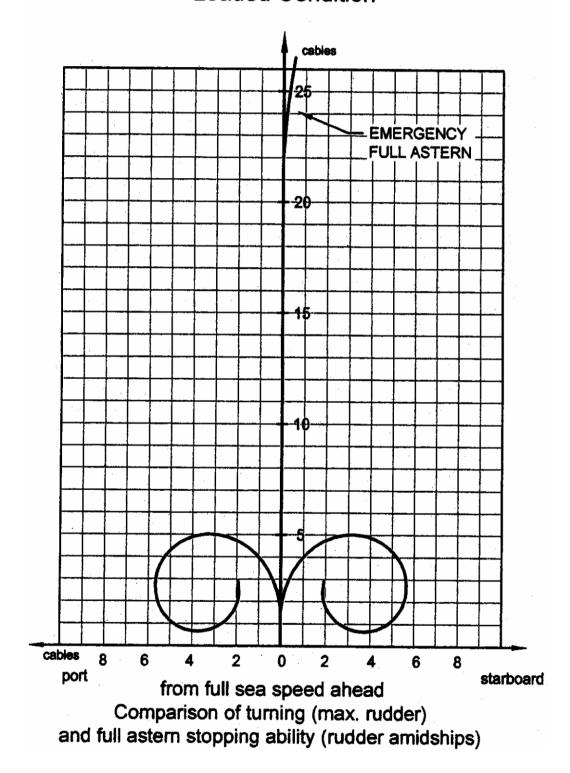




PART 6
Emergency Stopping in Loaded Conditions

EMERGENCY MANEUVERS

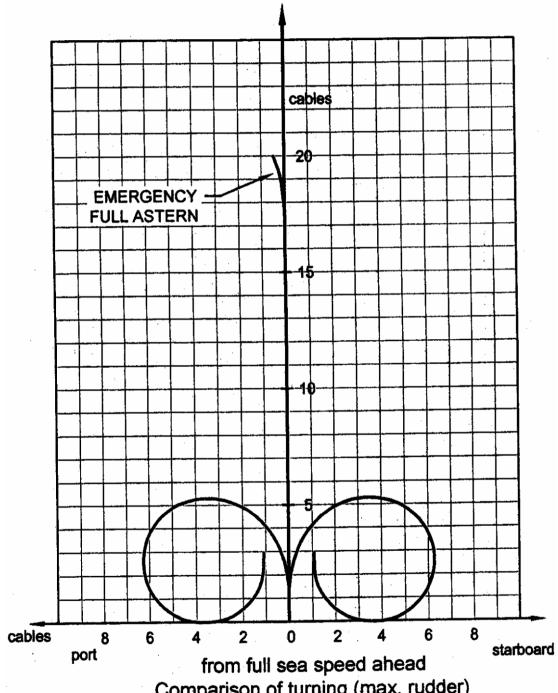
Estimated from Sea Trials
Loaded Condition



PART 7
Emergency Stopping in Ballast Conditions

EMERGENCY MANEUVERS

Estimated from Sea Trials
Ballast Condition

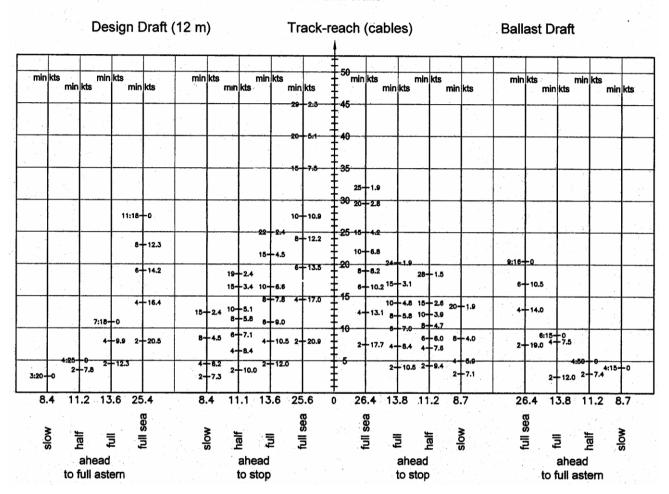


Comparison of turning (max. rudder) and full astern stopping ability (rudder amidships)

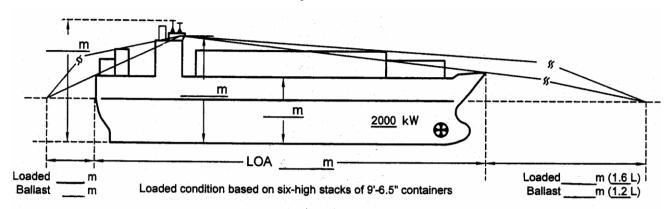
PART 8 Stopping Characteristics

STOPPING CHARACTERISTICS

From Sea Trails



PART 9 Visibility Characteristics



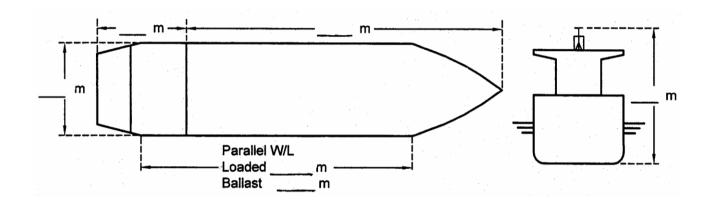


FIGURE 2 Sample of One Sided Pilot Card

PILOT CARD				
Ship's Name Original Name				
Flag C	all sign	Age	nt	
Tonnage: G Deadweight				
Draft Aft	_ m. Dra	aft Forward	m.	
SHIP	'S PAR	TICULAR	s	
Length overall	m.	Breadt	h m.	
Increase of dra	ught, per	degree hee	1: m.	
Thruster Power Available Com	Stern:	kW (_ kW (_ kW (_	HP)	
	lbous bow able Pitc	yes/no h Prop yes	/no	
parallel Part m.				
Type of engine		Manufacturer	·	
Maximum power: Ahead	kW (HP) Astern	_ kW (HP)	
Maneuvering enigne order	RPM/pitch	Speed, Loaded	knots	
Full ahead Half ahead Slow ahead Dead slow ahead				
Dead slow astern			tern sec	
Slow astern Half astern Full astern		Full ahead to full aste	rn sec	
rull astern		Max cont. no. Critical RPM		
Maximum revolutions	available: A	Ahead A	astern	
STEERING CHARACTERISTICS				
Rudder rate °/sec Maximum angle °				
Steady turn diameter ship lengths (deep water)				
STATUS OF SHIP'S EQUIPMENT				
Anchor manned and ready to let go yes/no				
Gyro error to port/starboard				
Problems with any other equipment				

FIGURE 3 Sample of Double Sided Pilot Card

PILOT CARD			
Ship's Name Original Name			
Flag Call sign Agent			
Tonnage: Gross Net Deadweight tonnes			
Draft Aft m. Draft Forward m.			
SHIP'S PARTICULARS			
Length overall m. Breadth m.			
Increase of draught, per degree heel: m.			
Thruster Bow: kW (HP)			
Power Stern: kW (HP)			
Available Combined: kW (HP)			
Bulbous bow yes/no Controllable Pitch Prop yes/no			
m. m. m.			

FIGURE 3 (continued) Sample of Double Sided Pilot Card

Type of engine		Manufacturer	·	
Maximum power: Ahead kW (HP) Astern kW (HP)				
Maneuvering	RPM/pitch	Speed, knots		
enigne order		Loaded	Ballast	
Full ahead				
Half ahead				
Slow ahead Dead slow ahead				
Dead Slow anead				
Dead slow astern		Time limit as	tern sec	
Slow astern		Full ahead to		
Half astern			rn sec	
Full astern		Max cont. no. Critical RPM	starts	
		CIICICAI KFM		
Maximum revolutions	available: A	Ahead A	stern	
CTTTTTN	C CHAD	ACTERIS:	PT CC	
SIEEKIN	G CHAR	ACIEKIS.	TTC5	
Rudder rate	_ °/sec	Maximum and	gle °	
Steady turn dia	meter	ship le	ngths	
(deep water)				
STATUS O	F SHIP	'S EOUI	PMENT	
		~ -		
Anchor manned and ready to let go yes/no				
Gyro error to port/starboard				
Problems with any other equipment				



APPENDIX 8 Human Element (Factor) Considerations

1 General

Vessel maneuverability and human task performance from the bridge interact significantly. A review of accident data suggests that a majority of shipping accidents involving collisions and groundings stem from actions performed on the navigation bridge. Bridge human errors related to situation awareness and information management, often involving misunderstandings related to vessel control and response dynamics, are common.

These instances of human error can be reduced through:

- Bridge Resource Management:
 - Ship and Masters standing orders related to vessel control
 - Crew familiarization with the vessel and its handling, control and response characteristics (especially for oncoming Mates, Masters and Pilots)
 - Vessel handling communications (among bridge and deck crew, pilot and shore facilities)
 - Command authority and leadership
- Ergonomic design of vessel controls (helm, bridge wing controls), displays (rate of turn indicators, helm order indicators, piloting displays, radars), and the general layout of the pilot house and bridge wings
- Procedures (command authority, command handoff, crew familiarization)

In recognition of the importance of these factors to vessel control, ABS has developed the following documentation to assist vessel designers in the application of ergonomic principles to the design, arrangement, and operations of vessels:

- ABS Guidance Notes on the Application of Ergonomics to Marine Systems
- ABS Guidance Notes on Ergonomic Design of Navigation Bridges
- ABS Guide for Crew Habitability on Ships

These documents introduce ergonomics principles and guidance that can be considered during the design, and operation, of a vessel. The ergonomics Guidance Notes can be used to design a vessel's bridge to facilitate bridge crew task performance during vessel control and maneuvering.

These are available, free of charge, from "http://www.eagle.org".

Each is briefly described below.

2 ABS Guidance Notes on the Application of Ergonomics to Marine Systems

The ABS Guidance Notes on the Application of Ergonomics to Marine Systems [ABS 2003b] provides designers with ergonomic principles for the design and layout of equipment and systems and how they interface with crewmembers, the human-system interface. Examples of human-system interfaces include: controls, displays, alarms, video-display units, computer workstations, labels, ladders, stairs and overall workspace arrangement. Without consideration of personnel during design, interfaces may not meet the needs and expectations of personnel and human errors may result.

Ambient environmental conditions in work and living spaces are also important. Conditions that personnel are subjected to onboard vessels and offshore installations will affect their performance, as well as the ability to rest and recover from fatigue. Such conditions include vibration, noise, indoor climate and lighting.

3 ABS Guidance Notes on Ergonomic Design of Navigation Bridges

To help facilitate safety, the design and operation of a vessel's navigation bridge should reflect sound and accepted ergonomic principles. These Guidance Notes [ABS 2003a] provides data regarding:

- General ergonomic design guidance (design principles) for navigation bridges
- Specific bridge design guidance gleaned from international sources, such as the International Maritime Organization (IMO) and the International Association of Classification Societies
- A process to identify individual vessel bridge requirements to guide application of ergonomic design principles

Ergonomic guidance contained in this document includes:

- Bridge arrangement and layout
- Console and workspace design
- Work environment
- Ergonomic design and evaluation process

4 ABS Guide for Crew Habitability on Ships

The objective of designing for crew habitability is to apply criteria or limits that provide the best overall shipboard conditions, given design constraints and budget that support good human performance, mental alertness and basic levels of comfort in order to promote the general well-being of crewmembers. Design emphasis should be paid to the accommodations where the crew live and work, as well as levels of living and working ambient environmental conditions (i.e., whole-body vibration, noise, indoor climate and lighting).

This Guide [ABS 2001] contains accommodations and ambient environmental criteria that can be controlled, measured and assessed in crew accommodations spaces. The evaluation methodologies were written from a sufficiently rigorous yet practical perspective. The criteria are intended to provide vessel Owners and operators with the means to promote enhanced levels of crew habitability. This Guide offers an optional class notation of Habitability (**HAB**) or Habitability Plus (**HAB+**).

5 Bridge Resource Management Practices

To facilitate the safe control of a vessel, the operation and management of information from a vessel's navigational bridge should reflect sound and accepted information management practices. These will lead to enhanced safety and situation awareness. Information management includes:

- The clarity, timeliness and accuracy of communication
- The accuracy of perception of the environment, displays and equipment
- The ability of the crew to identify and understand the safety significance of the information at hand
- Accurate knowledge of the immediate fitness of the vessel and crew to respond in a safe and effective manner



APPENDIX 9 References

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