Guide for

Spectral-Based Fatigue Analysis for Floating Production, Storage and Offloading (FPSO) Installations

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SPECTRAL-BASED FATIGUE ANALYSIS FOR FLOATING PRODUCTION, STORAGE AND OFFLOADING (FPSO) INSTALLATIONS
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Foreword

This *Guide for Spectral-Based Fatigue Analysis for Floating Storage and Offloading (FPSO) Installations*, herein referred to as the "Guide", provides information on the method to perform spectral fatigue analysis for ship-type "Floating Production Installations". This type of offshore installation is usually referred to as a "Floating Storage and Offloading (FSO) System"; or "Floating Production, Storage and Offloading (FPSO) System". FPSO is the term that will be used in this Guide to denote these ship-type Floating Production Installations. Spectral fatigue analysis performed for FPSOs in accordance with the procedures and criteria in this Guide will be identified in the *Record* by the notation **SFA**.

The Rules and Guides for Classification for which this Guide is considered to be most relevant are:

- ABS *Rules for Building and Classing Mobile Offshore Units*
- ABS *Rules for Building and Classing Offshore Installations*
- ABS *Rules for Building and Classing Floating Production Installations*

Additionally, the use of the Guide relies on reference to the ABS *Guide for the Fatigue Assessment of Offshore Structures*. This Guide specifically relates to the latest editions of the above-mentioned Rules and Guides. The use and relevancy of the Guide to other editions of these references, or with other ABS criteria, should be established in consultation with ABS.

This Guide is based on an earlier publication entitled ABS *Guidance Notes on Spectral-Based Fatigue Analysis for Floating Offshore Structures* (March 2005). The present document supersedes the earlier one.

ABS welcomes comments and suggestions for the improvement of the Guide. Comments or suggestions can be sent electronically to *rsd@eagle.org*.

GUIDE FOR

SPECTRAL-BASED FATIGUE ANALYSIS FOR FLOATING PRODUCTION, STORAGE AND OFFLOADING (FPSO) INSTALLATIONS

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SECTION 1
Introduction

1 Background and Applicability

This Guide provides a detailed description of the method to perform spectral fatigue analysis for ship-type offshore installations in order to obtain the optional classification notation **SFA**. Spectral fatigue analysis relies on the presumed linearity of wave-induced loads with respect to waves. This condition is sufficiently satisfied for ship-type offshore installations; such as Floating Production, Storage and Offloading (FPSO) and Floating Storage and Offloading (FSO) systems. For a ship-type hull, linear diffraction forces are the dominant component of wave load. The application of the spectral fatigue analysis method to a ship-type hull is presented in this Guide. For other hull configurations composed of members with relatively large cross sectional dimensions; such as a Tension Leg Platform, Spar, or Column Stabilized units, a spectral approach is often employed; but with appropriate modifications to account for the influence of nonlinear drag forces that tend to become more important as cross sectional member dimensions decrease. This Guide assumes that any required modification to linearize the wave-induced load effects has been satisfactorily accomplished.

This Guide employs basic concepts and terminology that were defined in the ABS *Guide for the Fatigue Assessment of Offshore Structures* (2003). In that reference it is stated that:

"*Fatigue assessment* denotes a process where the *fatigue demand* on a structural element (e.g., a connection detail) is established and compared to the predicted *fatigue strength* of that element. One way to categorize a fatigue assessment technique is to say that it is based on a direct calculation of *fatigue damage* or expected *fatigue life*. Three important methods of assessment are called the *Simplified Method*, the *Spectral Method* and the *Deterministic Method*. Alternatively, an indirect fatigue assessment may be performed by the *Simplified Method*, based on limiting a predicted (probabilistically defined) stress range to be at or below a permissible stress range. There are also assessment techniques that are based on *Time Domain* analysis methods that are especially useful for structural systems that are subjected to nonlinear structural response or nonlinear loading."

In this Guide, the fatigue assessment technique that is presented is a direct calculation method based on the spectral analysis method, which can produce a fatigue assessment result in terms of either expected damage or life. The fatigue strength of structural details is established using the S-N curve approach that is specified in the referenced Guide.

It should be borne in mind that for the hull structure of an Offshore Installation, wave-induced loading is usually the dominant source of fatigue damage. However some types of floating offshore structures may also be subjected to significant fatigue loading from other loading sources. This can be true for hull types that undergo frequent loading and discharge of produced fluids. For example, in FPSO and FSO systems, such load changes can induce large ranges of hull girder stress and secondary stress (albeit at lower cycles than direct wave loads). Such load cycle fatigue is also addressed in this Guide. In addition, fatigue loading may also be induced by the operation of equipment associated with the function of the Offshore

Installation. The extent to which the spectral-based fatigue method can or will be adapted to take into account these "non-wave" sources of fatigue damage must be further considered by the designer.

3 FPSO Areas for Fatigue Assessment

There are two general categories of FPSO structural details for which fatigue assessments are required. The first type relates to conventional tanker hull details and is indicated in 1/3.1. For some of these details, in addition to wave loads, low cycle produced fluid (cargo) loading and offloading induced loads should be considered in the fatigue assessment. The second type of details is specific to an FPSO as indicated in 1/3.3. For some of the latter type, other kinds of loads, (e.g., low-frequency loads or operational dynamic loads) should be included in the fatigue assessments.

3.1 Hull Structure

General guidance on areas of the hull where fatigue assessment should be performed is as follows:

3.1.1 Connections of Longitudinal Stiffeners to Transverse Web/Floor and to Transverse Bulkhead

3.1.1(a) 2 to 3 selected side longitudinal stiffeners in the region from 1.1 maximum draft to about 0.33 maximum draft in the midship region and also in the region between 0.15*L* and 0.25*L* from F.P., respectively.

3.1.1(b) 1 to 2 stiffeners selected from each of the following groups:

- Deck longitudinals, bottom longitudinals, inner bottom longitudinals and longitudinals on side longitudinal bulkheads.
- One longitudinal on the longitudinal bulkheads within 0.1*D* from the deck is to be included.

For these structural details, the fatigue assessment is to be first focused on the flange of the longitudinal stiffener at the rounded toe welds of attached flat bar stiffeners and brackets.

Then, the critical spots on the web plate cut-out, on the lower end of the stiffener, as well as the weld throat, are also to be checked for the selected structural detail.

Where the stiffener end bracket arrangements on two sides of a transverse web are different, both configurations are to be checked.

3.1.2 Shell, Bottom, Inner Bottom or Bulkhead Plating at Connections to Webs or Floors (for Fatigue Assessment of Plating)

3.1.2(a) 1 to 2 selected locations of side shell plating near the summer LWL amidships and between 0.15*L* and 0.25*L* from F.P. respectively.

3.1.2(b) 1 to 2 selected locations in way of bottom and inner bottom amidships.

3.1.2(c) 1 to 2 selected locations of lower strakes of side longitudinal bulkhead amidships.

3.1.3 Connections of the Slope Plate to Inner Bottom and Side Longitudinal Bulkhead Plating at the Lower Cargo Tank Corners

One selected location amidships at transverse web and between webs, respectively.

3.1.4 End Bracket Connections for Transverses and Girders

1 to 2 selected locations in the midship region for each type of bracket configuration

3.1.5 Other Regions and Locations

Other regions and locations, highly stressed by fluctuating loads, as identified from structural analysis

3.3 FPSO-Specific Structural Areas

The adequacy of the following FPSO-specific areas for fatigue should be suitably demonstrated:

- Position mooring/hull interface, if spread moored (5A-1-4/3 of the ABS *Rules for Building and Classing Floating Production Installations (FPI Rules)*)
- Turret and its interface with hull, if turret moored (6/17.7 and 6/17.9 of the ABS *Guide for Position Mooring Systems*)
- Riser porches
- The details, below and on the deck of the hull, comprising the supports of the topside structures. (The interface details between the hull structure and equipment skids and support frames deserve particular attention.)
- Additional areas, as applicable, including: flare tower foundation, crane pedestals, helideck to deck connections and deck penetrations.

5 Tanker Conversion

When an FPSO is converted from a trading tanker, the fatigue damage accumulated during the "trading tanker" phase is to be deducted when establishing the remaining fatigue life for future service as an FPSO.

When calculating the fatigue damage for past services, the wave conditions of specific routes the vessel has experienced in past service can be employed, instead of using the wave condition representing unrestricted service as may have been done for classification as a tanker.

When calculating the fatigue damage accumulated during the "trading tanker" phase, the effects of vessel speed (encounter frequency) should be included (i.e., in the evaluation of stress RAOs and the number of stress cycles).

7 General Comments about the Spectral-based Method

Spectral-based Fatigue Analysis is a complex and numerically intensive technique. As such, there is more than one variant of the method that can be validly applied in a particular case. This Guide is not intended to preclude the use of any valid variant of a Spectral-based Fatigue Analysis method by "over specifying" the elements of an approach. However, there is a need to be clear about the basic minimum assumptions that are to be the basis of the method employed and some of the key details that are to be incorporated in the method to produce results that will be acceptable to ABS. For this reason, most of the remainder of this Guide addresses these topics.

The main assumptions underlying the Spectral-based Fatigue Analysis method are listed below:

i) Ocean waves are the main source of the fatigue-inducing loads acting on the structural system being analyzed. The fatigue damage from other loading sources can be considered separately.

ii) In order for the frequency domain formulation and the associated probabilistically-based analysis to be valid, load analysis and the associated structural analysis are assumed to be linear. Hence, scaling and superposition of stress transfer functions from unit amplitude waves are considered valid.

iii) Nonlinearities, brought about by nonlinear roll motions and intermittent application of loads such as wetting of the side shell in the splash zone, are treated by correction factors.

iv) Structural dynamic amplification, transient loads and effects such as springing are insignificant for a typical FPSO hull structure, and hence, use of quasi-static finite element analysis is valid, and the fatigue inducing stress variations due to these types of load effects can be ignored.

Also, for the particular method presented in Appendix A3, it is assumed that the short-term stress variation in a given sea-state is a random narrow-banded stationary process. Therefore, the short-term distribution of stress range can be represented by a Rayleigh distribution.

The key components of the Spectral-based Fatigue Analysis method for the selected structural locations can be categorized into the following components:

- Establish Fatigue Demand
- Determine Fatigue Strength or Capacity
- Calculate Fatigue Damage or Expected Life

These analysis components can be expanded into additional topics, as follows, which become the subject of particular Sections in the remainder of this Guide.

The topic, "Establish Fatigue Demand" is covered in Sections 2 through 7 and part of Appendix A3. The topics "Determine Fatigue Strength or Capacity" and "Calculate Fatigue Damage or Expected Life" are covered in Sections 8 and 9, respectively. Reference can be made to 1/9 FIGURE 1 for a schematic representation of the Spectral-based Fatigue Analysis Procedure.

A purposeful effort is made in this Guide to avoid complicated formulations, which will detract from the concepts being presented. The most complex formulations are those relating to the calculation of fatigue damage resulting from the predicted stress range. These formulations are presented in Appendix A3. It is often at this formulation level that valid variations of a method may be introduced, and for that reason, it is emphasized that the contents of Appendix A3 are provided primarily to illustrate principle, rather than as mandatory parts of the Spectral-based Fatigue method.

9 Data to be submitted

The submittals for review should include:

i) A contract which clearly defines owner's specification other than standard requirement, or other critical information

ii) Site metocean data

iii) Principal Dimensions, Lines and Trim/Stability booklet

iv) Key drawings (General Arrangement, Midship section, Shell expansion, Construction profile and Deck plan)

v) More drawings for Forebody/Aftbody, Typical bulkhead and Engine room

vi) DLA analysis report detailing findings and identifying any inconsistencies, assumptions, and corrective actions

vii) Seakeeping input/output files including DLPs' RAO and their extreme values

viii) Structural FE model and its analysis results

and

ix) SFA Procedure document

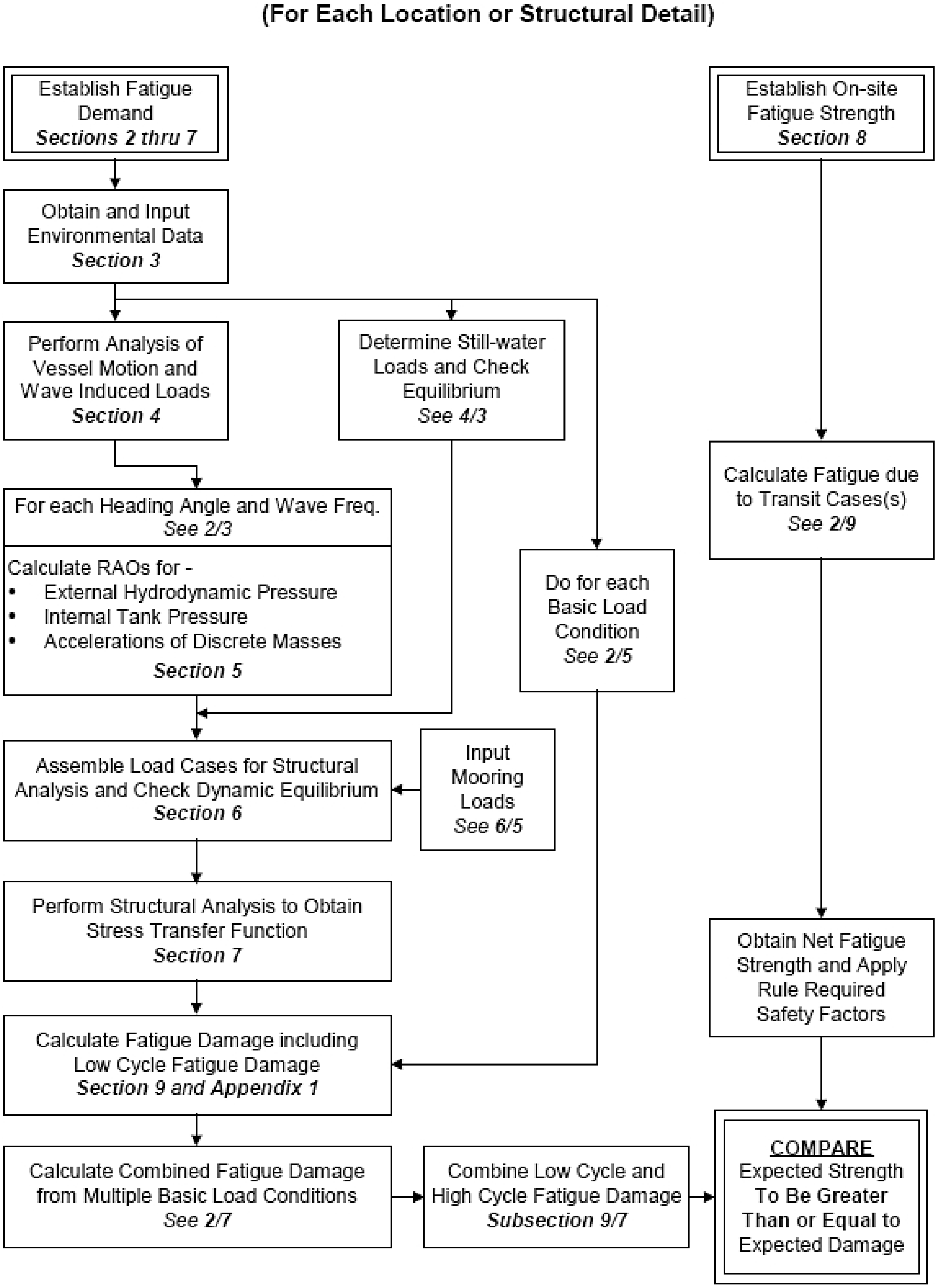
x) Site metocean data, motion analysis, RAO's etc.

xi) Fatigue screening report to indicate details to be checked and those to be omitted, if screening analysis is performed

xii) S-N curve selection for analyzed details

xiii) SFA report detailing findings and identifying any inconsistencies, assumptions, and corrective actions

FIGURE 1
Schematic Spectral-based Fatigue Analysis Procedure



SECTION 2

Establishing Fatigue Demand

1 Introduction

Sections 2 through 7 address the procedures used to estimate the stress transfer functions or stress RAOs at a structural location that is the object of the fatigue assessment.

3 Stress Transfer Function

With ocean waves considered the main source of fatigue demand, the fundamental task of a spectral fatigue analysis is the determination of the stress transfer function, $H_\sigma(\omega|\theta)$, which expresses the relationship between the stress at a particular structural location and the unit amplitude wave of wave frequency (ω) and wave heading (θ).

It is preferred that a structural analysis be carried out at each frequency, heading angle, "Basic Loading Condition" (see Subsection 2/5) and vessel speed, if applicable, employed in the spectral analysis, and that the resulting stresses are used to directly generate the stress transfer function.

Normally, the frequency range to be used is 0.2 to 1.80 radians/second, in increments not larger than 0.1 rad/s. However, depending on the characteristics of the response, it may be necessary to consider a different frequency range. The wave heading range is 0 to 360 degrees, in increments not larger than 30 degrees.

Note:

The pertinent frequency range and increment applicable to other types of floating offshore installations may be different.

5 Basic Loading Conditions

The Basic Loading Conditions relate to the probable variations in loading that the hull structure of the FPSO will experience during its on-site service life and for the transit case(s). The main parameters defining a Basic Loading Condition are tank loading/ballast arrangements, hull draft and trim, and significant variations in topside equipment loads. These parameters have a direct influence on the "static" stress components of the hull's response, but they also affect the wave-induced cyclic stress experienced at a structural location. There are two direct ways that this influence is felt. First, this influence is felt in the magnitudes and distributions of masses and restoring forces in the determination of global and local accelerations and rigid body displacements, which in turn affect the wave-induced load effects employed in the structural analysis. Secondly, the variation of draft affects the areas of the hull that will be subjected to direct external pressures, and the magnitude and distribution of these pressures.

7 Combined Fatigue from Multiple Basic Loading Conditions

Because of the variability in Basic Loading Conditions and its effects on the fatigue damage predictions, it is necessary to consider more than one basic case in the fatigue analysis. As a minimum for the analysis of post-installation on-site conditions, two cases are to be modeled and used in the Spectral-based Fatigue Analysis process. The two required cases are ones resulting from, and representing, the probable deepest and shallowest drafts, respectively, that the installation is expected to experience during its on-site service life.

Note:

Suggested Approach: In some (so-called "Closed Form") formulations to calculate fatigue demand, the fraction of the total time on-site for each Basic Loading Condition is used directly. In this case, potentially useful information about the separate fatigue damage from each basic loading condition is not obtained. Therefore, it is suggested that the fatigue damage from each basic loading condition be calculated separately. Then the "combined fatigue life" is calculated as a weighted average of the reciprocals of the lives resulting from considering each case separately. For example, if two basic loading conditions are employed, and the calculated fatigue life for a structural location due to the respective basic loading conditions are denoted L_1 and L_2, and it is assumed that each case is experienced for one-half of the FPSO's on-site service life, then the combined fatigue life, L_C is:

$$L_C = 1/[0.5(1/L_1) + 0.5(1/L_2)]$$

As a further example, if there were three basic loading conditions L_1, L_2, L_3 with exposure time factors of 40, 40 and 20 percent, respectively; then the combined fatigue life, L_C is:

$$L_C = 1/[0.4(1/L_1) + 0.4(1/L_2) + 0.2(1/L_3)]$$

9 Transit Cases

The fatigue demand arising from anticipated FPSO transit cases (usually only the FPSO voyage to the installation site) is to be determined.

During assessment of the fatigue damage accumulated during transit cases, the effects of vessel speed are to be included in the evaluation of stress RAOs and the number of stress cycles.

Note:

As in the previous Note, it is suggested that the fatigue demand produced by the transit case(s) be calculated separately.

The extent to which fatigue analysis is to be performed for the transit cases associated with an FPSO with a classification notation **Disconnectable** (see 1-1-2/5.1 of the *FPI Rules*) will be specially considered.

SECTION 3
Environmental Conditions

1 General

Fatigue damage of a structure is caused by fluctuating loads that occur during the structure's service life. For offshore structures, the most dominant source of fluctuating load is waves. However, in some particular cases other sources, such as vortex induced vibrations (VIV), wind, current and operational loads may become significant.

A structure will experience various fluctuating loads during its service life. To describe these sources adequately is the first, crucial step in the fatigue assessment of the structure. Obviously, it is impossible to predict or describe with certainty the expected environmental conditions the structure will experience during its service life. However, it is possible to define a series of conditions and establish statistically the probability of each condition happening to the structure during its life. A fatigue analysis can then be performed based on this kind of statistical description of environmental conditions.

3 Waves

During the service life of an FPSO, it will experience a huge number of waves, from very small wavelets to possibly giant waves. A practical way to describe these unceasingly changing waves is to divide them into various categories (sea states), and use short-term wave statistics to depict each sea state and long-term wave statistics, usually in the form of a wave scatter diagram and rosette, to delineate the rate at which a sea state occurs.

In a similar way, there are two levels in the description of wave directionality (i.e., wave directional spectrum or wave spreading for short-term, and wave rosette for long-term, respectively).

3.1 Wave Spectra (Short-term Wave Statistics)

3.1.1 Unidirectional Spectra

A wave spectrum describes the energy distribution among wave components of different frequencies of a sea state. Wave spectra can be obtained directly from measured data. However, various mathematical formulae of wave spectra have been available based on analysis of measured data, such as ISSC Wave Spectrum, Bretschneider Spectrum (or Pierson-Moskowitz (P-M) spectrum), JONSWAP spectrum and Ochi's six-parameter spectrum. These spectrum formulae are suitable for different sea states.

A fully-developed sea is a sea state that will not change if wind duration or fetch is further increased (for a fixed wind speed). The Bretschneider spectrum is applicable to fully-developed seas. For most of the ships and offshore structures in ABS's classification, either the Bretschneider spectrum for open ocean areas with fully-developed seas, or the JONSWAP spectrum for fetch-limited regions is used, respectively. For example, the Bretschneider wave spectrum is usually employed to describe tropical storm waves, such as those generated by hurricanes in the Gulf of

Mexico or typhoons in the South China Sea. The JONSWAP wave spectrum is used to describe winter storm waves of the North Sea. In some cases, it can also be adjusted to represent waves in Offshore Eastern Canada and swells, such as those in West Africa and Offshore Brazil. A suitable wave spectrum should be chosen based on a partially or fully developed sea state for fatigue strength assessment. In general, the Bretschneider spectrum has a greater frequency bandwidth than the JONSWAP spectrum. Therefore, the selection of a spectrum should be based on the frequency characteristics of the wave environment.

The above-described two spectra are single-modal spectra, which are usually used to represent pure wind waves or swell-only cases. When wind waves co-exist with swells (i.e., there are multi-modes in the spectrum), no single-modal spectrum can match the spectral shape very well. In this case, recourse can be made to the use of the Ochi-Hubble 6-Parameter Spectrum.

3.1.2 Directional Spectra (Wave Spreading)

There is a simple case where the observed wave pattern at a fixed point neglects different directions of wave components. This is equivalent to assuming that all wave components travel in the same direction. These waves are called 'long-crested' since the wave motion is two-dimensional and the wave crests are parallel. Waves produced by swell are almost long-crested in many situations since the crests of the wave system observed outside the storm area (beyond the fetch area) which produced them become nearly parallel as the observation point recedes from the storm area.

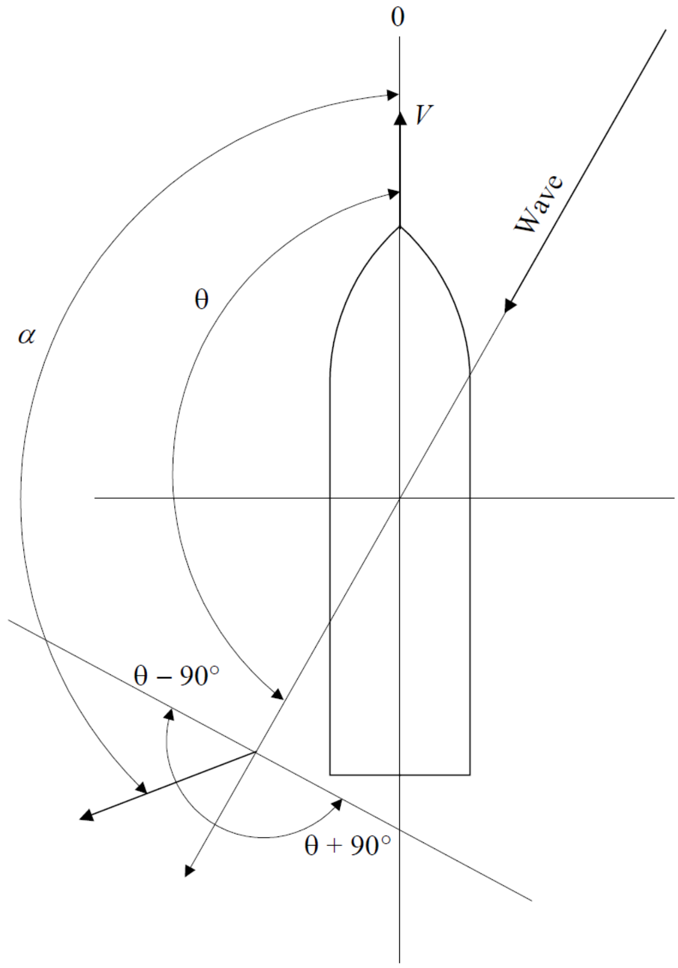
The waves in the ocean are more likely to travel in many directions; therefore, the combined wave system will be short-crested. The spreading of wave directions should be taken into account to describe the short-crestedness.

The wave energy spectrum can be obtained by integrating the spreading wave spectrum over the range of directions from $-\theta_{max}$ to $+\theta_{max}$ (θ_{max} can be typically taken as 90°). The general expression for wave spreading is given by

$$S_\eta(\omega) = \int_{-\theta_{max}}^{\theta_{max}} S_\eta(\omega, \theta)\, \mathrm{d}\,(\alpha - \theta)$$

where α denotes the predominant wave direction and θ is the wave spreading angle, as shown in Section 3, Figure 1.

FIGURE 1
Definition of Spreading Angles



In general, the cosine spreading function for the wave spectrum can be used as:

$$S_\eta(\omega, \theta) = D\cos^n\left[\frac{\pi}{2\theta_{max}}(\alpha - \theta)\right]S_\eta(\omega)$$

where D is a normalizing constant that ensures that the spreading function $G(\omega,\alpha - \theta)$ integrates to 1 and n is the wave spreading parameter, which is a positive integer.

3.3 Wave Spectral Formulation

The shape of a spectrum supplies useful information about the characteristics of the ocean wave system to which it corresponds. There exist many wave spectral formulations (e.g., Bretschneider spectrum, Pierson-Moskowitz spectrum, ISSC spectrum, ITTC spectrum, JONSWAP spectrum, Ochi-Hubble 6-parameter spectrum, etc.).

The Bretschneider spectrum or two-parameter Pierson-Moskowitz spectrum is the spectrum recommended for open-ocean wave conditions (e.g., the Atlantic Ocean).

$$S_\eta(\omega) = \frac{5H_S^2\omega_p^4}{16\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \quad \text{in m}^2/(\text{rad/s})(\text{ft}^2/(\text{rad/s}))$$

or

$$S_\eta(\omega) = \frac{H_S^2}{4\pi\omega^5}\left(\frac{2\pi}{T_Z}\right)^4 \exp\left[-\frac{1}{\pi}\left(\frac{2\pi}{T_Z}\right)^4 \omega^{-4}\right] \quad \text{in m}^2/(\text{rad/s})(\text{ft}^2/(\text{rad/s}))$$

where

ω_p	=	modal (peak) frequency corresponding to the highest peak of the spectrum , in rad/s
H_s	=	significant wave height, in m (ft)
ω	=	circular frequency of the wave, in rad/s
T_z	=	average zero up-crossing period of the wave, in seconds

The JONSWAP spectrum is derived from the Joint North Sea Wave Project (JONSWAP) and constitutes a modification to the Pierson-Moskowitz spectrum to account for the regions that have geographical boundaries that limit the fetch in the wave generating area (e.g., the North Sea).

$$S_\eta(\omega) = \frac{5H_S^2\omega_p^4}{16\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^\alpha(1-0.287\ln\gamma) \quad \text{in} \quad \text{m}^2/(\text{rad/s})\left(\text{ft}^2/(\text{rad/s})\right)$$

where

α	=	$\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]$
σ	=	$\begin{cases} 0.07 & \text{when} \quad \omega \le \omega_p \\ 0.09 & \text{when} \quad \omega > \omega_p \end{cases}$
ω	=	circular frequency of the wave, in rad/s
γ	=	peakedness parameter, typically 1 ~ 7,

Here, the factor (1 – 0.287 ln γ) limits its practical application, because for γ = 32.6, the spectral value from above formula becomes zero. For a peakedness larger than 7, it is recommended that an adjustment to the formula has to be made. The formula of the JONSWAP spectrum can be then given by:

$$S_\eta = \frac{\alpha g^2}{\omega^5}\exp\left[-1.25\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^a \quad \text{in} \quad \text{m}^2/(\text{rad/s})\left(\text{ft}^2/(\text{rad/s})\right)$$

where

γ	=	peakedness parameter, representing the ratio of the maximum spectral density to that of the corresponding Pierson-Moskowitz spectrum. This means that for γ = 1 the JONSWAP spectrum defaults to the Pierson-Moskowitz spectrum
g	=	gravitational acceleration = 9.8 m/s^2 (32.2 ft/s^2)
α	=	parameter to be determined as a function of the significant wave height, through the expression provided in the formula of H_s below, since the integral is a function of α
H_s	=	$4\sqrt{\int_0^\infty S_\eta(\omega)d\omega}$

The Ochi-Hubble 6-Parameter spectrum covers shapes of wave spectra associated with the growth and decay of a storm, including swells. As may be seen in some wave records, the variability in the form of spectra can be great. Multi-modal spectra are common, and a single-modal Bretschneider form may not match the shape of such spectrum in an accurate manner. In order to cover a variety of shapes of wave spectra associated with the growth and decay of a storm, including the existence of swell, the following 6-parameter spectrum was developed by Ochi and Hubble:

$$S_\eta(\omega) = \frac{1}{4}\sum_{j=1}^{2} \frac{\left(\frac{4\lambda_j+1}{4}\omega_{pj}^4\right)^{\lambda_j}}{\Gamma(\lambda_j)} \times \frac{H_{sj}^2}{\omega^{4\lambda_j+1}} e^{-\frac{4\lambda_j+1}{4}\left(\omega_{pj}/\omega\right)^4} \quad \text{in} \quad \mathrm{m}^2/(\mathrm{rad/s})\left(\mathrm{ft}^2/(\mathrm{rad/s})\right)$$

where j = 1, 2 stands for lower (swell part) and higher (wind seas part) frequency components. The six parameters, H_{s1}, H_{s2}, ω_{p1}, ω_{p2}, λ_1, λ_2, are determined numerically to minimize the difference between theoretical and observed spectra.

The design sea state may come from intensification of the local wind seas (waves) and/or swell propagating with different directions. In general, both are statistically independent. The wind seas are often characterized with the Bretschneider or the JONSWAP spectrum while the Gaussian distribution function can be used to describe swells. The spectral formulation for the swell can be represented by the Gaussian-Swell spectrum:

$$S_\eta(\omega) = \frac{(H_s/4)^2}{2\pi\delta\sqrt{2\pi}}\exp\left[-\frac{\left(\omega-\omega_p\right)^2}{2(2\pi\delta)^2}\right] \quad \text{in m}^2/(\mathrm{rad/s})(\mathrm{ft}^2/(\mathrm{rad/s}))$$

where

H_s	=	significant wave height, in m (ft)
δ	=	peakedness parameter for Gaussian spectral width

3.5 Wave Scatter Diagram and Rosette (Long-term Wave Statistics)

Long-term descriptions of the wave environment in the form of “wave scatter diagram” and “rosette” are required for long-term statistical analysis of structural response, such as the predictions of extreme response and the fatigue assessment.

3.5.1 Wave Scatter Diagram

A wave scatter diagram consists of a table of the probabilities of occurrence of various “sea states”. Each cell in the table contains information on three data items, namely (1) the significant wave height, H_s, (2) the characteristic wave period, T, and (3) the fraction of the total time or probability of occurrence for the sea state defined by spectrum with parameters H_s and T. The characteristic wave period usually can be given as peak period, average period or zero up-crossing periods. Attention should be paid to which characteristic wave period is specified in a wave scatter diagram so that it will be consistent with the wave period in the wave spectrum formulation.

3.5.2 Wave Rosette

A wave rosette (also called long-term wave directionality) describes the probability of each heading angle (the main wave direction) at a site. Directional convention should be noticed in using the rosette (e.g., for NOAA wave data, index 1 represents wave coming from true north and as the index increases, the wave direction changes clockwise). Directionality has significant effects on structural response. It is recommended that a realistic wave rosette be used in the fatigue analysis. In case the wave rosette is not available, it is reasonable to assume equal probability of all heading angles in open ocean conditions. However, for a moored offshore structure, the waves may have strong directional characteristics that should be accounted for.

5 Currents

Current in the ocean can be any one or combination of wind-driven, thermohaline, tidal and storm surge currents. At a location (installation site), the current is defined by its speed and directional profiles through the water depth. Currents may change with time, from hourly to seasonally.

Although current itself can be treated as producing essentially static loads, it can induce or intensify certain kinds of dynamic loads and fatigue damage (especially for slender structures such as mooring lines and risers). There are mainly two ways current can affect fatigue: 1) the presence of a current can increase cyclic drag loading of waves due to the nonlinear coupling of current velocity and wave orbital velocity. It is recommended that current be considered if its magnitude is comparable with the wave orbital velocity for those waves that make the greatest contributions to the fatigue damage; 2) Current may also create cyclic “lifting” loads due to vortex shedding, which can cause significant fatigue damage.

7 Wind

Another source of fatigue-inducing loads is wind. Wind produces low-frequency drift motions, which can produce low-frequency fatigue effects in mooring lines and risers. These fatigue-inducing effects of wind can be included in the fatigue assessment as described in 4/5.5. Also wind gusts, vortex shedding and other dynamic wind effects can create significant fatigue damage, especially to superstructure components. However, these wind gust and dynamic effects on fatigue are not typically considered in classification.

SECTION 4

Motion Analysis and Wave-induced Loads

1 General

This Section gives general criteria on the parameters to be obtained from the vessel motion analysis and the calculation of wave-induced load effects. In the context of a Spectral-based Fatigue Analysis, the main objective of motion and load calculations is the determination of *Response Amplitude Operators* (RAOs), which are mathematical representations of the vessel responses and load effects to unit amplitude sinusoidal waves. The motion and load effects RAOs are to be calculated for ranges of wave frequencies and wave headings, as indicated in Subsection 2/3.

Aside from vessel motions, the other wave-induced load effects that need to be considered in the Spectral-based Fatigue Analysis of an FPSO are the external wave pressures, internal tank pressures due to tank fluid accelerations, inertial forces on the masses of structural components and significant items of equipment and mooring loads. Additionally, there may be situations where partial models of the FPSO's structural system are used. In such a case, hull girder shear forces and bending moments are to be determined to appropriately represent the boundary conditions at the ends of the partial models.

Note:

Fatigue damage due to the sloshing of fluid in partially-filled tanks is not within the scope of the **SFA** classification notation. However, the designer is encouraged to perform and submit such calculations if deemed important for the FPSO, as decided by the designer or by ABS.

3 Still-water Loads

The motion and load calculations are to be performed with respect to static initial conditions representing the vessel geometry and loadings, (see Subsection 2/5). With the input of hull loadings, the hull girder shear force and bending moment distributions in still water are to be computed at a sufficient number of transverse sections along the hull's length, in order to accurately take into account discontinuities in the weight distribution. A recognized hydrostatic analysis program is to be used to perform these calculations. By iteration, the convergence of the displacement, Longitudinal Center of Gravity (LCG) and trim should be checked to meet the following tolerances:

Displacement:	±1%
Trim:	±0.5 degrees
Draft:	
Forward	±1 cm (0.4 in.)
Mean	±1 cm (0.4 in.)
Aft	±1 cm (0.4 in.)
LCG:	±0.1% of length

SWBM:	±5%
SWSF:	±5%

Additionally, the longitudinal locations of the maximum and the minimum still-water bending moments and, if appropriate, that of zero SWBM should be checked to assure proper distribution of the SWBM along the vessel's length.

5 Essential Features of Motion and Wave Load

5.1 General Modeling Considerations

The representation of the hull should include the masses of the topside equipment and their supporting structure. The model should also consider the interaction with the mooring system, and as appropriate, the effects of risers, the effects of the Dynamic Positioning system and the operations of offloading or support vessels. The motion analysis should appropriately consider the effect of shallow water on vessel motions. There should also be sufficient compatibility between the hydrodynamic and structural models so that the application of fluid pressures onto the finite element mesh of the structural model can be done appropriately.

For the load component types and structural responses of primary interest, analysis software formulations derived from linear idealizations are deemed to be sufficient. However, the use of enhanced bases for the analysis, especially to incorporate nonlinear loads (for example hull slamming) is encouraged, if this proves to be necessary for the specific design being evaluated. The adequacy of the employed calculation methods and tools is to be demonstrated to the satisfaction of ABS.

5.3 Diffraction-Radiation Methods

Computations of the wave-induced motions and loads should be carried out using appropriate, proven methods. Preference should be given to the application of seakeeping analysis codes utilizing three-dimensional potential flow-based diffraction-radiation theory. All six degrees-of-freedom rigid-body motions of the vessel should be accounted for, and effects of water depth should be considered. These codes, based on linear wave and motion amplitude assumptions, make use of boundary element methods with constant or higher order sink-source panels over the entire wetted surface of the hull on which the hydrodynamic pressures are computed.

5.5 Low Frequency Motions

The low frequency motions of a floating structure are induced by higher-order wave forces and wind loads. The low frequency drift motion of an FPSO is an example of this kind of motion. Although low frequency motions usually have negligible effects on most structural details of the hull, they become significant, and may even dominate the fatigue of structural components such as the mooring system, risers and their interfaces with the hull.

The low frequency motions of a floating structure are to be analyzed for each sea state using a recognized hydrodynamic analysis program. All motion constraints imposed by, for instance, mooring lines, risers, control umbilical and positioning thrusters, should be considered. Although the low-frequency motions may be calculated in either time domain or frequency domain, the time domain simulations are recommended in the sense that the frequency domain method cannot account for the nonlinearity and time-varying drift force.

There are typically two approaches, which may be employed when the time domain method is used to evaluate the low frequency motions.

i) *Approach I.* The calculations of wave-frequency and low-frequency motions are completely decoupled. The time domain global performance analysis is carried out over the drift frequency range only and the wave frequency responses are filtered out. The overall motions are obtained by combining the low-frequency motions with the wave-frequency motions, which are calculated using diffraction-radiation methods (RAOs of the motions) for the given sea states. A relatively large time step may be used in the time domain analysis. Subsection 9/3 discusses the combination of wave-frequency and low-frequency responses for the fatigue damage calculation.

ii) *Approach II.* Another approach, which is theoretically and numerically more rigorous, is to perform a time domain simulation including both wave-frequency and low-frequency motions simultaneously. The rainflow counting method is used to obtain the number of cycles for each stress range. The disadvantage of this approach, however, is that a much smaller time step should be used to achieve the sufficient accuracy.

SECTION 5

Wave-induced Load Components

1 General

Wave-induced loads on a buoyant structure are complicated because, in addition to producing direct forces (e.g., wave pressures on the external surface of the hull), there are indirect force components produced by the rigid body motions of the vessel. The motions result in inertial forces and rotational components of the (quasi-statically considered) loads. These two motion-related load components are referred to below as the "inertial" and "quasi-static" load components. For a moored, buoyant structure such as an FPSO, added complexity arises from the mooring system, which produces reaction load components.

The treatment of the various load and motion effects is typically done through the use of their real and imaginary parts that are employed separately in structural analyses. In a physical sense, the real and imaginary parts correspond to two wave systems that are 90 degrees out of phase relative to each other.

The following Subsections list the primary wave-induced load components that are to be considered in the Spectral-based Fatigue Analysis of an FPSO. Using the methods and calculation tools that are described in Section 4, the Response Amplitude Operators (RAOs) for the listed components are obtained.

3 External Pressure Component

3.1 Total Hydrodynamic Pressures

The total hydrodynamic pressure includes the direct pressure components due to waves and the components due to hull motions. The components of the hydrodynamic pressure should be determined from the model and calculation procedure mentioned in Section 4.

3.3 Intermittent Wetting

Ship motion analysis based on linear theory will not predict the nonlinear effects near the mean waterline due to intermittent wetting. In actual service, this phenomenon is manifested by a reduction in the number of fatigue cracks at side shell plating stiffeners located near the waterline compared to those about four (4) or five (5) bays below. To take into account the pressure reduction near the mean waterline due to this nonlinearity, the following reduction factor can be used:

$RF = 0.5[1.0 + \tanh(0.35d)]$ for d in meters

$RF = 0.5[1.0 + \tanh(0.11d)]$ for d in feet

where d is the depth, in meters (feet), of the field point below the still-water waterline.

Note:

In order to correctly implement the intermittent wetting effects, the size of hydrodynamic panel of side shell near waterline should be appropriately modeled with consideration of longitudinal spacing. It is recommended that the size of panel be no greater than two times of side longitudinal spacing in the vertical direction.

3.5 Pressure Distribution on Finite Element Models

The pressure distribution over a hydrodynamic panel model may be too coarse to be used directly in the structural FEM analysis. Therefore, as needed, the pressure distribution is to be interpolated (3-D linear interpolation) over the finer structural mesh.

5 Internal Tank Pressure Component

As stated in Subsection 5/1, the vessel motion-related internal tank pressure is composed of quasi-static and inertial components. The quasi-static component results from gravity for the instantaneous roll and pitch of the vessel. The inertial component is due to the acceleration of the fluid caused by vessel motion in six degrees of freedom. The vessel motion should be obtained from analysis performed in accordance with Section 4.

The internal tank pressure (quasi-static + inertial) for each of the tank boundary points is calculated by the following:

$$p \quad = \quad p_o + \rho h_i[(g + a_V)^2 + (g_T + a_T)^2 + (g_L + a_L)^2]^{1/2}$$

where

p	=	internal tank pressure at a tank boundary point
p_o	=	either the vapor pressure or the pressure setting on pressure/vacuum relief valve
ρ	=	liquid density, cargo or ballast
h_i	=	internal pressure head defined by the height of projected liquid column in the direction of a resultant acceleration vector. For a completely filled tank, the pressure head is to be measured from the highest point of the tank to the load point (see Section 8, Figure 1). For a partially filled tank, the pressure head is to be measured from the free surface level to the load point (see Section 8, Figure 2). The free surface is defined as the liquid surface normal to the resultant acceleration vector. In Section 8, Figures 1 and 2, only vertical and transverse accelerations are considered for illustration purpose.
g	=	acceleration of gravity
g_L, g_T	=	longitudinal and transverse components of gravitational acceleration relative to the vessel-fixed coordinate system due to roll and pitch inclinations
	=	$(-g\sin\phi, \quad g\sin\theta)$
θ	=	roll angle
ϕ	=	pitch angle
a_L, a_T, a_V	=	longitudinal, transverse and vertical components of local accelerations caused by vessel motions relative to the vessel-fixed coordinate system at the center of gravity of tank contents

The internal pressure at the tank boundary points are to be linearly interpolated and applied to all of the nodes of the structural analysis model defining the tank boundary.

7 Loads from the Motions of Discrete Masses

Vessel motions produce loads acting on the masses of “lightship” structure and equipment. The motion induced acceleration is determined for each discrete mass from the formula:

$$(a_L, a_T, a_V) = \vec{a} + \vec{\Theta} \times \vec{R}$$

where

(a_L, a_T, a_V)	=	longitudinal, transverse and vertical components of local accelerations at the CG of tank content
$\vec{a}$	=	surge, sway and heave acceleration vector
$\vec{\Theta}$	=	roll, pitch and yaw acceleration vector
$\vec{R}$	=	distance vector from the vessel's center of gravity to the CG of tank content

It might be noted that the nonlinear term due to the centripetal acceleration of rotational motion is neglected in the above equation.

Using the real and imaginary parts of the complex accelerations calculated above, the motion-induced inertial load is computed, as described below.

The vertical component of dynamic load due to vertical acceleration may be expressed by the following equation:

$$F_V = ma_V$$

where

m	=	the discrete mass under consideration
a_V	=	local vertical acceleration

The transverse component of dynamic load due to transverse acceleration may be expressed by the following equation:

$$F_T = m(g_T + a_T)$$

where

g_T	=	transverse component of gravitational acceleration relative to the vessel-fixed coordinate system due to roll inclination
	=	$g\sin\theta$
α_T	=	local transverse acceleration

The longitudinal component of dynamic load due to longitudinal acceleration may be expressed by the following equation:

$$F_L = m(g_L + a_L)$$

where

g_L	=	longitudinal component of gravitational acceleration relative to the vessel-fixed coordinate system due to pitch inclination
	=	$-g\sin\phi$
a_L	=	local longitudinal acceleration

The real and imaginary parts of the motion-induced loads from each discrete mass in all three directions are calculated and applied to the structural model.

SECTION 6

Loading for Global Finite Element Method (FEM) Structural Analysis Model

1 General

For each heading angle and wave frequency at which the structural analysis is performed (see Subsection 2/3), two load cases corresponding to the real and imaginary parts of the frequency regime wave-induced load components are to be analyzed. Then, for each heading angle and wave frequency, the frequency-dependent wave-induced cyclic stress transfer function, $H_{\sigma}(\omega \,|\, \theta)$, is obtained for the Basic Vessel Loading Condition and vessel speed, if applicable.

When inputting the pressure loading components, care is to be taken in the interpolation near regions where pressure changes sign.

3 Number of Load Cases

In order to generate the stress transfer function, the number of combined load cases for each Basic Loading Condition can be relatively large. When the structural analysis is performed for 33 frequencies (0.2 to 1.80 rad/s at a 0.05 increment) and 12 wave headings (0 to 360 degree at a 30-degree increment), the number of combined Load Cases is 792 (considering separate real and imaginary cases). If there are three (3) Basic Loading Conditions, the total number of load cases is (3 × 792) = 2376.

However, a significant reduction in the number of heading angles, hence load cases, to be analyzed is possible in the "on-site" analysis of an FPSO system with a "weathervaning" turret mooring. Where justified by the environmental data, a minimum of 5 heading angles, predominant heading and 30 and 60 degrees off either side of predominant heading, will be considered sufficient. In this case, with 3 Basic Loading Conditions the number of load cases for analysis is (33 × 2 × 5 × 3) = 990.

Loads cases for unusual design are subject to special considerations.

5 Mooring Loads

Mooring loads are primarily elastic reactions resisting the combined effects of wave-induced forces and motions, and current, wind, etc., effects on the FPSO hull. The effects of mooring can be considered in three regimes of hull motion: *first-order* (wave frequency), *second-order* (low frequency or slowly varying) and *steady offset* due to wind and wave. These frequency-related components are to be obtained using a recognized vessel mooring analysis method.

The results of the mooring analysis and the environmental data on the directionality of the prevalent load effects are to be used to establish the mooring loads to be included in the structural analyses of Section 7.

7 Equilibrium Check

The applied hydrodynamic external pressure and the mooring loads should be in equilibrium with all other loads applied. The unbalanced forces in three global directions for each load case should be calculated and checked. For the head sea condition, the unbalanced force should not exceed one percent of the displacement. For oblique and beam sea conditions, it should not exceed two percent of the displacement. These residual forces could be balanced out by adding suitably distributed inertial forces (so called "inertial relief") before carrying out the FEM structural analysis.

SECTION 7

Structural Modeling and Analysis

1 General

The stress transfer function, $H_\sigma(\omega|\theta)$, for a location where the fatigue strength is to be evaluated, should be determined by the finite element method (FEM) of structural analysis using a three dimensional (3-D) model representing the entire hull structure, the topside equipment support structure and the interface with the mooring system, and as applicable, the risers. The Load Cases to be used in the structural analysis should be those obtained in accordance with Section 6. Special attention should be paid to the modeling of the topsides deck stool supports, stiffness of a turret mooring system and the transmission of mooring loads into the hull.

As necessary to evaluate the fatigue strength of local structure, finer mesh FEM analyses should also be performed. Results of nodal displacements or forces obtained from the overall 3-D analysis model should be used as boundary conditions in the subsequent finer mesh analysis of local structures.

Specialized fine mesh FEM analysis is required in the determination of stress concentration factors associated with the "hot-spot" fatigue strength evaluation procedures (see Subsection 7/9).

3 Areas for Fatigue Strength Evaluations

Refer to Subsection 1/3.

5 3-D Global Analysis Modeling

The global structural and load modeling should be as detailed and complete as practicable. For the Spectral-based Fatigue Analysis of a FPSO structure, net scantlings, which are defined as gross scantlings minus the Nominal Design Corrosion Values specified in 5A-3-1/1.7 of the *FPI Rules* for double hull structure and 5A-3-6/1.1 of the *FPI Rules* for single hull and double side single bottom structure, are to be used. For more details of global FE modeling, refer to 5A-3-4/11 of the *FPI Rules*. The calculated stress ranges need to be reduced by using a factor of 0.95, which is an adjustment factor to reflect a mean wasted condition.

For modeling convenience, FE modeling based on gross or as-built scantling can be used as an option in SFA analysis. No adjustment factor is applied if FE model is based on gross or as-built scantling.

In making the model, a judicious selection of nodes, elements and degrees of freedom is to be made to represent the stiffness and inertial properties of the hull. Lumping of plating stiffeners, use of equivalent plate thickness and other techniques may be used to keep the size of the model and required data generation within manageable limits.

The finite elements, whose geometry, configuration and stiffness closely approximate the actual structure, are of three types:

i) Truss or bar elements with axial stiffness only

ii) Beam elements with axial, shear, bending and torsional stiffness, and

iii) Thin plate and shell elements, either triangular or quadrilateral.

Mesh design, the discretization of a structure into a number of finite elements, is one of the most critical tasks in finite element modeling and often a difficult one. The following parameters need to be considered in designing the layout of elements: mesh density, mesh transitions and the stiffness ratio of adjacent elements. As a general rule, a finer mesh is required in areas of high stress gradient. The performance of elements degrades as they become more skewed. If the mesh is graded, rather than uniform, as is usually the case, the grading should be done in a way that minimizes the difference in size between adjacent elements.

Most analysts rely on preprocessors to develop the finite element mesh. Automatic mesh generators yield adequate meshes. However, in very demanding configurations, the mesh generator may produce a poor mesh. In such situations, the mesh should be manually produced to improve the mesh quality.

In modeling complex structural assemblies, there is a possibility of constructing models where adjacent structural elements have very different stiffness. To prevent large numerical errors, a conservative stiffness ratio of the order of 10^4 or more between members making up a model should be avoided.

7 Analyses of Local Structure

More refined local stress distributions should be determined from the fine mesh FEM analysis of local structure. In the fine mesh models, care is to be taken to accurately represent the structure's stiffness, as well as its geometry. Boundary displacements obtained from the 3-D global analysis are to be used as boundary conditions in the fine mesh analysis. In addition to the boundary constraints, the pertinent local loads should be reapplied to the fine mesh models.

The models are to be constructed with linear quadrilateral and triangular elements (shell elements being flat bending plate elements arranged in the mid-plane of the structural components) and one-dimensional rod and beam elements. As shown in 7/9 FIGURE 1, the areas around the expected stress concentrations in each model (e.g., bracket heel, bracket toe, etc.) are to be carefully meshed with quadrilateral shell elements of approximate size $t \times t$, where t is the minimum plate thickness in the vicinity of a particular stress concentration. Edges of brackets and cutouts are to be meshed with dummy rod elements of approximate length t and cross-sectional area 0.01 cm^2.

The welds are usually not modeled as illustrated in 7/9 FIGURE 1, except for special cases where the results are affected by high local bending (e.g., due to an offset between plates, such as doubling, or due to a small free plate length between adjacent welds such as at lug (or collar) plates, as shown in 7/9 FIGURE 2). In this case, the weld may be included by vertical or inclined plate elements having appropriate stiffness or by introducing multiple constrained elements to couple node displacements.

9 Hot Spot Stress Determination

The differences between a Nominal Stress Approach and a Hot-Spot Stress Approach and the selection of S-N curves in the respective approaches are described in Section 2 of the ABS *Guide for the Fatigue Assessment of Offshore Structures*.

When employing the so-called "Hot-Spot" Stress Approach (for example, to determine the fatigue strength at the toe of a fillet weld), it is necessary to establish a procedure to be followed to characterize the expected fatigue strength. The two major parts of the procedure are (a) the selection of an S-N Data Class (see Section 8) that applies in each situation; and (b) specifying the fine mesh FEM model adjacent to the weld toe detail and how the calculated stress distribution is extrapolated to the weld toe (hot-spot) location. The 7/9 FIGURE 3 shows an acceptable method that can be used to extract and interpret the "near weld toe" element stresses and to obtain the (linearly) extrapolated stress at the weld toe. When stresses are obtained in this manner, the use of the E class S-N curve for plated details is considered to be most appropriate.

FIGURE 1
Fine Mesh FEM Model

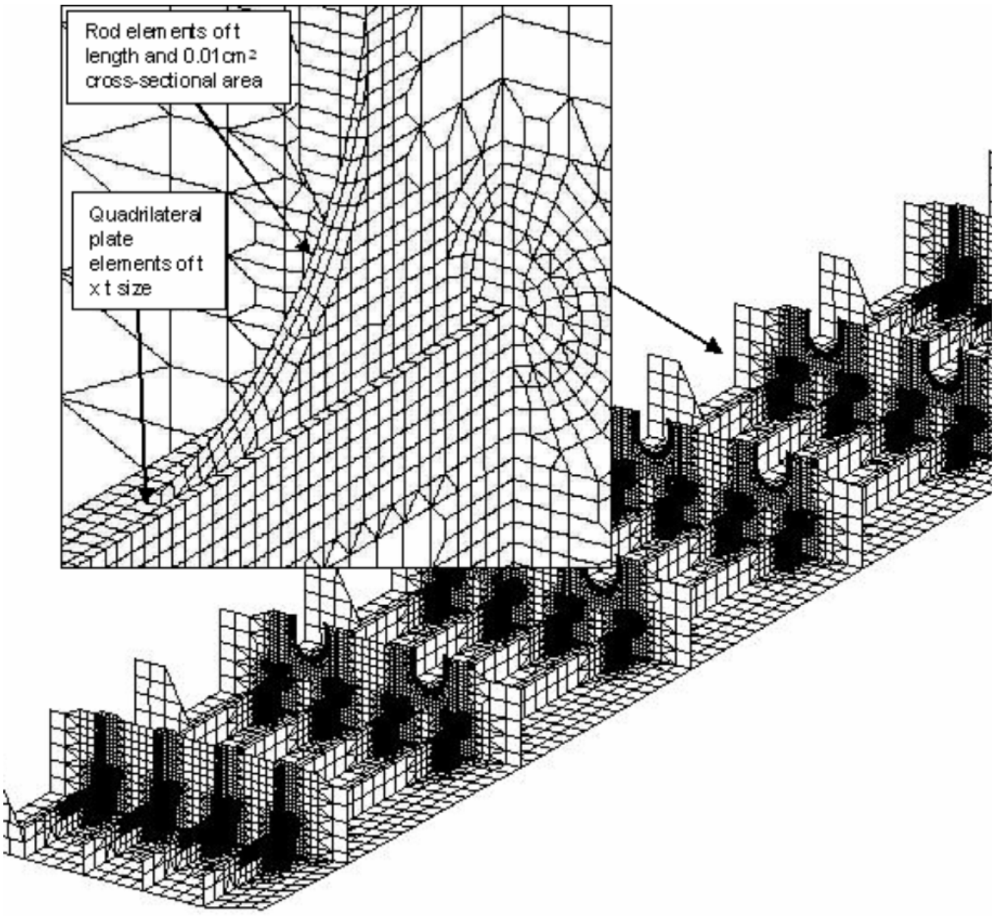
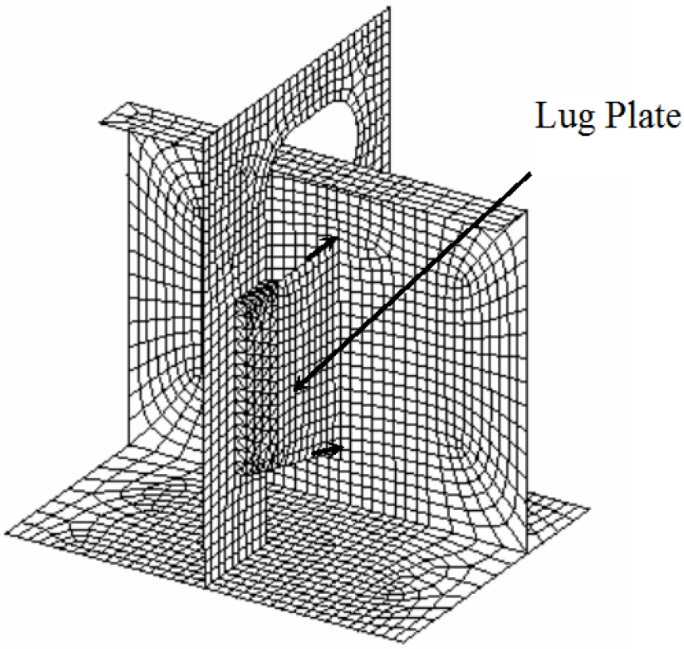


FIGURE 2
Local Structural FE Model: Welds Modeled



The algorithm described below may be used to obtain the hot spot stress for the point at the toe of a weld, as shown in 7/9 FIGURE 3.

Consider the four points, P_1 to P_4, measured by the distances X_1 to X_4 from the weld toe, designated as the origin of the coordinate system (7/9 FIGURE 4). These points are the centroids of four neighboring finite elements, the first of which is adjacent to the weld toe. Assuming that the applicable surface component stresses, S_i, at P_i have been determined from FEM analysis, the corresponding stresses at "hot spot" (i.e., the stress at the weld toe) can be determined by the following procedure:

1) Select two points, A and B, such that points A and B are situated at distances $3t/2$ and $t/2$ from the weld toe; i.e.:

$$X_A = 3t/2$$

$$X_B = t/2$$

where t denotes the thickness of the member to which elements 1 to 4 belong.

2) For a given point X, compute the values of four coefficients, as follows:

$$C_1(X) = [(X - X_2)(X - X_3)(X - X_4)]/[(X_1 - X_2)(X_1 - X_3)(X_1 - X_4)]$$

$$C_2(X) = [(X - X_1)(X - X_3)(X - X_4)]/[(X_2 - X_1)(X_2 - X_3)(X_2 - X_4)]$$

$$C_3(X) = [(X - X_1)(X - X_2)(X - X_4)]/[(X_3 - X_1)(X_3 - X_2)(X_3 - X_4)]$$

$$C_4(X) = [(X - X_1)(X - X_2)(X - X_3)]/[(X_4 - X_1)(X_4 - X_2)(X_4 - X_3)]$$

3) The corresponding stress at the given point can be obtained by interpolation as:

$$S_L = C_1S_1 + C_2S_2 + C_3S_3 + C_4S_4$$

4) Apply step (2) and (3) to Point A and Point B. The stress at Point A and Point B can be obtained by interpolation, i.e.:

$$S_A = C_1(X_A)S_1+C_2(X_A)S_2+C_3(X_A)S_3+C_4(X_A)S_4$$

$$S_B = C_1(X_B)S_1+C_2(X_B)S_2+C_3(X_B)S_3+C_4(X_B)S_4$$

5) The corresponding stress at hot spot, S_{hot}, is given by

$$S_{hot} = (3S_B - S_A)/2$$

FIGURE 3
Weld Toe Extrapolation Points

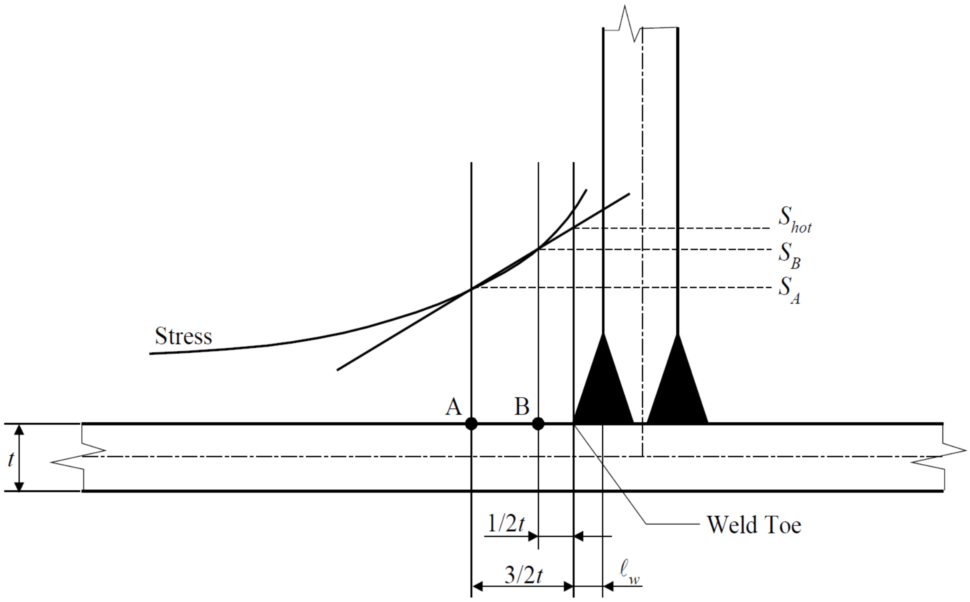
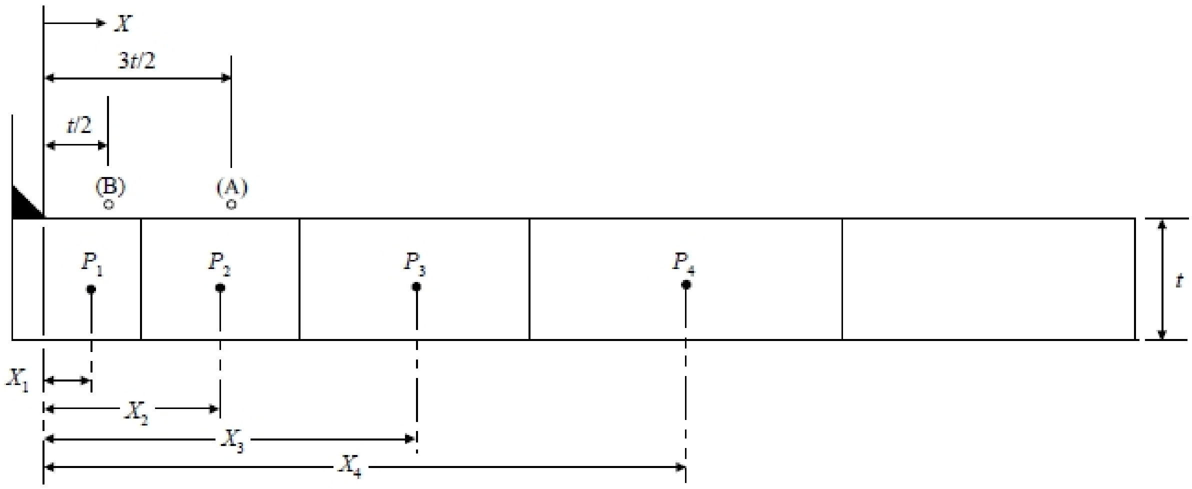


FIGURE 4
Elements Adjacent to Weld Toe



SECTION 8
Fatigue Strength

1 General

The previous Sections of the Guide have addressed establishing the stress transfer function or stress RAOs for locations in the structure for which the adequacy of fatigue life is to be evaluated. The following steps to evaluate the stress range distribution are described in Appendix A3 of this Guide. The capacity of a structural location to resist fatigue damage is characterized by the use of S-N Data, which are described in Sections 2 and 3 of the ABS *Guide for the Fatigue Assessment of Offshore Structures*.

Using the S-N approach, fatigue strength (capacity) is usually characterized in one of two ways. One way is called a *nominal stress approach*. In this approach, the acting cyclic stress (demand) is considered to be obtained adequately from the nominal stress distribution in the area surrounding the particular location for which the fatigue life is being evaluated. The other way of characterizing fatigue strength (capacity) at a location is the *"hot-spot" approach* (see Subsection 7/9). The hot-spot approach is needed for locations where complicated geometry or relatively steep local stress gradients would make the use of the nominal stress approach inappropriate or questionable. Reference should be made to Section 2 of the ABS *Guide for the Fatigue Assessment of Offshore Structures* for further explanation and application of these two approaches.

3 S-N Data *(1 February 2013)*

Section 3 of the ABS *Guide for the Fatigue Assessment of Offshore Structures* provides S-N curves for non-tubular details, tubular joints and cast steel components. Each set of curves can be adjusted for thickness effect and adjustments are provided to reflect corrosion effects. Three corrosive conditions are considered: in-air, cathodically protected in seawater, and free corrosion in seawater. For cast steel components "in-air" S-N curve is given.

There are other adjustments that could be considered to increase fatigue capacity above that portrayed by the cited S-N data. These include adjustments for "mean stress" effects, a high compressive portion of the acting cyclic stress and the use of "weld-improvement" techniques. The first two of these adjustments are not permitted, primarily because they may violate the calibration and validation that was performed in the development of the recommended fatigue strength criteria.

The use of a weld-improvement technique, such as weld toe grinding or peening to relieve ambient residual stress, can be effective in increasing fatigue life. However, credit should not be taken for such a weld improvement in the design phase of the structure. Consideration for granting credit for the use of weld-improvement techniques should be reserved for situations arising during construction, operation or future reconditioning of the structure. An exception may be made if the target design fatigue life cannot be satisfied by other preferred design measures such as refining layout, geometry, scantlings and welding profile to minimize fatigue damage due to high stress concentrations. Grinding or ultrasonic/hammer peening can be used to improve fatigue life in such cases. The calculated fatigue life is to be greater than 2/3 times the design fatigue life years excluding the effects of life improvement techniques.

Grinding is preferably to be carried out by rotary burr and to extend below the plate surface in order to remove toe defects and the ground area is to have effective corrosion protection. The treatment is to produce a smooth concave profile at the weld toe with the depth of the depression penetrating into the plate surface to at least 0.5 mm below the bottom of any visible undercut. The depth of groove produced is to be kept to a minimum, and, in general, kept to a maximum of 1 mm. In no circumstances is the grinding depth to exceed 2 mm or 7% of the plate gross thickness, whichever is smaller. Grinding has to extend to areas well outside the highest stress region.

The finished shape of a weld surface treated by ultrasonic/hammer peening is to be smooth and all traces of the weld toe are to be removed. Peening depth below the original surface is to be maintained at least 0.2 mm. Maximum depth is generally not to exceed 0.5 mm. Provided these recommendations are followed, a credit of 2 on fatigue life may be permitted when suitable toe grinding or ultrasonic/hammer peening are provided.

Where an improvement technique is applied, full details of the grinding standard including the extent, profile smoothness particulars, final weld profile, and improvement technique workmanship and quality acceptance criteria are to be clearly shown on the applicable drawings and submitted for review together with supporting calculations indicating the proposed factor on the calculated fatigue life.

SECTION 9

Fatigue Life (Damage) Calculation and Acceptance Criteria

1 General

Mathematically, spectral-based fatigue analysis begins after the determination of the stress transfer function. Wave data are then incorporated to produce stress response spectra, which are used to derive the magnitude and frequency of occurrence of local stress ranges at the locations for which fatigue damage is to be calculated. Wave data are represented in terms of a wave scatter diagram and a wave energy spectrum. The wave scatter diagram consists of sea states, which are short-term descriptions of the sea in terms of joint probability of occurrence of a significant wave height, H_s, and a characteristic period.

An appropriate method is to be employed to establish the fatigue damage resulting from each considered sea state. The damage resulting from individual sea states is referred to as "short-term". The total fatigue damage resulting from combining the damage from each of the short-term conditions can be accomplished by the use of a weighted linear summation technique (i.e., Miner's rule).

Appendix A3 contains a detailed description of the steps involved in a suggested Spectral-based Fatigue Analysis method that follows the basic elements mentioned above. ABS should be provided with background and verification information that demonstrate the suitability of the analytical method employed.

3 Combination of Wave-Frequency and Low-Frequency Responses in Wave-induced Fatigue Damage Calculation

When the process that induces variable stresses in a structural detail contains wave-frequency and low-frequency components, the process is considered to be wide-banded. Although the Wirsching's rainflow counting correction can be applied to account for a wide band process, the formulae are calibrated only to a wave frequency process.

When wave-frequency and low-frequency stress responses are obtained separately, simple summation of fatigue damage from the two frequency bands does not count the effects of simultaneous occurrence of the two frequency bands processes. This method is therefore non-conservative and should not be used.

There is an alternative method, which is both conservative and easy to use, that is known as the combined spectrum method. In this method, the stress spectra for the two frequency bands are combined. The RMS and the mean up-crossing frequency of the combined stress process are given, respectively, as

$$\sigma_c = (\sigma_w^2 + \sigma_\ell^2)^{1/2}$$

$$f_{0c} = (f_{0w}^2\sigma_{wc}^2 + f_{0\ell}^2\sigma_\ell^2)^{1/2}/\sigma_c$$

where

σ_w = RMS of the wave-frequency stress components

σ_ℓ = RMS of the low-frequency stress components

f_{0w} = mean up-crossing frequency of the wave-frequency stress components

$f_{0\ell}$ = mean up-crossing frequency of the low-frequency stress components

For each sea state, the fatigue damage for the combined wave-frequency and low-frequency process is obtained by substituting the above quantities for the combined process into the closed-form formula of spectral fatigue given in Appendix A3.

However, if both frequency components of stress range are significant, the above-mentioned combination method may be too conservative since the wave-frequency contribution is expected to dominate, thus controlling the mean up-crossing frequency of the combined stress process. To eliminate the conservatism, a correction factor given below can be applied to the calculated fatigue damage of the sea state:

$$\lambda = \frac{f_{0p}}{f_{0c}}\left[\lambda_\ell^{m/2+2}\left(1-\left(\lambda_w/\lambda_\ell\right)^{1/2}\right)+\left(\pi\lambda_w\lambda_\ell\right)^{1/2}\frac{m\Gamma(m/2+1/2)}{\Gamma(m/2+1)}\right]+\frac{f_{0w}}{f_{0c}}\lambda_w^{m/2}$$

where

λ_ℓ = σ_ℓ^2/σ_c^2

λ_w = ${\sigma_w}^2/{\sigma_c}^2$

f_{0p} = $\left(\lambda_\ell^2 f_{0\ell}^2+\lambda_\ell\lambda_w f_{0w}^2\right)^{1/2}$

m = slope parameter of the lower cycle segment of the S-N curve

$\Gamma()$ = complete gamma function

An alternative, more accurate method of fatigue damage calculation is to simulate the combined stress process in the time domain, and employ rainflow counting to count the stress cycles for each sea state. The accumulative fatigue damage is the weighted summation of the damages from all sea states considering the probability of occurrence of each sea state, as given in the wave scatter diagram.

5 Low Cycle Fatigue Damage

5.1 Low Cycle Fatigue Load

When fatigue is of concern, structural responses are assumed to result from two external sources, the wave loading on the offshore structure and the process of loading and offloading of produced cargo resulting in uneven buoyancy. This loading/offloading process produces slow varying static loads including oscillatory still water bending moment (SWBM) and still water pressure. Some structural components experience cyclic plasticity when the combination of the two load sources produces cyclic stresses that exceed the yield strength of the material used. Typically this occurs at the toe of a weld. Described in this section is the process of defining damage due to low cycle fatigue.

5.3 Loading Conditions

Static cyclic loads including still-water bending moments and static pressure due to cargo loading and offloading are considered.

5.5 Stress Range Calculation

5.5.1 Elastic Hot Spot Stress Range Calculation *(2018)*

In the following, all reference to stress is to be interpreted as the elastic hot spot stress at the toe of a weld in question. Also, at the outset, it will be assumed that the S-N curve defining fatigue strength is given in pseudo hot spot stress. In the elastic high cycle range a pseudo hot spot stress will be the same as an elastic hot spot stress. They will differ in the low cycle range.

As shown in 9/5.5.1 FIGURE 1, the stress process in certain structural components of an offshore installation can be considered as a superposition of wave induced stresses, $S_W(t)$, and stresses associated with static load, $S_B(t)$. The cycles of S_B result from the loading/offloading process.

The total or net stress process will be:

$$S(t) = S_B(t) + S_W(t)$$

FIGURE 1
Sample Functions of S_W and S_B

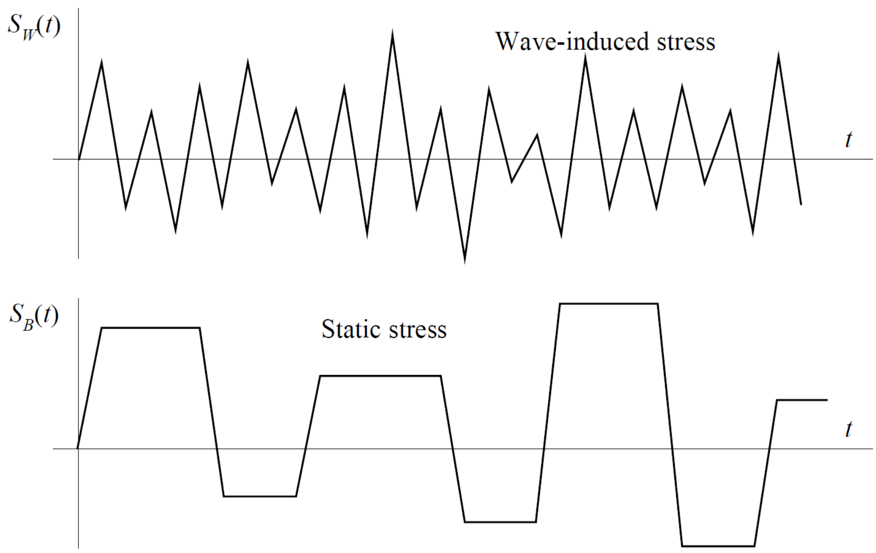
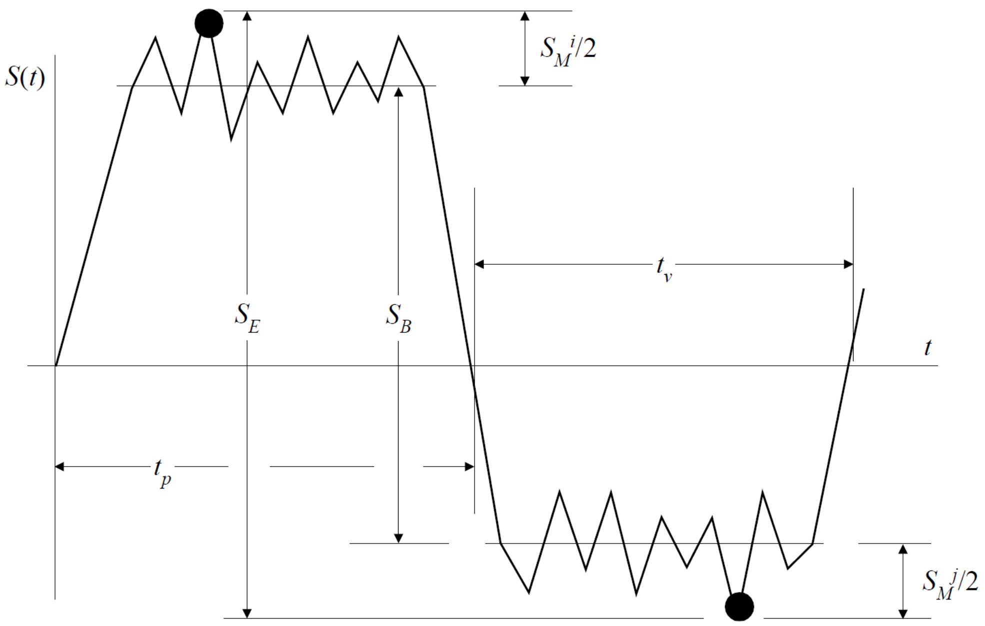


FIGURE 2
A Single Loading/Offloading Cycle



In one cycle of the static process, as shown in 9/5.5.1 FIGURE 2, the total stress range associated with this cycle is S_E,

$$S_E = S_B + 0.5(S_M^i + S_M^j)$$

where

S_B	=	static stress range for this cycle
S_M^i	=	median of the largest stress range of wave induced load for *i*-th load condition
S_M^j	=	median of the largest stress range of wave induced load for *j*-th load condition

From extreme value theory, the median largest stress range S_M^i in *n* cycles is given as:

$$\frac{S_M^i}{\delta} = \left[-\ln\left(1 - 0.5^{1/n}\right)\right]^{1/\gamma}$$

where γ and δ are the long term stress shape and scale factors, respectively. δ can be determined statistically from long term records of stress ranges or can be calculated by the formula:

$$\delta = \frac{S_R}{[\ln(N_S)]^{1/\gamma}}$$

where S_R is the stress range associated with a probability of exceedance of $1/N_S$, and N_S is equal to 10^4.

n may be computed by taking the estimated time for a half cycle divided by the estimated wave period.

The number of cycles for installation's loading and unloading, n_{LCF}, is assumed to be not less than 1200 for 20 years. The actual cycles of loading/offloading may be used for historical sites in FPSO phase.

Assume there are 10^8 wave cycles within 20 years, n is then equal to:

$$\frac{10^8}{2n_{LCF}}$$

In general, it is expected that the time in tension will not equal the time in compression. For a conservative analysis, the larger of the two might be selected.

5.5.2 Pseudo Hot Spot Stress Range Calculation

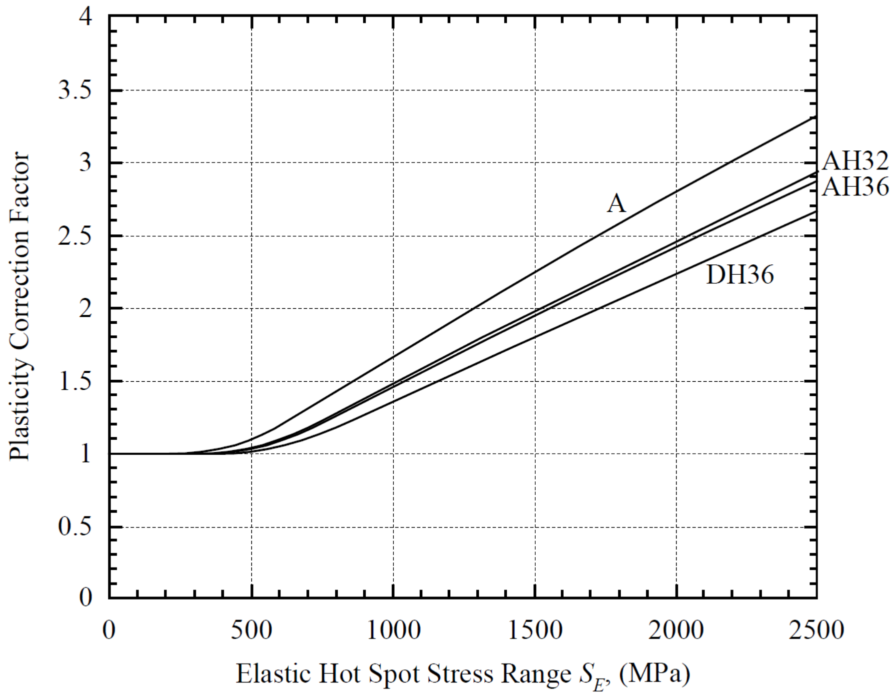
To transform elastic hot spot stress range to pseudo hot spot stress range, a plasticity correction factor, k_e, is defined as:

$$k_e = \frac{S_L}{S_E}$$

where S_L is the pseudo hot spot stress range.

A plot of k_e as a function of S_E is given in 9/5.5.2 FIGURE 3.

FIGURE 3
k_e as a Function of S_E



An approximate analytical formula derived from the above curves can be used:

$k_e = 0.5 + k_m S_E$, but should not be less than 1.0

Values of k_m

Material	Mild	HT32	HT36	HT40
k_m	11.20×10^{-4}	9.60×10^{-4}	9.40×10^{-4}	8.56×10^{-4}

5.5.3 Low Cycle S-N Curve and Damage Calculation *(2018)*

The design S-N curve in the low cycle region is defined in 9/5.5.3 FIGURE 4. It may be considered to be a modified D-Curve.

The low cycle fatigue (LCF) design S-N curve is given as:

$$NS^q = B \qquad \text{for } 100 < N < 10^4$$

where

q = 2.4

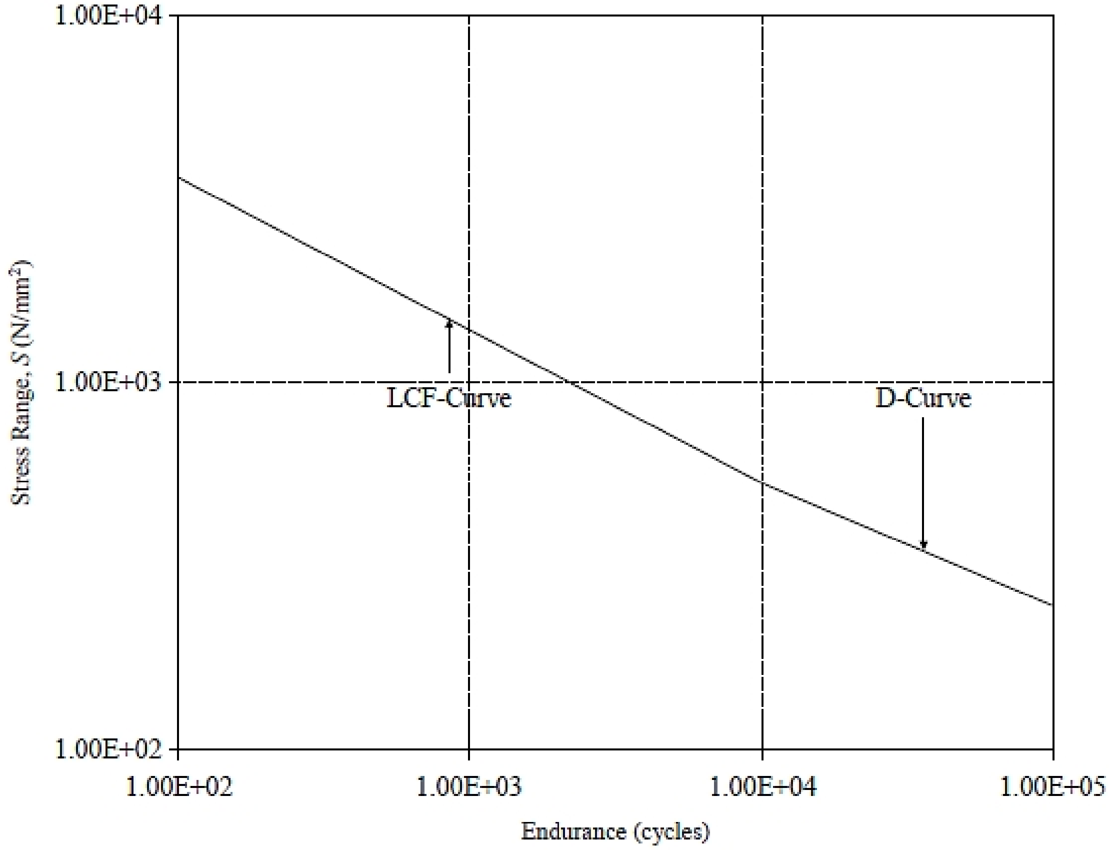
B = 3.51×10^{10} (MPa units)

It is assumed that the LCF design S-N curve is applicable to static induced stresses. Basic application of Miner's rule produces the expression of static stress damage DM_{LCF} is:

$$DM_{LCF} = \frac{N_{LCF} S_L^q}{B}$$

n_{LCF} is the total cycles of loading/offloading, which is not to be less than 1200 for a ship-type installation to be operated for 20 years. The actual cycles of loading/offloading may be used for historical sites in FPSO phase.

FIGURE 4
Low Cycle Fatigue Design Curve



7 Combined Low Cycle and High Cycle Fatigue Damage

The total fatigue damage due to both low cycle and high cycle stress can be calculated by

$$DM_{comb} = \frac{\left(DM_{LCF}^2 + 2\delta DM_{LCF} DM_{HCF} + DM_{HCF}^2\right)}{\sqrt{DM_{LCF}^2 + DM_{HCF}^2}}$$

where

δ = 0.02

DM_{LCF} = low cycle fatigue damage

DM_{HCF} = wave induced high cycle fatigue damage

9 Acceptance Criteria

The criteria are presented as a comparison of fatigue strength of the structure (capacity), and fatigue inducing loads (demands), in the form of a fatigue damage parameter, *DM*. The calculated fatigue damage, *DM*, being equals to 1, for the required fatigue life of a FPSO, refer to 5A-1-3/3.11, 5A-1-4/7.5 and 5A-2-1/3 of the *FPI Rules*, corresponds to a fatigue life of 20 years.

APPENDIX 1
Wave Data

The Spectral-based Fatigue Analysis of a vessel that is classed for "unrestricted service" should be based on the "wave scatter" diagram data given below.

TABLE 1
ABS Wave Scatter Diagram for Unrestricted Service Classification

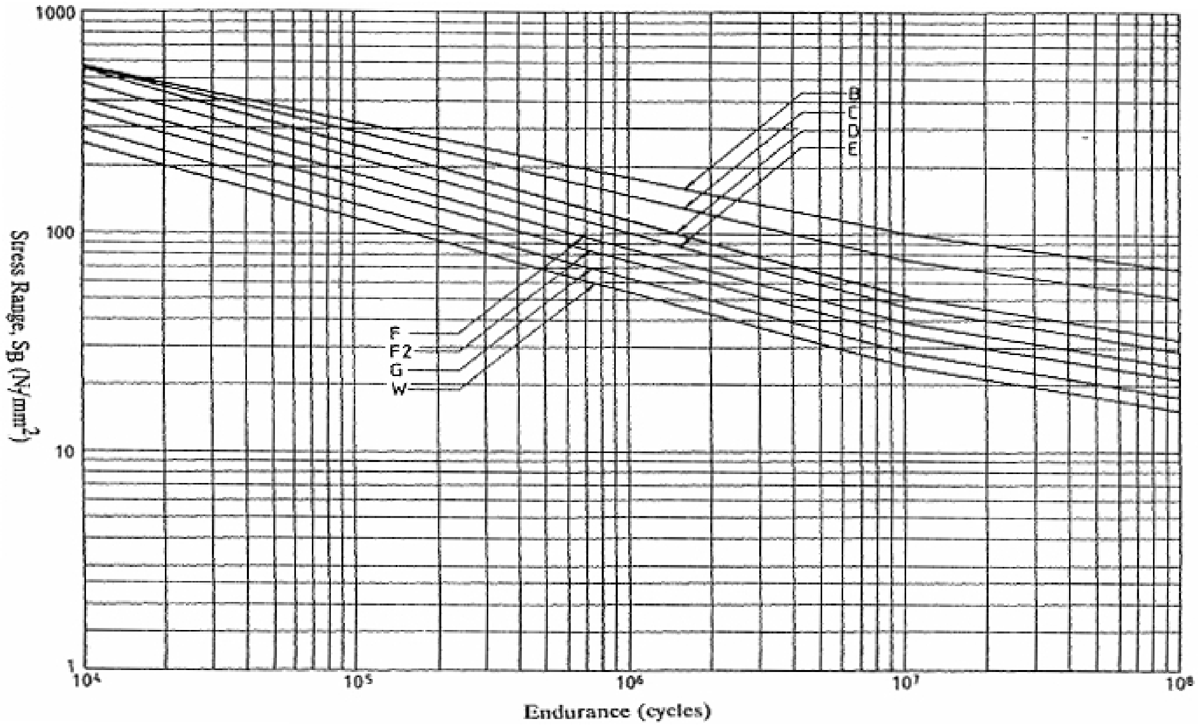
* Wave heights taken as significant values, H_s

** Wave periods taken as zero crossing values, T_Z

*Wave Height (m)**	*Wave Period (sec)***											*Sum Over All Periods*
	3.50	*4.50*	*5.50*	*6.50*	*7.50*	*8.50*	*9.50*	*10.50*	*11.50*	*12.50*	*13.50*	
0.5	8	260	1344	2149	1349	413	76	10	1			5610
1.5		55	1223	5349	7569	4788	1698	397	69	9	1	21158
2.5		9	406	3245	7844	7977	4305	1458	351	65	10	25670
3.5		2	113	1332	4599	6488	4716	2092	642	149	28	20161
4.5			30	469	2101	3779	3439	1876	696	192	43	12625
5.5			8	156	858	1867	2030	1307	564	180	46	7016
6.5			2	52	336	856	1077	795	390	140	40	3688
7.5			1	18	132	383	545	452	247	98	30	1906
8.5				6	53	172	272	250	150	65	22	990
9.5				2	22	78	136	137	90	42	15	522
10.5				1	9	37	70	76	53	26	10	282
11.5					4	18	36	42	32	17	7	156
12.5					2	9	19	24	19	11	4	88
13.5					1	4	10	14	12	7	3	51
> 14.5					1	5	13	19	19	13	7	77
Sum over All Heights	8	326	3127	12779	24880	26874	18442	8949	3335	1014	266	100000

Basic Design S-N Curves

FIGURE 1
S-N Curves



The S-N Curves are represented by the following equation:

$S^m N = A$

where

S	=	stress range
N	=	number of cycles to failure
A, m	=	parameters representing the intercept and inverse slope of the upper (left) portion of the S-N Curve. These change at $N = 10^7$ cycles to C and r, respectively. Values of these parameters are given in the following table.

TABLE 1
Parameters For Basic S-N Design Curves

Class	$N \leq 10^7$		$N > 10^7$	
	***A* (For MPa units)**	***m***	***C* (For MPa units)**	***r***
B	1.013×10^{15}	4	1.020×10^{19}	6
C	4.227×10^{13}	3.5	2.584×10^{17}	5.5
D	1.519×10^{12}	3	4.239×10^{15}	5
E	1.035×10^{12}	3	2.300×10^{15}	5
F	6.315×10^{11}	3	9.975×10^{14}	5
F2	4.307×10^{11}	3	5.278×10^{14}	5
G	2.477×10^{11}	3	2.138×10^{14}	5
W	1.574×10^{11}	3	1.016×10^{14}	5

Refer to Part 5C of the *Marine Vessel Rules* on the categorization of structural details into the indicated classes.

Notes for Application of Classes:

Class B: Parent material with automatic flame-cut edges ground to remove flame cutting drag line.

Class C: Parent material with automatic flame-cut edges and full penetration butt welds ground flush in way of hatch corners in container carriers or similar deck areas in other vessel types.

Class D: Full penetration butt welds in way of hatch corners in container carriers or similar deck areas in other vessel types.

APPENDIX 3

Outline of a Closed Form Spectral-based Fatigue Analysis Procedure

Notes:

1 This Appendix is referred to in Section 9. It is provided to describe the formulations comprising a Spectral-based Fatigue Analysis approach. However, it is often at this formulation level that valid variations of a method may be introduced. For this reason, it is emphasized that the contents of this Appendix are provided primarily to illustrate principle, rather than to give mandatory steps for the Spectral-based Fatigue method.

2 The procedure described below considers the use of a wave scatter diagram that represents long-term wave data at the installation site that have been "normalized" to represent a period of one-year. Where a different base period for the wave scatter diagram is employed, the procedure must be suitably modified.

1 General

In the "short-term closed form" approach described below, the stress range is normally expressed in terms of probability density functions for different short-term intervals corresponding to the individual cells or bins of the wave scatter diagram. These short-term probability density functions are derived by a spectral approach based on the Rayleigh distribution method, whereby, it is assumed that the variation of stress is a narrow-banded random Gaussian process. To take into account effects of swell, which are not accounted for when the wave environment is represented by the scatter diagram, Wirsching's "rainflow correction" factor is applied in the calculation of short-term fatigue damage. Having calculated the short-term damage, the total fatigue damage is calculated through their weighted linear summation (using Miner's rule). Mathematical representations of the steps of the Spectral-based Fatigue Analysis approach just described are given next.

3 Key Steps in Closed Form Damage Calculation

1) Determine the complex stress transfer function, $H_\sigma(\omega\,|\,\theta)$, at a structural location of interest for a particular load condition. This is done in a direct manner where structural analyses are performed for the specified ranges of wave frequencies and headings, and the resulting stresses are used to explicitly generate the stress transfer function. See Sections 2 to 7.

2) Generate a stress energy spectrum, $S_\sigma(\omega\,|\,H_s, T_z, \theta)$, by scaling the wave energy spectrum $S_\eta(\omega\,|\,H_s, T_z)$ in the following manner:

$$S_\sigma(\omega|H_s, T_z, \theta) = |H_\sigma(\omega|\theta)|^2 \cdot S_\eta(\omega|H_s, T_z) \qquad (1)$$

3) Calculate the spectral moments. When vessel speed V is considered (i.e., transit case or past service of tanker conversion), the n-th spectral moment, m_n, is calculated as follows:

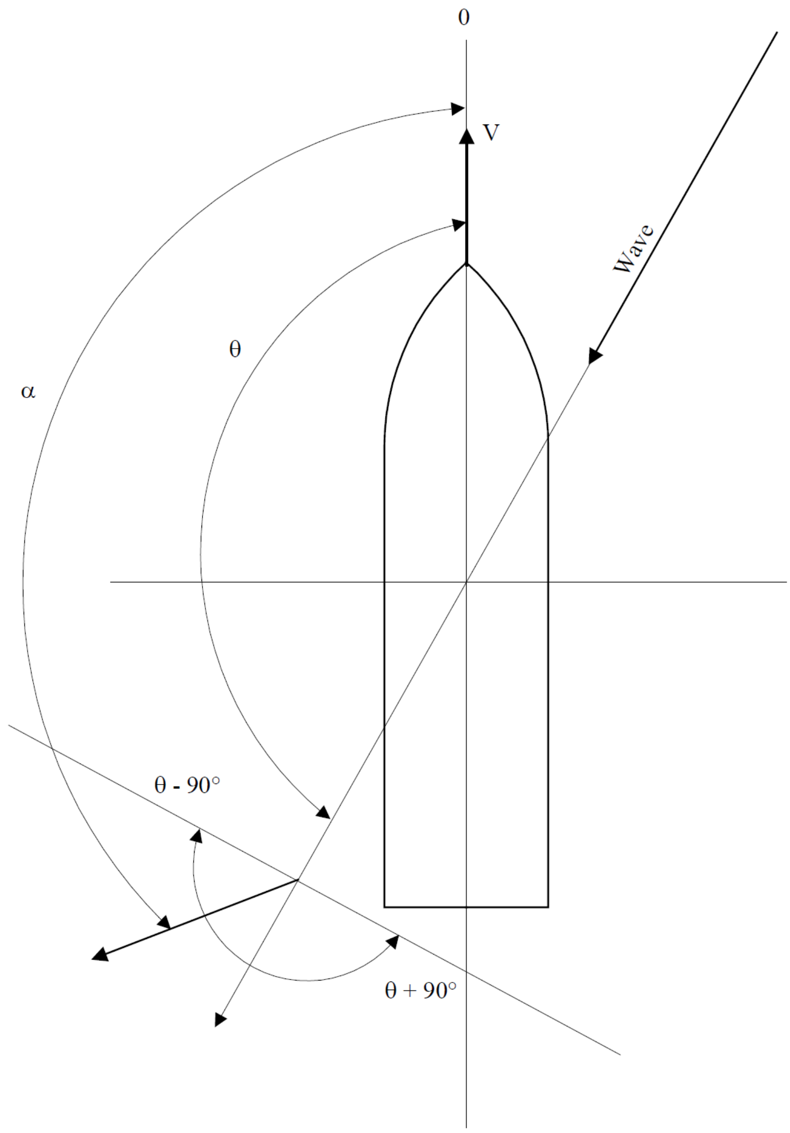
$$m_n = \int_0^\infty \left(\omega - V\omega^2\cos\theta/g\right)^n S_\sigma(\omega|H_s, T_z, \theta)\,d\omega \qquad (2)$$

Most fatigue damage is associated with low or moderate seas, hence confused short-crested sea conditions must be allowed. Confused short-crested seas result in a kinetic energy spread which is modeled using the cosine-squared approach, $(2/\pi)\cos^2\alpha$. Generally, cosine-squared spreading is assumed from -90 to +90 degrees on either side of the selected wave heading (refer to A3/3.3 FIGURE 1). Applying the wave spreading function modifies the spectral moment as follows:

$$m_n = \int_{\theta - 90}^{\theta + 90} \left(\frac{2}{\pi}\right)\cos^2(\alpha - \theta) \cdot \left(\int_0^\infty \left(\omega - V\omega^2\cos\alpha/g\right)^n S_\sigma(\omega|H_s, T_z, \alpha)d\omega\right)d\alpha \qquad (3)$$

The above integral is usually performed for each cell in the wave scatter diagram. However, the number of times to perform integration can be dramatically reduced if it is noted that for the cells of the same T_z, the n-th spectral moments are scalable to H_s^2 since the wave spectra are proportional to H_s^2.

FIGURE 1
Spreading Angles Definition



4) Using the spectral moments, the Rayleigh probability density function (pdf) describing the short-term stress-range distribution, the zero up-crossing frequency of the stress response and the bandwith parameter used in calculating Wirsching's "rainflow correction" are calculated as follows:

Rayleigh pdf:

$$g(S) = \frac{S}{4\sigma^2}\exp\left[-\frac{S^2}{8\sigma^2}\right] \quad (4)$$

Zero-up crossing frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} \quad (5)$$

Bandwidth Parameter:

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \quad (6)$$

where

S	=	stress range (twice the stress amplitude)
σ	=	$\sqrt{m_0}$

m_0, m_2 and m_4 are the spectral moments.

5) Calculate cumulative fatigue damage based on Palmgren-Miner's rule, which assumes that the cumulative fatigue damage (D) inflicted by a group of variable amplitude stress cycles is the sum of the damage inflicted by each stress cycle (d_i), independent of the sequence in which the stress cycles occur:

$$D = \sum_{i=1}^{N_{total}} d_i = \sum_{i=1}^{N_{total}} \frac{n_i}{N_i} \quad (7)$$

where

n_i	=	number of stress cycles of a particular stress range
N_i	=	average number of loading cycles to failure under constant amplitude loading at that stress range according to the relevant S-N curve
N_{total}	=	total number of stress cycles.

Failure is predicted to occur when the cumulative damage (D) over N_{total} loading cycles exceeds a critical value equal to unity. The short-term damage incurred in the i-th sea-state assuming an S-N curve of the form $N = KS^{-m}$ is given by:

$$D_i = \left(\frac{T}{K}\right)\int_0^\infty S^m f_{0i} p_i g_i ds \quad (8)$$

where

D_i	=	damage incurred in the i-th sea-state
m, K	=	physical parameters describing the S-N curve
T	=	target fatigue life
f_{0i}	=	zero-up-crossing frequency of the stress response
p_i	=	joint probability of H_s and T_z
g_i	=	probability density function governing S
S	=	stress range

Summing D_i over all of the sea-states in the wave scatter diagram leads to the total cumulative damage, D. Therefore:

$$D = \left(\frac{f_0 T}{K}\right)\int_0^\infty S^m[\sum f_{0i}p_i g_i/f_0]ds \qquad (9)$$

where

D	=	total cumulative damage
f_0	=	"average" frequency of S over the lifetime
	=	$\Sigma_i p_i f_{0i}$

Introducing the long-term probability density function, $g(S)$, of the stress range as:

$$g(S) = \frac{\sum_i f_{0i}p_i g_i}{\sum_i f_{0i}p_i} \qquad (10)$$

and N_T equal to the total number of cycles in life time = f_0T, the expression for total cumulative damage, D can be re-written as:

$$D = \left(\frac{N_T}{K}\right)\int_0^\infty S^m g(S)dS \qquad (11)$$

The minimum target fatigue life is twenty years. Having calculated the damage, fatigue life would then be equal to $20/D$. Changing the minimum target fatigue life to higher values is done accordingly.

5 Closed Form Damage Expression

For all one-segment linear S-N curves, the closed form expression of damage, D as given by Equation 9, is as follows:

$$D = \frac{T}{K}(2\sqrt{2})^m\Gamma(m/2+1)\Sigma_i\lambda(m,\varepsilon_i)f_{0i}p_i(\sigma_i)^m \qquad (12)$$

where

σ_i = σ in Equation (4)

λ = Wirsching's rainflow factor, defined as:

$$\lambda(m, \varepsilon_i) = a(m) + [1 - a(m)][1 - \varepsilon_i]^{b(m)} \qquad (13)$$

where

$a(m)$ = $0.926 - 0.033m$

$b(m)$ = $1.587m - 2.323$

ε_i = Spectral Bandwidth (Equation 6)

For the combined wave-frequency and low-frequency process, use the λ specified in Subsection 9/3 in this Guide.

For bi-linear S-N curves where the negative slope changes at point $Q = (S_q, 10^q)$ from m to $m' = m + \Delta m$ ($\Delta m > 0$) and the constant K changes to K', the expression for damage, as given in Equation 12, is as follows:

$$D = \frac{T}{K}(2\sqrt{2})^m \Gamma(m/2 + 1) \sum_i \lambda(m, \varepsilon_i) \mu_i f_{0i} p_i (\sigma_i)^m \qquad (14)$$

where μ_i is the endurance factor having its value between 0 and 1 and measuring the contribution of the lower branch to the damage. It is defined as:

$$\mu_i = 1 - \frac{\int_0^{S_q} s^m g_i ds - \left(\frac{K}{K'}\right) \int_0^{S_q} s^{m + \Delta m} g_i ds}{\int_0^{\infty} s^m g_i ds} \qquad (15)$$

If $g(S)$ is a Rayleigh distribution, then μ_i is:

$$\mu_i = 1 - \frac{\gamma(m/2 + 1, v_i) - (1/v_i)^{\Delta m/2} \gamma(m'/2 + 1, v_i)}{\Gamma(m/2 + 1)} \qquad (16)$$

where

v_i = $(1/8)\,[S_q/\sigma_i]^2$

γ = incomplete gamma function

= $\gamma(a, x) = \int_0^x u^{a-1} \exp(-u) du$