



GUIDANCE NOTES ON

DYNAMIC ANALYSIS PROCEDURE FOR SELF-ELEVATING UNITS

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ABS Plaza
16855 Northchase Drive
Houston, TX 77060 USA**

Updates

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- February 2014 version plus Corrigenda/Editorials

Foreword

The guidance contained herein should be used in conjunction with the *ABS Rules for Building and Classing Mobile Offshore Drilling Units* for the purpose of ABS Classification of a **Self-Elevating Unit**. The guidance indicates acceptable practice in a typical case for types of designs that have been used successfully over many years of service. The guidance may need to be modified to meet the needs of a particular case, especially when a novel design or application is being assessed. The guidance should not be considered mandatory, and in no case is this guidance to be considered a substitute for the professional judgment of the designer or analyst. In case of any doubt about the application of this guidance ABS should be consulted.

A self-elevating unit is referred to herein as an “SEU”, and the *ABS Rules for Building and Classing Mobile Offshore Drilling Units*, are referred to as the “*MODU Rules*”.

These Guidance Notes become effective on the first day of the month of publication.

Users are advised to check periodically on the ABS website www.eagle.org to verify that this version of these Guidance Notes is the most current.

We welcome your feedback. Comments or suggestions can be sent electronically by email to rsd@eagle.org.



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SECTION 1 Introduction (1 February 2014)

1 Background

These Guidance Notes present acceptable practice for an important aspect in the Classification of **self-elevating units (SEUs)**. The technical criteria contained in **the original version of these Guidance Notes published in 2004 were** based on the results of a Joint Industry Project sponsored by Owners, Designers, Builders, Operators and Classification Societies. The criteria were subsequently published as Reference 1. That reference is specifically aimed at providing assessment criteria for the site-specific use of the **SEU**. **Reference 1 was also used in the development of the International Standards Organization (ISO) Standard 19905-1 [Reference 6] for site-specific assessment of mobile offshore units – Jack-Ups. There were changes in both required and acceptable methods of assessment when the ISO 19905-1 standard was developed from Reference 1. The changes that are relevant to Classification were incorporated into this revision of these GNs.**

The fundamental difference between site-specific evaluation and Classification is that the latter is not site-specific in nature. Instead, the Owner specifies conditions for which the unit is to be reviewed for Classification. The basic dimensions of the envelope of conditions that the Owner may specify for Classification are:

- i) Water depth (plus air gap and penetration depth into the seabed)
- ii) Environmental conditions of wind, wave and current
- iii) Total elevated load
- iv) Spudcan-soil rotational stiffness

(The last item **is** a consideration introduced by ABS in 2003 when dynamic response is assessed for Classification.)

Therefore, a major theme of these Guidance Notes is to **designate** the portions of the criteria in **References 1 and 6** that **can** be applied without modification and the portions of the criteria that may need to be adapted for Classification purposes.

3 Basic Concepts of the Inclusion of Dynamic Effects into Structural Analysis

Because the natural period of an **SEU** is typically in the range of 5 to 15 seconds, there may be a concern that there will be dynamic amplification (resonance) with waves in this period range. It is therefore often desirable to account for the dynamic effects of the **SEU** in the elevated condition due to waves (and waves with current).

The basic approach most commonly used to include dynamic effects into structural analysis is characterized as a “quasi-static” method, which entails a two-step procedure. In the first step, a *Dynamic Analysis* model of the structural system is analyzed. Then, the static response to the same loads is obtained using the same model. A Dynamic Amplification Factor (DAF) is obtained as the ratio of the **most probable maximum extreme (MPME)** of a response when dynamics is considered to the **most probable maximum extreme (MPME)** of the same response statically considered. DAFs can be **obtained** for various structural responses, such as the global overturning moment of the unit, base shear force or the lateral displacement of the elevated hull (i.e., surge and sway). From the DAFs, an “inertial load set” is established that simulates the dynamic effects. The loads considered to produce the dynamic response are those induced by waves or waves acting with current. Usually, it is sufficient that the level of structural system idealization used to determine DAFs is, as often described, an “equivalent model”, which is an “equivalent 3-leg idealization” coupled with an “equivalent hull structural model”. The need to appropriately account for the stiffness of the leg-to-hull interaction and **spudcan-soil** interaction adds some minor complexity to this simplified modeling approach.

In the second step, the “inertial load set” is imposed, along with all of the other coexisting loads, onto the usual, detailed static structural model that is used to perform the “unity checking” for structural acceptance based on the Rules. Because this model now includes the “inertial load set” to simulate the dynamic response, it is often also referred to as the *Quasi-Static* model.

The two-step procedure is summarized as:

- i) Use an “equivalent” model to perform a random wave dynamic analysis deriving the **DAFs and subsequently the** inertial load set caused by wave-induced structural dynamics.

Alternatively, the DAFs may also be estimated using the single degree-of-freedom (SDOF) approach given in 4/5.1.2 as an alternative to the random wave dynamic analysis, above. However, care should be exercised since the SDOF approach may significantly over or underestimate the DAF. See the limitations of the SDOF approach for deriving the DAFs given in 4/5.1.4.

- ii) Use a “detailed” model to perform, with static gravity and wind loads and quasi-static wave loads plus the derived inertial load set, a static structural analysis deriving the stresses for unity checks in accordance with the ABS strength requirements in the *MODU Rules* for the leg chords, braces and the jacking pinions.

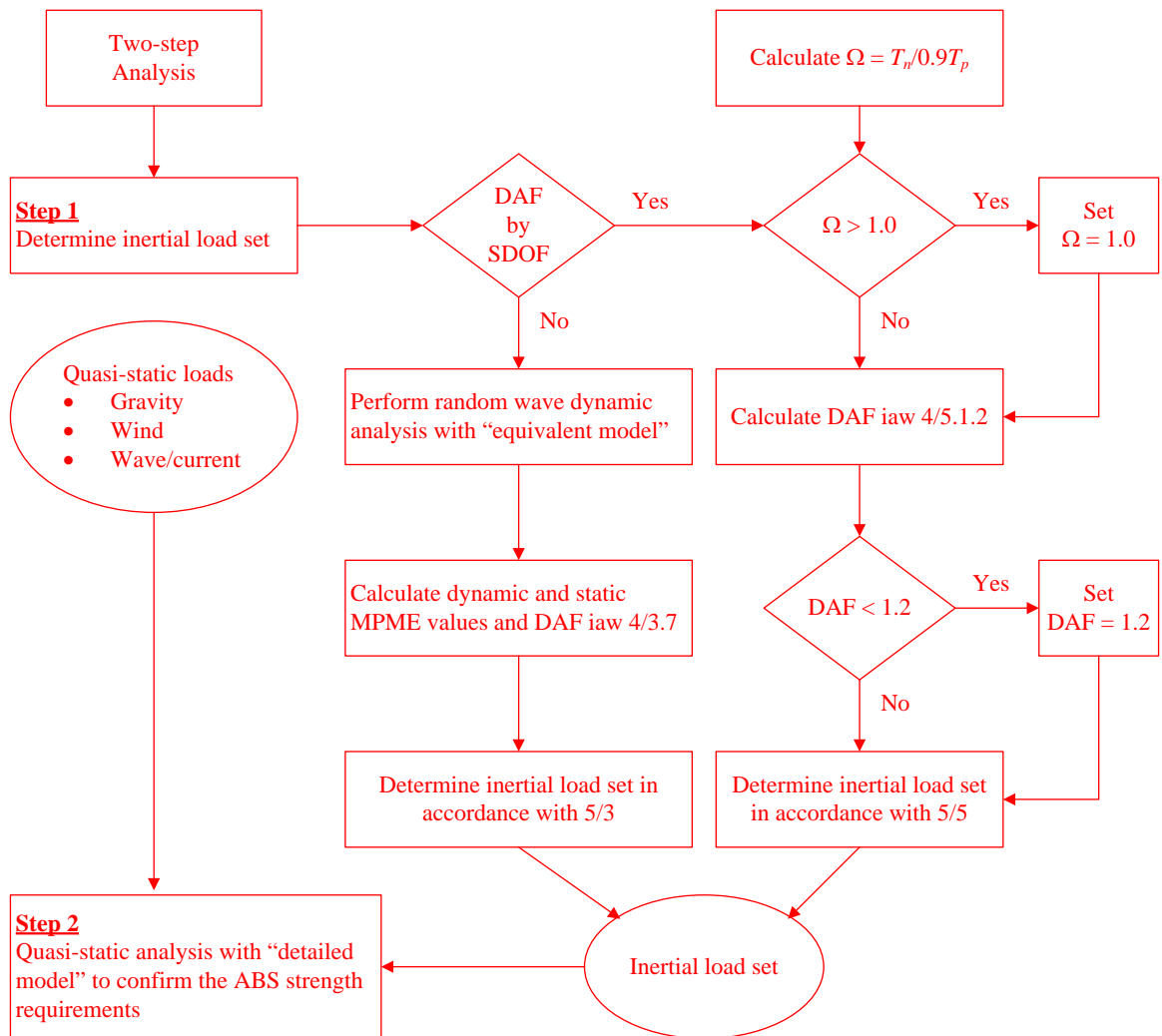
The flowchart of the two-step procedure is shown in Section 1, Figure 1. More details on the modeling procedure and the determination and application of the inertial load set are given in these Guidance Notes as follows:

- Specification of Wave Parameters and Spudcan-Soil Stiffness Section 2
- Dynamic Analysis Modeling Section 3
- Dynamic **Response** Analysis Methods Section 4
- **Dynamic Amplification Factor and Inertial Load Set** Section 5

5 Exception

Since 2008 the ABS MODU Rules require that wave induced dynamic response is to be included in the SEU design, except when the dynamic amplification factor (DAF) obtained from SDOF given in 4/5.1 is less than 1.1 considering the SEU as pin-ended at least 3 m (10 ft) below sea bed. However, caution should still be exercised since the SDOF approach may underestimate the dynamic response when the ratio of the natural period of the SEU to the wave period exceeds unity (1.0) or is less than 0.6. See also the limitations for use SDOF given in 4/5.1.4.

FIGURE 1
Flowchart of the Two-Step Procedure (1 February 2014)





SECTION **2 Specification of Wave Parameters and Spudcan-Soil Stiffness** (1 February 2014)

1 Introduction

Environmental and geotechnical data are inherent to site-specific design and analysis. In the Classification of a MOU, the environmental conditions (such as wave, current and wind) that are used in design are selected by the Owner and become a basis of the unit's Classification. It is an assumption of Classification that the Owner will not operate the unit in environmental and other conditions that produce loads that are worse than those reviewed for Classification. This principle carries over to the dynamic response assessment.

In Classification, it is usual that the design storm is expressed deterministically, via the parameters (H_{max} , T_{ass}). However, procedures used to explicitly compute dynamic response mostly rely on a spectral representation of the design-level sea states, so guidance is provided below in Subsection 2/3 on characterizing the design storm sea state in terms of (H_s , T_p) and the defining spectral formulation.

Also in Classification, the MODU Rules have specified that the bottoms of the legs should be assumed to penetrate to a depth of at least 3 meters below the seabed, and that each leg end (i.e., spudcan) is pinned (i.e., free to rotate about the axes normal to the leg's longitudinal axes, but fixed against displacements). Since (2003), a change was made to the MODU Rules that affects this practice. When the Owner wishes to credit spudcan-soil rotational stiffness at the bottom of each leg, this can be done in a manner as outlined in Subsection 2/5 below.

3 Spectral Characterization of Wave Data for Dynamic Analysis

The wave conditions typically specified for Classification are regular waves. The deterministic parameters (H_{max} , T_{ass}) of the regular wave need to be restated as wave spectral parameters (H_s , T_p) for the dynamic analysis.

Where suitable data is not available, the following procedures may be used to convert the deterministic wave parameters to spectral parameters:

$$\begin{aligned}
 H_{srp} &= H_{max}/1.75 \quad (\text{for cyclonic areas}) \\
 &= H_{max}/1.86 \quad (\text{for non-cyclonic areas}) \dots\dots\dots(2.1)
 \end{aligned}$$

$$H_s = [1.0 + (10H_{srp}/T_p^2) e^{(-d/25)}] \times H_{srp} \dots\dots\dots(2.2)$$

$$\begin{aligned}
 T_p &= 1.05T_{ass} && \text{when } 4.00\sqrt{H_{srp}} < T_p < 4.72\sqrt{H_{srp}} \\
 &&& \text{but if } T_p > 4.72\sqrt{H_{srp}}, \text{ then use } T_p = 4.72\sqrt{H_{srp}} \\
 &&& \text{if } T_p < 4.00\sqrt{H_{srp}}, \text{ then use } T_p = 4.00\sqrt{H_{srp}}
 \end{aligned}$$

where

- H_{srp} = significant wave height, in meters, of the three-hour storm for the assessment return period
- H_s = effective significant wave height, in meters
- d = water depth, in meters ($d > 25$ m)
- T_p = peak period associated with H_{srp} (also used with H_s), in seconds

Equation (2.2) is the Wheeler stretching adjustment that accounts some nonlinear effects around the free surface in shallower water depth. The JONSWAP spectrum with a peak enhancement factor of 3.3 and the above calculated H_s and T_p should be used to represent the considered sea state. The short-crestedness of waves should not be considered.

5 Spudcan-Soil Rotational Stiffness (SC-S RS)

Since 2003, the ABS *MODU Rules* permits consideration of “spudcan-soil rotational stiffness” for cases involving dynamic response. The maximum extent to which this rotational stiffness can be applied to the system, $K_{rs, fixed}$, is defined by the following equation.

$$K_{rs, fixed} = EI / (L C_{min})$$

where

- E = Young’s modulus, 209GPa for steel
- I = moment of inertia, in m^4
- L = the sum of the distance, in m, from the underside of the hull to seabed plus the seabed penetration (minimum 3 meters) $\geq 4.35(I/A_s)^{0.5}$
- C_{min} = $(1.5 - J)/(J + F)$
- J = $1 + [7.8 I/(A_s L^2)]$
- F = $12 I F_g / (A Y^2)$
- A = axial area of the equivalent leg, in m^2
- A_s = shear area of the leg, in m^2
- Y = the distance, in m, between the centerline of one leg and a line joining the centers of the other two legs for a 3-leg unit; the distance, in m, between the centers of leeward and windward rows of legs; in the direction of being considered
- F_g = 1.125 for a three leg unit and 1.0 for a four leg unit

The Owner may select values of SC-S RS ranging from zero (the pinned ended condition) up to the maximum value indicated.



SECTION 3 Dynamic Analysis Modeling (1 February 2014)

1 Introduction

To determine a DAF, a simplified Dynamic Analysis model, as indicated below, may be used. The usual level of modeling employed in this case is designated as an “equivalent model”. Inaccurate or inappropriate modeling can have a major **effect** on the calculated structural responses, therefore, special care should be exercised to assure that the modeling and application of the dynamic loading is done **appropriately**. The stiffness of the Dynamic Analysis model should also be consistent with that of the “**detailed**” model used for the Quasi-Static structural **analysis** to check the adequacy of the structure by the permissible stress unity check criteria of the *MODU Rules*.

3 Stiffness Modeling

The level of stiffness modeling of the “equivalent model” for dynamic analysis discussed in this section includes

- Leg stiffness
- **Hull stiffness**
- Leg-to-hull connection **stiffness** (stiffness of jacking system, proper load transfer direction of guides, pinions and clamps, etc.)
- **P-Delta effect**
- **Foundation stiffness (leg-to-seabed interactions)**

3.1 Leg Stiffness

The stiffness of a leg is characterized by the following equivalent cross sectional properties:

- Cross sectional area
- Moment of inertia
- Shear area
- Torsional moment of inertia

The dominant factor affecting the **leg** stiffness is leg bending, but other compliance should be incorporated, such as the shear deflection of legs. The shear deflection of most members is small, but it can be significant in a ‘**lattice**’ structure. Therefore, shear deflection of legs should be properly incorporated in the analysis model.

In an equivalent model, a leg can be modeled by a series of collinear beams. The cross sectional properties of the beams may be derived by employing the formulas given in **Subsection A1/3** or by applying various unit **load cases** to the detailed leg model, **following the procedure given in Subsection A1/5**. If the properties are calculated with the formulas, they may change along the axis of the leg because the properties of the members constituting the leg may vary along the axis of the leg. Although it is not required to model each bay of the leg with a beam element, doing this will facilitate a more accurate mass distribution along the leg.

A spudcan may usually be modeled as a rigid member.

3.3 Hull Stiffness

Hull structure can be modeled as a grillage of beam members. The properties of the beam may be calculated based on the depth of the bulkheads and side shell and the effective width of deck and bottom plating.

The overall structural stiffness or, in turn, the natural period of a unit is less sensitive to hull stiffness. Therefore, the grillage of beams can simply consist of several beam members at each location of the bulkheads and side shell. When considering the contribution from deck(s) and bottom plating, the effective width of deck(s) or bottom plating assigned to a beam member is so determined that the overlapping plan area reaches minimum (i.e., to minimize the areas whose contribution is either not included or included twice). This overlapping will happen when the axes of adjacent beams are not parallel to each other.

The second moment of area of the hull is normally much higher than that of the leg. A common error is to not make the rotational stiffness and “in plane” bending stiffness of equivalent hull members high enough.

3.5 Leg-to-Hull Connection Stiffness

The leg-to-hull connection is very important to the dynamic analysis. The compliance of the connection is due to a number of factors:

- There may be a global rotation of the leg between the guides due to compliance of the jacking/holding system.
- There may be a global rotation of the leg between the guides due to the local deflection of the guide structure.
- Local deflection of the leg chords, induced by the guide reactions, may lead to an effective rotation of the leg. Also, deformation of the chord wall itself will produce additional leg rotation.

Due to this compliance of the connection, the rotational, horizontal and vertical stiffness of the connection should be modeled with adequate accuracy. A rigid connection is usually not considered acceptable unless the justification of this simplification is provided.

In an equivalent model, the rotational stiffness of the connection may be represented by linear rotational springs and the horizontal and vertical stiffness by linear translational springs. The stiffness of the springs may be derived by employing the formulas given in Subsection A2/3 or by applying various unit load cases to the detailed leg-to-hull connection model, provided the detailed model appropriately represents the stiffness of the connection, following the procedures given in Subsection A2/5.

3.7 P-Delta Effect – ($P-\Delta$)

The actual structure will be less stiff than estimated from a linear analysis because of displacement dependent effects, $P-\Delta$ or Euler amplification. This will tend to increase the deflection of the structure, thereby reducing its effective stiffness. Therefore, the $P-\Delta$ effect should be accounted for in the Dynamic Analysis model.

A common way to account for the $P-\Delta$ effect is the geometric stiffness method. In this method, negative stiffness correction terms are introduced into the global stiffness matrix of the Dynamic Analysis model. In order to do this, springs of negative stiffness are connected between each spring’s fixed reaction point and a point on each leg where the hull intersects the leg. The negative stiffness for horizontal displacements is given by:

$$K_{pd} = -P_g/L$$

where

P_g = total effective gravity load on each leg, including hull weight and weight of the leg above the hull and leg joint point

L = distance from the spudcan reaction point to the hull vertical center of gravity

3.9 Foundation Stiffness

Additional stiffness to represent the Spudcan-Soil Rotational Stiffness may be included in the model to the extent indicated in [Subsection 2/5](#).

One way to implement this in the equivalent model is for each leg to have a **pair of orthogonal rotational springs** of specified stiffness **horizontally** connected to the reaction point on the leg and an “earth” point where all degrees of freedom are fixed.

5 Modeling the Mass

The mass that will be dynamically excited and the distribution of that mass should be represented accurately in the Dynamic Analysis model. Items that should be considered include:

- The elevated mass (arising from hull self-weight; mass of additional equipment, variable mass from drilling equipment and consumables and other supplies)
- Leg mass, added mass and any entrained and entrapped (water) mass
- Spudcan mass and entrapped (water) mass

Usually, no mass from functional loads will need to be considered as participating in the dynamic response.

Leg mass can be modeled as nodal **masses** along the **leg**. A mass for each bay is adequate for the dynamic analysis. Added mass and any entrained/entrapped mass should be included. If more accurate information about mass distribution is not available, elevated weight may be modeled as nodal masses acting on the hull at its connection to legs.

7 Hydrodynamic Loading

The hydrodynamic loads to be considered in the dynamic analysis are those induced by waves and waves acting with current. The basis of the hydrodynamic loading is Morison’s equation, as applied to the Dynamic Analysis model. Equivalent drag and mass coefficients should be developed for the “equivalent leg” idealization of the leg, and as applicable, the spudcan, etc. **Formulas for deriving the equivalent drag and mass coefficients of the leg are presented in Appendix 3**. The current profile should be as specified for Classification, with stretching and compression effects as specified in [Subsection A3/9](#). The hydrodynamic load **calculation** should consider the relative velocities between the wave and the structure.

When deriving the hydrodynamic properties, such as equivalent diameter, area, drag and mass coefficients of a leg, it is important to account for all members, such as chords, horizontal members, diagonal members, span breakers, etc., in a bay of the leg and their orientations. Some of the properties, i.e., drag coefficient, are storm-heading-dependent.

Where the dynamic analysis is performed considering sea state simulation using random wave generation procedures, as described in Section 4, Airy wave theory can be used to develop the hydrodynamic forces.

When determining loads due to the simultaneous occurrence of waves and current using Morison’s equation, the current velocity is to be added vectorially to the wave particle velocity before the total force is computed.

9 Damping

Damping can have a significant **effect** on the response. The total damping ratio to be used in the dynamic response analysis (expressed as a percentage of the critical damping) is defined as:

$$\zeta = c/c_{cr} \cdot 100 \quad \%$$

where

c = system damping

c_{cr} = critical damping = $2\sqrt{m \cdot k}$

m = effective mass of the system

k = effective stiffness of the system

The **total** damping ratio should not be taken more than **7%**. The three main sources of damping are:

- Structural, including holding system, normally taken as 2% maximum on an independent leg **SEU**.
- **Small strain foundation**, normally taken as 2% maximum for an **SEU** with independent legs.
- Hydrodynamic, **if** the relative velocity term **is** incorporated into **the** dynamic analysis, damping to account for hydrodynamic damping should not be considered. **However, when using the approach that does not consider the relative velocity term, a maximum additional hydrodynamic damping of 3% can be assumed.**



SECTION 4 Dynamic Response Analysis Methods

1 General (1 February 2014)

An SEU responds dynamically to waves. This behavior should be modeled appropriately in the SEU's global strength analysis by including the static and dynamic contributions. Fully detailed random wave dynamic analysis in the time domain may be pursued to obtain the static and dynamic responses for design of an SEU's global strength. However, the "inertial load set" approach described in 1/3 is most often used in practical design, and yields sufficiently good results in normal circumstances. In this approach, the random wave dynamic analysis is performed only for determining appropriate values for DAFs and for subsequently capturing the dynamic contributions as inertial loads using the determined DAFs.

The random wave dynamic analysis approach is based on considering the wave (sea-state) as a random quantity. Using a time domain approach, the most probable maximum extreme (MPME) values of selected static and dynamic responses are obtained. The DAF is the ratio of the MPME of the dynamic response to that of the static response. The MPME is the mode, or highest point, of the *probability density function* (PDF) for the extreme of the response being considered. This is a value with an approximately 63% chance of exceedance, corresponding to the 1/1000 highest peak level in a sea-state with a 3-hour duration. There are several methods to predict a selected extreme response, as will be addressed later in Subsection 4/3.

A simpler method referred to as the *Single Degree of Freedom* (SDOF) Approach can also be used for deriving the DAFs, which will be discussed later in Subsection 4/5. Due to the limitations of the single-degree-of-freedom (SDOF) approach the random-wave-time-domain approach is the preferred one to be applied for deriving the DAFs.

3 Random Wave Dynamic Analysis in Time Domain

3.1 General (1 February 2014)

The "equivalent" model indicated in Section 3 is usually employed in time domain analysis. In time domain simulation, a Gaussian random sea state is generated, and the time-step for the simulation is required to be sufficiently small. The duration of the simulation(s) should also be sufficiently long for the method being used to reliably determine the extreme values of the responses being sought.

The overall methodology is to determine the Most Probable Maximum Extreme (MPME) values of the dynamic and static responses in the time domain. The ratio of these two values – defined as DAF – represents the ratio by which the static response, obtained using a high order wave theory and the maximum wave height, should be increased in order to account for dynamic effects. A DAF can be calculated for each individual global response parameter, (e.g., base shear, overturning moment or hull sway). Usually, DAF of overturning moment is higher than the other two.

3.3 Random Wave Generation

The wave elevation may be modeled as a linear random superposition of regular wave components, using information from the wave spectrum. The statistics of the underlying random process are Gaussian and fully known theoretically. An empirical modification around the free surface may be needed to account for free surface effects (Wheeler stretching, Equation 2.2). The following criteria are to be satisfied for the generated random waves.

3.3.1 Wave Components

The random wave generation should use at least 200 wave components with divisions of equal wave energy. It is recommended that smaller energy divisions be used in high frequency regions of the spectrum, where the enforcement and cancellation frequencies are located.

3.3.2 Validity of Generated Sea State

The generated random sea state must be Gaussian and should be checked for validity, as follows:

- Correct mean wave elevation
- Standard deviation = $(H_s/4) \pm 1\%$
- $-0.03 < \text{skewness} < 0.03$
- $2.9 < \text{kurtosis} < 3.1$
- Maximum crest elevation = $(H_s/4) \sqrt{2 \ln(N)}$ (error within -5% to +7.5%),

where

$$N = \text{number of wave cycles in the time series being qualified, } N \approx \text{Simulation Duration}/T_z$$

$$T_z = \text{zero up-crossing period of the wave}$$

3.3.3 Random Seed Effect

Depending on the method used to predict extreme responses and DAF, the random seed effect can be significant. Care should be taken to ensure that the predicted results are not affected by the selection of random seeds.

3.5 Calculation of Structural Response

The structural response should be obtained using the Dynamic Analysis model discussed in Section 3. The analysis model (i.e., the equivalent model with proper loading and boundary conditions) is to be solved using a reliable solver having the capability to do time domain calculations and response statistics calculations. Special attention is to be paid to the topics listed below.

3.5.1 Validity of the Natural Periods of Equivalent Model

The natural periods of a structure are the most important indicators of the dynamic characteristics of the structure. If the computed natural periods are not reasonable, there must be something wrong with the established equivalent model, either its stiffness distribution or its mass distribution, or both. Therefore, the check of natural periods is an indispensable step in the dynamic analysis.

The natural periods of the established equivalent model can be found by solving the eigen-value problem, and the fundamental natural period should be checked against that estimated from the SDOF approach in 4/5.1.

3.5.2 Number of Simulations and Simulation Duration (1 February 2014)

There are four prevalent methods, as listed in 4/3.7, which can be used to establish the needed MPME values of the response from the time domain analysis. Each of these extreme value prediction methods has specific needs regarding the recommended number and duration of the simulations that should be performed to establish a sufficient statistical basis on which to obtain the MPME value. Therefore, the recommended number and duration of the simulations given below should be followed in the calculation of structural response.

- i) *Drag-Inertia Parameter Method:* Simulation time of at least 60 minutes; four simulations with different control parameters, (i.e., fully dynamic, quasi-static, quasi-static with C_d (drag coefficient) = 0 and quasi-static with C_{Me} (inertia coefficient) = 0)
- ii) *Weibull Fitting method:* Simulation time of at least 60 minutes; number of simulation ≥ 5 .
- iii) *Gumbel Fitting method:* Simulation time of at least 180 minutes; number of simulation ≥ 10 .
- iv) *Winterstein/Jensen method:* Simulation time of at least 180 minutes; number of simulation = 1.

More detailed descriptions of these four methods are provided in 4/3.7.

It should be noted that the “static response analysis” described here and in 4/3.7.1 is performed using the Dynamic Analysis model, but with the mass and damping terms set to zero. This analysis is performed to establish DAFs. It should not be confused with the analysis that is described later with the more detailed model that is used for a Quasi-Static structural analysis to obtain the “unity-checks”, as described in Section 5.

3.5.3 Time Step of the Simulations

The integration time-step should be less than, or equal to, the smaller of the following equations, unless it can be shown that a larger time-step leads to no significant change in results.

$$T_z/20 \quad \text{or} \quad T_n/20$$

where

- T_z = zero up-crossing period of the wave
- = $T_p/1.406$ for the Pierson-Moskowitz spectrum
- T_n = first mode natural period of the SEU

3.5.4 Transients

Transient response is to be discarded by removing the first 100 seconds of the response time series before predicting the extreme responses.

3.5.5 Relative Velocity

It is expected that the relative velocity between the wave particle and structural velocities will be included in the hydrodynamic force formulations used in the time domain analysis.

3.7 Prediction of Extreme Responses

Although the waves are considered linear and statistically Gaussian, the structural response of an SEU is likely to be non-Gaussian due to non-linear drag force and free surface effects which are included in the wave kinematics calculations. The statistics of such a non-Gaussian process are generally not known theoretically, but the extremes are generally larger than the extremes of a corresponding Gaussian random process. For a detailed investigation of the dynamic behavior of an SEU, the non-Gaussian effects should be included. The four prevalent methods elaborated below are considered acceptable for this purpose.

3.7.1 Method I – Drag/Inertia Parameter Method (1 February 2014)

The drag/inertia parameter method is based on the assumption that the extreme value of a standardized process can be calculated by splitting the process into two parts, evaluating the extreme values of each and the correlation coefficient between the two, then combining as:

$$(mpm_R)^2 = (mpm_{R1})^2 + (mpm_{R2})^2 + 2\rho_{R12}(mpm_{R1})(mpm_{R2}) \dots \dots \dots (4.1)$$

The extreme values of the dynamic response can therefore be estimated from the extreme values of the static response, which is obtained by solving the dynamic equation with both mass term and damping term equal to zero, and the so-called “inertia” response, which is in fact the difference between the dynamic response and the static response. The correlation coefficient of the static and “inertia” responses is calculated as:

$$\rho_R = \frac{\sigma_{Rd}^2 - \sigma_{Rs}^2 - \sigma_{Ri}^2}{2\sigma_{Rs}\sigma_{Ri}} \dots \dots \dots (4.2)$$

The extreme value of the “inertia” response can be reasonably expressed as:

$$mpm_{Ri} = 3.7 \sigma_{Ri} \dots \dots \dots (4.3)$$

In the drag/inertia method the extreme value used is the MPME value of the response and the method requires the response of the SEU to be determined for four conditions. In all four cases the storm simulation (random seed) should be identical, but with different components of the loading and/or response simulated. The responses considered will usually be total wave and current base shear and total wave and current overturning moment, for computing the base shear and overturning moment DAFs, respectively.

The four cases to be simulated are full dynamic response, full static response, static response to inertia only wave loading (setting $C_d = 0$) and static response to drag only loading (setting $C_m = 0$). From these the inertial response is obtained as the full dynamic response minus the full static response. The means and standard deviations of the response are extracted from the time domain responses and the DAFs computed as illustrated in Section 4, Figure 1.

The drag-inertia method given here includes a final step to scale the DAF based on the period ratio T_n/T_p . This step is included to ensure that the DAF values are not underestimated for cases where T_n approaches T_p , see Reference 7. The equation for the scaling factor is given in Section 4, Figure 1, and it is illustrated in Section 4, Figure 2.

FIGURE 1
The Drag-Inertia Method Including DAF Scaling Factor (1 February 2014)

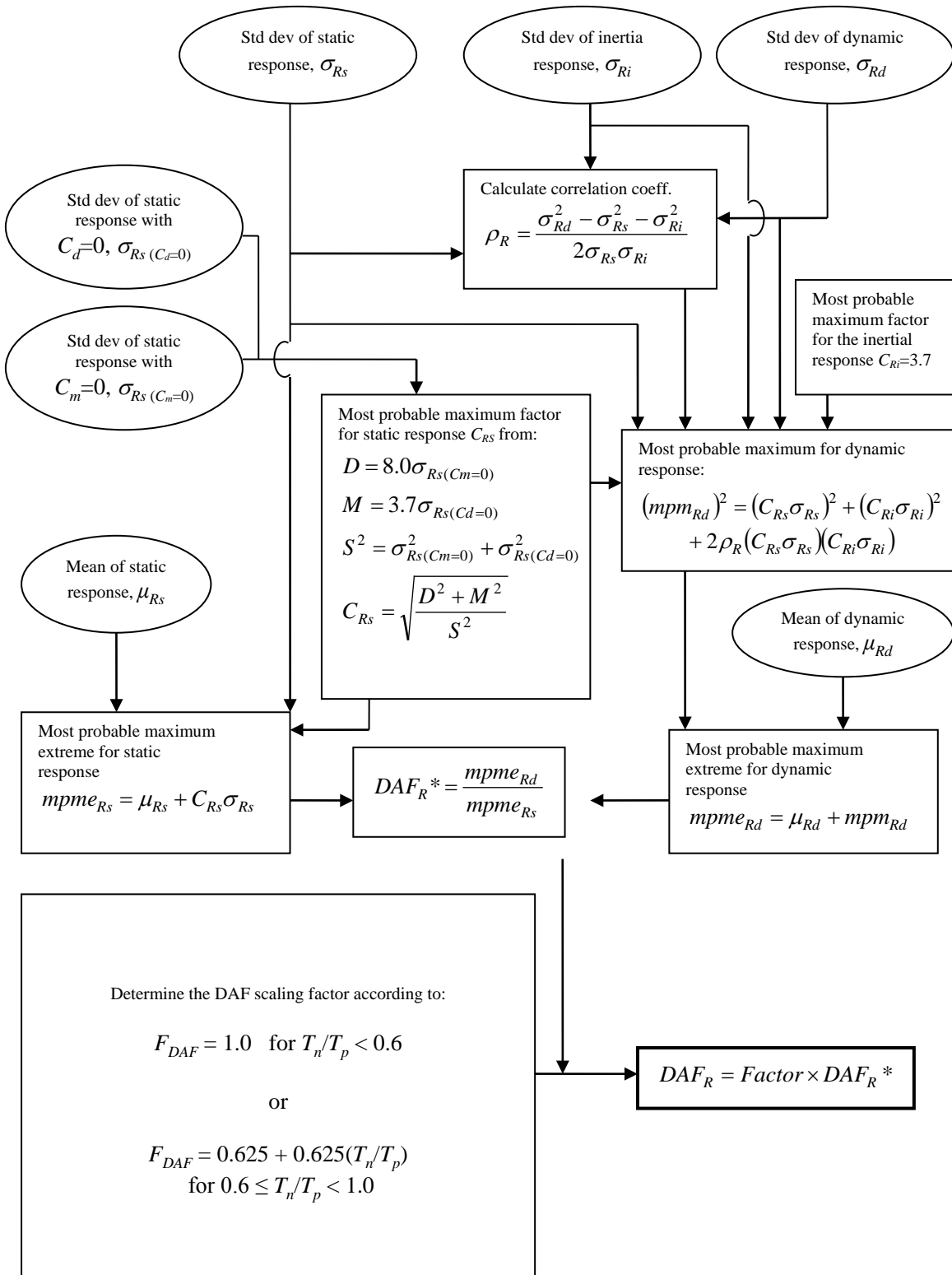
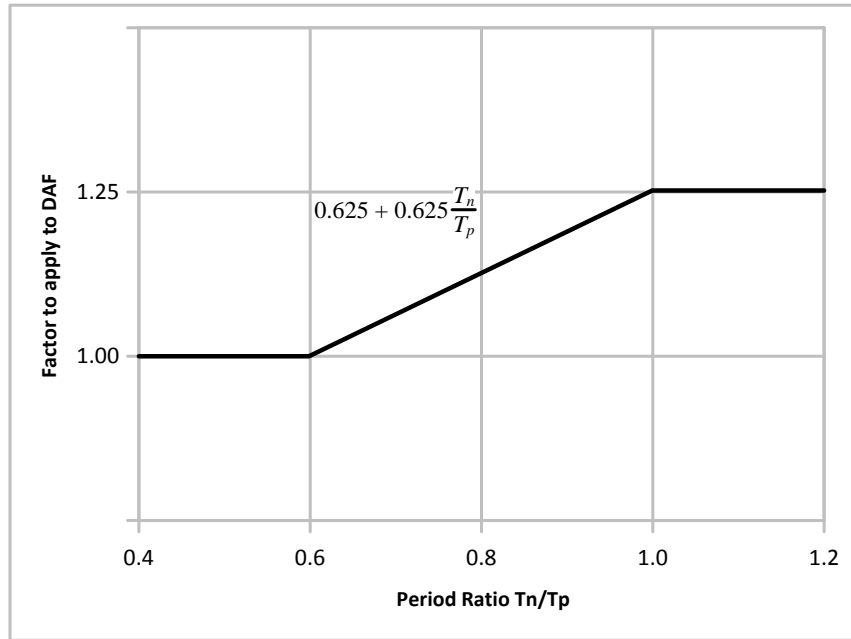


FIGURE 2
Graphical Representation of DAF Scaling Factor, F_{DAF} ,
Applied in the Drag-Inertia Method (1 February 2014)



3.7.2 Method II – Weibull Fitting (1 February 2014)

Weibull fitting is based on the assumption that for a drag dominated structure, the cumulative distribution of the maxima of the structural response can be fitted to a Weibull class of distribution:

$$F_R = 1 - \exp\left[-\left(\frac{R - \gamma}{\alpha}\right)^\beta\right] \dots\dots\dots (4.4)$$

The extreme value for a specified exceedance probability (e.g., 1/N) can therefore be calculated as:

$$R = \gamma + \alpha[-\ln(1 - F_R)]^{1/\beta} \dots\dots\dots (4.5)$$

Using a uniform level of exceedance probability of 1/N, Equation (4.8) leads to

$$R_{MPME} = \gamma + \alpha[-\ln(1/N)]^{1/\beta} \dots\dots\dots (4.6)$$

The key issue for using this method is therefore to calculate the parameters α , β and γ , which can be established from regression analysis, maximum likelihood estimation or static moment fitting can estimate. For a 3-hour storm simulation, N is approximately 1000. The time series record is first standardized ($R^* = (R - \mu)/\sigma$), and all positive peaks are then sorted in ascending order.

As recommended in Reference 1, only peaks corresponding to a probability of non-exceedance greater than 0.2 are to be used in the curve fitting, and least square regression analysis is used for estimating Weibull parameters.

3.7.3 Method III – Gumbel Fitting (1 February 2014)

The Gumbel fitting method is based on the assumption that the three-hour extreme values follow the Gumbel distribution:

$$F(x_{extreme} \leq X_{MPME}) = \exp\left[-\exp\left(-\frac{1}{\kappa}(X_{MPME} - \psi)\right)\right] \dots\dots\dots (4.7)$$

The most probable maximum extreme discussed here corresponds to an exceedance probability of 1/1000 in a distribution function of individual peaks or to 0.63 in an extreme probability distribution function. The MPME of the response can therefore be calculated as:

$$\begin{aligned}
 X_{MPME} &= \psi - \kappa \ln\{-\ln[F(X_{MPME})]\} \\
 &= \psi - \kappa \ln[-\ln(0.37)] \approx \psi \dots\dots\dots (4.8)
 \end{aligned}$$

Now, the key issue is to estimate the parameters ψ and κ based on the response obtained from time-domain simulations. Reference 1 recommends that the maximum simulated value be extracted for each of the ten 3-hour response simulations, and that the parameters be computed by maximum likelihood estimation. Similar calculations should also be performed using the ten 3-hour minimum values. Although it is always possible to apply the maximum likelihood fit numerically, the method of moments may be preferred. See Reference 8.

For the Gumbel distribution, the mean and variance are given by

$$\begin{aligned}
 \text{Mean:} \quad \mu &= \psi + \gamma \cdot \kappa, \quad \gamma = \text{Euler constant (0.5772...)} \\
 \text{Variance:} \quad \sigma^2 &= \pi^2 \kappa^2 / 6
 \end{aligned}$$

By which means, the parameters ψ and κ can be directly obtained using the moment fitting method:

$$\kappa = \frac{\sqrt{6}\sigma}{\pi}, \quad \psi = \mu - 0.57722\kappa \dots\dots\dots (4.9)$$

3.7.4 Method IV – Winterstein/Jensen Method

The basic premise of Winterstein/Jensen method is that a non-Gaussian process can be expressed as a polynomial (e.g., a power series or an orthogonal polynomial) of a zero mean, narrow-banded Gaussian process (represented here by the symbol U), that is

$$R(U) = C_0 + C_1U + C_2U^2 + C_3U^3 \dots\dots\dots (4.10)$$

The same relationship exists between the MPMEs of the two processes. Since the MPME of Gaussian process U is theoretically known, the MPME of the non-Gaussian process can be calculated if the coefficients C_0, C_1, C_2, C_3 are determined.

3.7.4(a) *Determination of U_m* . Calculate the following statistical quantities of the time series for the response parameter R under consideration:

- μ_R = mean of the process
- σ_R = standard deviation
- α_3 = skewness
- α_4 = kurtosis

Then construct a standardized response process, $z = (R - \mu_R)/\sigma_R$. Using this standardized process, calculate the number of zero-upcrossings, N . In lieu of an actual cycle count from the simulated time series, $N = 1000$ may be assumed for a 3-hour simulation.

The most probable value, U_m , of the transformed process is computed by the following equation:

$$U_m = \sqrt{2 \log_e \left(N \cdot \frac{3 \text{ hours}}{\text{simulation time (in hours)}} \right)} \dots\dots\dots (4.11)$$

where U_m is the most probable value of a Gaussian process of zero mean, unit variance.

3.7.4(b) *Determination of C coefficients (1 February 2014).* One can establish the following equations for C_1 , C_2 and C_3 :

$$\begin{aligned} \sigma_R^2 &= C_1^2 + 6C_1C_3 + 2C_2^2 + 15C_3^2 \\ \sigma_R^3 \alpha_3 &= C_2(6C_1^2 + 8C_2^2 + 72C_1C_3 + 270C_3^2) \\ \sigma_R^4 \alpha_4 &= 60C_2^4 + 3C_1^4 + 10395C_3^4 + 60C_1^2C_2^2 + 4500C_2^2C_3^2 + 630C_1^2C_3^2 \\ &\quad + 936C_1C_2^2C_3 + 3780C_1C_3^3 + 60C_1^3C_3 \end{aligned}$$

Solve the equations with the initial guesses as:

$$\begin{aligned} C_1 &= \sigma_R K(1 - 3h_4) \\ C_2 &= \sigma_R K h_3 \\ C_3 &= \sigma_R K h_4 \end{aligned}$$

where

$$\begin{aligned} h_3 &= \alpha_3 / [4 + 2\sqrt{1 + 1.5(\alpha_4 - 3)}] \\ h_4 &= [\sqrt{1 + 1.5(\alpha_4 - 3)} - 1] / 18 \\ K &= [1 + 2h_3^2 + 6h_4^2]^{-1/2} \end{aligned}$$

Obtain

$$C_0 = \mu_R - \sigma_R K h_3$$

3.7.4(b) *Determination of R_{MPME} .* The most probable maximum extreme in a 3-hour storm, for the response under consideration, can be computed from the following equation:

$$R_{MPME} = C_0 + C_1 U_m^1 + C_2 U_m^2 + C_3 U_m^3 \dots \dots \dots (4.12)$$

5 Other Dynamic Analysis Methods (1 February 2014)

(1 February 2014) The random wave time domain method is the recommended approach for the dynamic analysis of an SEU. However, the analysis procedure is relatively complicated and under some circumstances, other methods can also generate results of sufficient accuracy. Besides, some results obtained from the simpler methods (e.g., natural period of the structure determined by SDOF approach) can be used to check the results of time domain analysis. For these reasons, the single degree of freedom approach is briefly discussed below. A frequency domain analysis method may be useful for limited preliminary or comparative studies of system responses. However a frequency domain analysis method is not recommended as the final basis of design.

5.1 Single-Degree-of-Freedom Approach

(1 February 2014) In a single-degree-of-freedom (SDOF) approach, the SEU is modeled as a simple mass/spring/damper system. Due to its simplicity, this approach is recommended for an initial evaluation of the dynamic amplification or for use with limitations given in 4/5.1.4.

5.1.1 Natural Period

The natural period of an SEU is an important indicator of the degree of dynamic response to be expected. The first and second vibratory modes are usually surge and sway (i.e., lateral displacements at the deck level). The natural periods of these two modes are usually close to each other. Which of the two is higher depends on which direction of the structure is less stiff. The third vibratory mode is normally a torsional mode. Since the period varies with the environmental load direction, care should be taken that the period used in analysis is consistent with the environmental load being considered.

An estimate of the first mode (fundamental) natural period, T_n , is obtained for a single-degree-of-freedom (SDOF) system, as follows:

$$T_n = \frac{1}{f} = 2\pi \sqrt{\frac{M_e}{K_e}}$$

where

- f = natural frequency
- M_e = effective mass associated with one leg
- K_e = effective stiffness associated with one leg, which suitably accounts for the bending, shear and axial stiffness of each leg, the stiffness of the hull-to-leg connection and the degree of spudcan-soil rotational restraint that is to be considered

The detailed information for the calculations of M_e and K_e can be found in Reference 1.

5.1.2 Calculation of the SDOF DAF

The Dynamic Amplification Factor (DAF) of a SDOF system under the influence of a sinusoidal (monotonic) forcing function is given by the following formula:

$$DAF = \frac{1}{\sqrt{(1 - \Omega^2)^2 + (2\zeta\Omega)^2}}$$

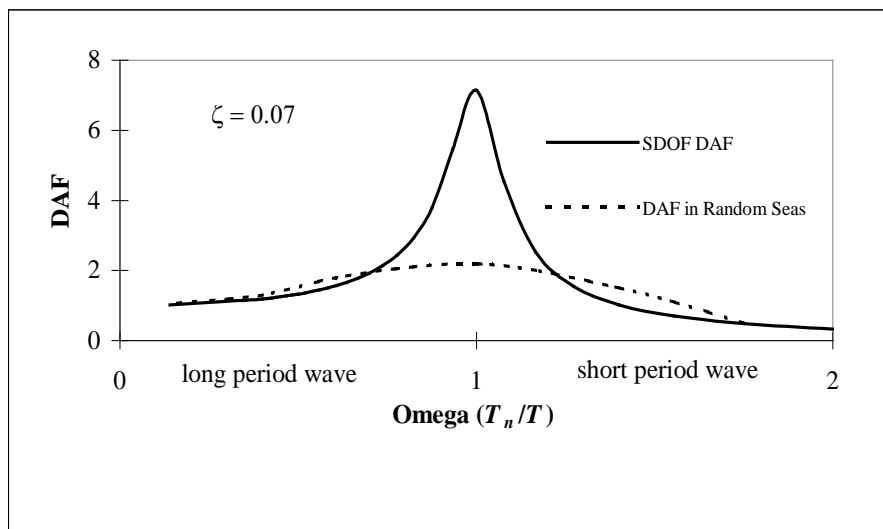
where

- Ω = $\frac{T_n}{T}$ = $\frac{\text{Natural period of the jackup}}{\text{Period of the applied load (wave period)}}$
- T = $0.9 T_p$
- ζ = damping ratio

As illustrated in Section 4, Figure 3, if the natural period of the SEU is equal to the period of the applied load (i.e., Ω is equal to 1.0), the DAF becomes just over 7 (when a damping ratio of 7% is used). Conversely, if there is a very large separation between the natural period and the load period, the DAF could be underestimated. An actual sea state can have a significant spread of energy over the period range, and the curve of DAF against Ω is likely to be much shallower than that predicted by the SDOF model. This is also illustrated in Section 4, Figure 3.

Care should be taken when determining the appropriate wave period to be used in an SDOF analysis. A range of wave periods should be investigated, along with a range of associated wave heights. The applicable sea states that result in maximum responses should be identified and used in the assessment of the adequacy of the structure's strength.

FIGURE 3
Dynamic Amplification Factor (SDOF)



5.1.3 Dynamic Load Application

The dynamic effect can be applied to the Quasi-Static model by applying an extra force representing the dynamically-induced inertial load set at the center of gravity of the hull structure. The procedure is presented in Section 5.

5.1.4 Limitations (1 February 2014)

The greatest problems with the SDOF approach are that it may grossly over-estimate the response when the natural period of the unit is close to the monotonic period of the applied load and may possibly underestimate the response when there are large differences in periods **or the natural period of the unit is longer than the period of the applied load ($\Omega > 1.0$)**. However, this method can give reasonable results when Ω is in the range of 0.4 to 0.8 and the current velocity is small relative to the wave particle velocity. Therefore, the following limitations should apply:

- i) The SDOF method can be unconservative for cases where the current velocity is large relative to the wave particle velocities. If the results of the analysis are close to the acceptance criteria further detailed analysis is recommended.
- ii) The SDOF method can be unconservative and should not normally be used in an extreme storm analysis when Ω is greater than 1.0 (i.e., when $T_n > 0.9T_p$). However, the SDOF analogy may be used when the calculated Ω is greater than 1.0 providing Ω is taken as 1.0.
- iii) A minimum value of 1.2 should be taken as the DAF for developing the inertial load set, regardless of the DAF calculated using the SDOF method.



SECTION 5 Dynamic Amplification Factor and Inertial Load Set (1 February 2014)

1 Introduction

The inertial load set required to perform a “two-step” analysis is calculated based on DAFs. The DAFs can be obtained from the random wave dynamic analysis or the SDOF approach. A commonly accepted way that the inertial load set is included in the “detailed” model for Quasi-Static structural analysis is as a concentrated load applied to the elevated hull structure. This idealization is most suitable for the case where the preponderance of the structural system’s total mass is in the hull, which is usually considered to be the case. If it were not the case, the complexity of the inertial load set would increase so that instead of a concentrated load, the inertial loads should be distributed in accordance with the mass distribution and vibratory mode shapes.

The inertial load sets calculated from the random wave dynamic analysis or the SDOF approach will not be the same, as is presented later in Subsections 5/3 and 5/5. Thus, when applying the inertial load set to the “detailed” model to simulate the dynamic response for a Quasi-Static structural analysis, special care should be exercised.

3 Inertial Load Set based on Random Wave Dynamic Analysis

The random wave dynamic analysis usually generates the DAFs for Overturning Moment (OTM) and Base Shear (BS) force. Thus, the magnitude of the concentrated inertial load set representing the dynamic response from waves (or waves acting with current) in the wave loading direction can be obtained from the following quantities:

The magnitude of the concentrated load representing the dynamic response from waves (or waves acting with current) in the wave loading direction can be obtained from the following quantities:

- d = vertical distance from the base of a leg to a location in the elevated hull structure where the concentrated inertial load is to be imposed.
- DAF_{OTM} = dynamic amplification factor for overturning moment obtained from the Dynamic Response analysis using the MPME values for the dynamic and statically considered simulated hydrodynamic loads on the unit
- DAF_{BS} = dynamic amplification factor for the base shear force obtained from the Dynamic Response analysis using the MPME values for the dynamic and statically considered simulated hydrodynamic loads on the unit
- OTM_{QS} = maximum, deterministic overturning moment from the considered wave (or wave acting with current) on the Quasi-Static structural model before the imposition of the inertial load set.
- BS_{QS} = maximum, deterministic shear force from the considered wave (or wave acting with current) on the Quasi-Static structural model before the imposition of the inertial load set.

The magnitude of the concentrated inertial force, F_I , and the correction moment, $OTM_{Correction}$, are then found, respectively, from the following equations:

$$F_I = (DAF_{BS} - 1) BS_{QS}$$

$$OTM_{Correction} = (DAF_{OTM} - 1) OTM_{QS} - F_I d$$

Depending on the purpose of the analysis, the correction moment can be applied as

- Horizontal or vertical couple in the hull (although these may cause additional stress in the hull structure) if the primary concern of the analysis is for leg and foundation.
- Concentrated moment at the base of the leg (although this may cause inaccuracies in the foundation model for other than pinned conditions) if the primary concern of the analysis is for hull.

5 Inertial Load Set based on SDOF Approach

When the **SDOF** approach presented in **Subsection 4/5** is applied, the procedure that **should** be followed to establish the inertial load set is as follows.

The magnitude of the **inertial load set** is determined from:

$$F_i = (DAF - 1) \times F_{wave\ amp}$$

where

F_i = inertial load set to be applied at the center of gravity of the hull

DAF = SDOF dynamic amplification factor

$F_{wave\ amp}$ = static amplitude wave force = $0.5(F_{max} - F_{min})$

F_{max}, F_{min} = maximum/minimum total combined wave and current force (or wave/current base shear) obtained from quasi-static structural analysis, using the appropriate sea state

7 Inertial Load Set Applications

The inertial load set is combined with all of the other **statically considered** loads, **such as those from wind, currents, deterministically considered wave, weights, functional loads, etc.** that should be included in the “detailed” **model for a quasi-static** structural analysis to obtain the stresses and deflections **for evaluations** with respect to the acceptance criteria given in the *MODU Rules*.

APPENDIX 1 Equivalent Section Stiffness Properties of a Lattice Leg (1 February 2014)

1 Introduction

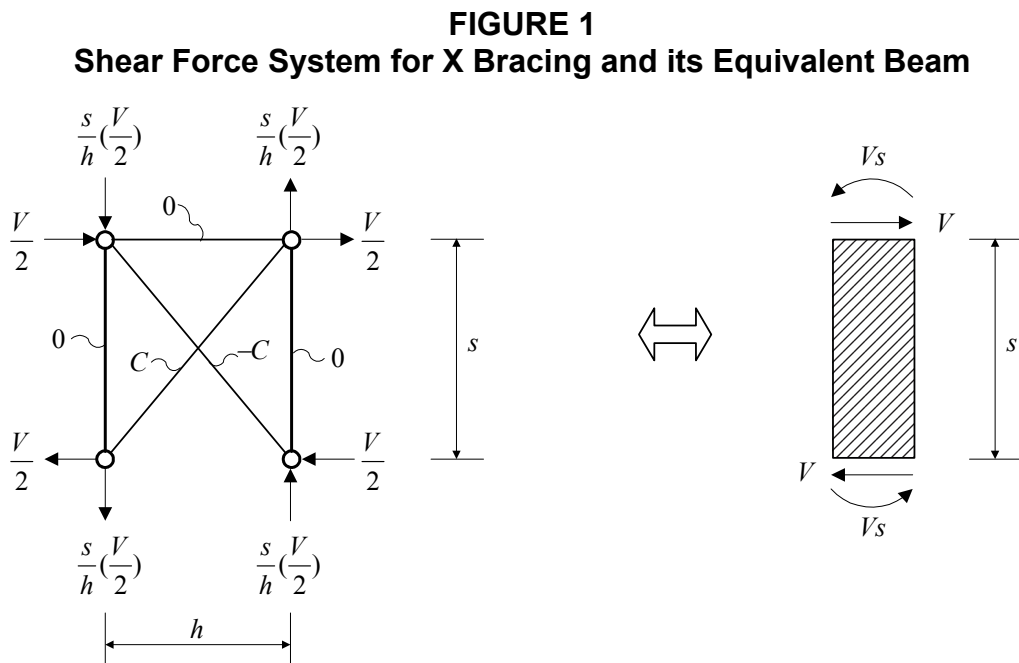
The equivalent section stiffness properties of lattice legs can be established by hand calculations using formulas or by applying unit load cases to a detailed leg model. These two methods are described in the following Subsections.

3 Formula Approach

In order to evaluate the equivalent section stiffness properties of 3D lattice legs, it is necessary first to identify the equivalent shear area of 2D lattice structures, which comprise each wall of the 3D lattice legs and the equivalent polar moment of inertia of the 3D lattice leg's cross-section. The equivalent shear area uses the equivalent 2D lattice shear area of the structure.

3.1 Equivalent Shear Area of 2D Lattice Structures

The equivalent shear area of a 2D lattice structure is evaluated by the principle of virtual work, as indicated in Reference 4. For example, Appendix 1, Figure 1 shows that the strain energy of the shear beam deformation is made equivalent to the complementary virtual work in the X bracing system.



The forces in the diagonals are $\pm C = \pm(V/2)d/h$, where d is the diagonal length, and the corresponding complementary energy for the 2D lattice truss is:

$$W^* = \frac{1}{2} \sum_i \frac{F_i^2 L_i}{EA_i} = 2 \left(\frac{1}{2} \frac{C^2 d}{EA_D} \right) = \frac{1}{4} \frac{V^2 d^3}{h^2 EA_D} \dots\dots\dots (A1.1)$$

where F_i , L_i , A_i are the force, length and area of the i -th member, and E is the modulus of elasticity. According to the principle of virtual forces, one obtains:

$$s \frac{V}{GA_Q} = \frac{\partial W^*}{\partial V} = \frac{1}{2} \frac{Vd^3}{h^2 EA_D} \dots\dots\dots (A1.2)$$

where

$$G = E/[2(1 + \nu)]$$

$$A_Q = \text{shear area of the equivalent member}$$

then:

$$A_Q = \frac{(1 + \nu)h^2 s}{\frac{d^3}{4A_D}} \dots\dots\dots (A1.3)$$

The formulae for four types of 2D lattice structures commonly employed in constructing the legs of an SEU are derived and listed in Appendix 1, Table 1. The shear areas calculated by these formulae are very close to those from the formulae presented in Reference 1 for typical SEUs in operation.

3.3 Equivalent Section Stiffness Properties of 3D Lattice Legs

The equivalent section stiffness properties of 3D lattice legs are obtained as follows:

- i) The cross-sectional area of a leg is the summation of the cross-sectional areas of all of the chords in the leg. The contribution from the braces is neglected.
- ii) The shear area of a leg’s cross-section in k direction (i.e., y or z direction) can be expressed as:

$$A_{Qk} = \sum_{i=1}^N A_Q \sin^2 \beta_i$$

where

$$A_Q = \text{equivalent shear area of 2D lattice structure}$$

$$\beta_i = \text{angle between } k \text{ direction and the normal direction of the } i\text{-th 2D lattice structure}$$

$$N = \text{total number of the 2D lattice structures in the leg (i.e., 3 or 4)}$$

- iii) The moment of inertia of the leg’s cross-section for k direction (i.e., y or z direction) is the summation of the cross-sectional area of a chord times the square of the distance from the chord center to the neutral axis of the leg’s cross-section in k direction for all chords. The contribution from the braces is neglected.
- iv) The polar moment of inertia of the leg’s cross-section is:

$$I_T = \sum_{i=1}^N A_Q \ell_i^2$$

where

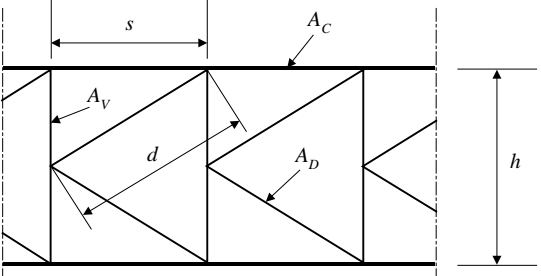
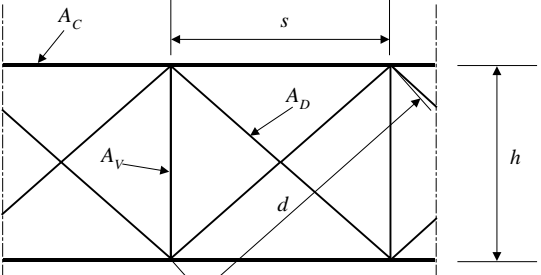
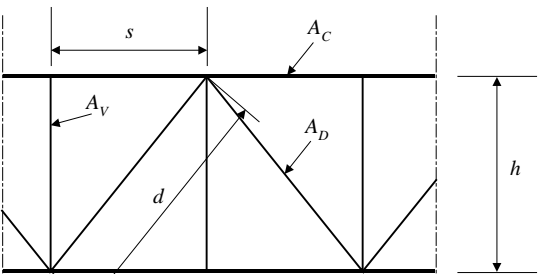
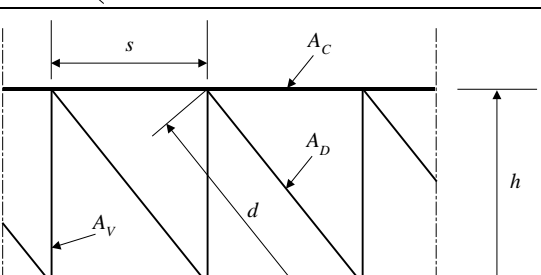
$$A_Q = \text{equivalent shear area of 2D lattice structure}$$

$$\ell_i = \text{distance from the } i\text{-th 2D lattice structure to the geometry center of the leg’s cross-section}$$

$$N = \text{total number of the 2D lattice structures in the leg (i.e., 3 or 4)}$$

Appendix 1, Table 2 presents the equivalent beam moment of inertial, which when multiplied by the modulus of elasticity provides the section stiffness properties of three types of leg configurations.

TABLE 1
Equivalent Shear Area of 2D Lattice Structures

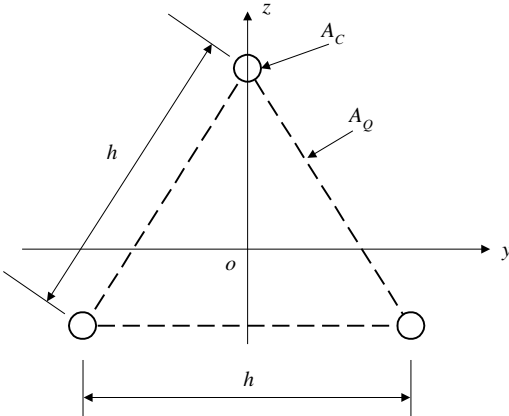
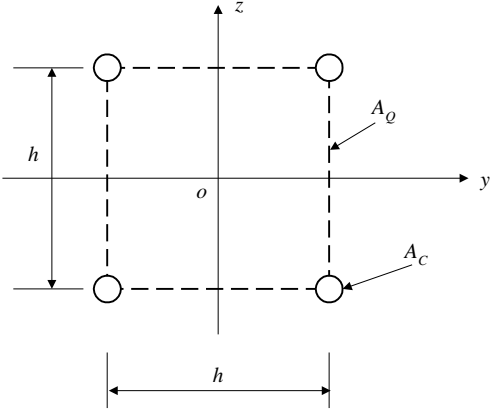
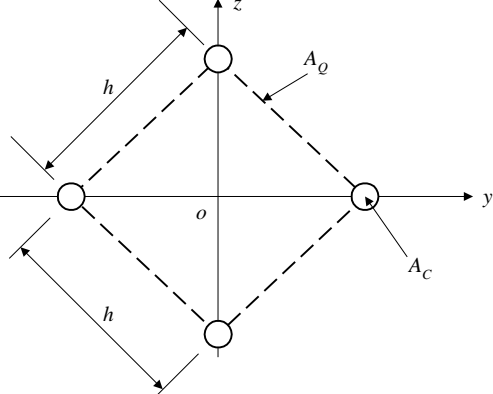
Structure	Equivalent Shear Area
	$A_Q = \frac{(1+\nu)h^2s}{\frac{d^3}{A_D} + \frac{h^3}{8A_V} + \frac{s^3}{4A_C}}$
	$A_Q = \frac{(1+\nu)h^2s}{\frac{d^3}{4A_D}}$
	$A_Q = \frac{(1+\nu)h^2s}{\frac{d^3}{2A_D} + \frac{s^3}{A_C}}$
	$A_Q = \frac{(1+\nu)h^2s}{\frac{d^3}{2A_D} + \frac{h^3}{2A_V} + \frac{s^3}{4A_C}}$

Note:

ν = Poisson ratio

A_k = cross sectional area of the corresponding member ($k = C, D$ or V)

TABLE 2
Equivalent Moment of Inertia Properties of 3D Lattice Legs

Leg Configuration	Equivalent Section Stiffness Properties
	$A = 3A_C$ $A_{Qy} = A_{Qz} = 3A_Q / 2$ $I_y = I_z = A_C h^2 / 2$ $I_T = A_Q h^2 / 4$
	$A = 4A_C$ $A_{Qy} = A_{Qz} = 2A_Q$ $I_y = I_z = A_C h^2$ $I_T = A_Q h^2$
	$A = 4A_C$ $A_{Qy} = A_{Qz} = 2A_Q$ $I_y = I_z = A_C h^2$ $I_T = A_Q h^2$

5 Unit Load Approach

The unit load method requires a detailed lattice leg model for deriving the equivalent section stiffness properties. The detailed leg model is restrained at the center of the lower guide or where the center of the fixation system is located. The detailed leg model acts as a cantilever beam. The unit load is applied at the free end of the model to calculate in accordance with elastic beam theory the slope and deflection where the unit load is applied. The following unit load cases are needed to determine the equivalent section stiffness properties.

5.1 Unit Axial Load Case

This case determines the axial area of the equivalent leg as:

$$A = \frac{FL}{E\Delta} \dots\dots\dots (A1.4)$$

where

- A = equivalent axial area of the lattice leg
- Δ = axial deflection of cantilever at point of load application
- F = applied axial load
- L = length of cantilever (from rigid support to point of load application)
- E = Young's modulus

5.3 Unit Shear Load Case

This case determines the area moment of inertia of the equivalent leg as:

$$I = \frac{PL^2}{2E\theta} \dots\dots\dots (A1.5)$$

where

- I = equivalent area moment of inertia of the lattice leg
- P = applied horizontal load
- L = length of leg from restrained point to load application point
- E = Young's modulus
- θ = end slope of cantilever at point of load application

This case also determines the shear area of the equivalent leg as:

$$A_s = \frac{PL}{\Delta_s G} \dots\dots\dots (A1.6)$$

where

- A_s = equivalent shear area of the lattice leg
- G = shear modulus
- P = applied horizontal load
- Δ_s = Δ_T - Δ_b (deflection due to shear at the point of load application)
- Δ_T = total deflection at the point of load application
- Δ_b = $\frac{PL^3}{3EI}$ (deflection due to bending at the point of load application)

5.5 Unit Torsional Moment Case

This case determines the torsional moment of inertia of the equivalent leg as:

$$J = \frac{M_T L}{G \cdot \phi} \dots\dots\dots (A1.7)$$

where

- J = equivalent torsional moment of inertia of the lattice leg
- M_T = applied torsional moment
- ϕ = resulted rotational angle about the axis of leg
- G = shear modulus
- L = length of leg from restrained point to load application point



APPENDIX 2 **Equivalent Leg-to-Hull Connection Stiffness Properties** (1 February 2014)

1 Introduction

The leg-to-hull connection of the “equivalent model” should represent the overall stiffness characteristics of the connection. The overall stiffness for rotation and translation of the connection can be derived by hand calculations using the empirical formulas or by applying unit load cases to two detailed leg models; one without the leg-to hull connection and the other with the leg-to-hull connection. These two methods are described in the following subsections.

3 Empirical Formula Approach

The stiffness of the equivalent hull-to-leg connection, K_{rh} , K_{vh} and K_{hh} , represent the interactions of the leg with the guides and the jacking and supporting system. The following approximations may be applied:

3.1 Horizontal Stiffness

$$K_{hh} = \infty \dots\dots\dots (A2.1)$$

3.3 Vertical Stiffness

$$K_{vh} = K_{Comb} \dots\dots\dots (A2.2)$$

where

K_{Comb} = effective stiffness due to the series combination of all vertical pinion or fixation system stiffness, allowing for combined action with shock-pads, where fitted

3.5 Rotational Stiffness

3.5.1 Unit with Fixation System

$$K_{rh} = F_n h^2 k_f \dots\dots\dots (A2.3)$$

where

- F_n = 0.5 for three chord leg
- = 1.0 for four chord leg
- h = distance between chord centers
- k_f = combined vertical stiffness of all fixation system components on one chord

3.5.2 Unit without Fixation System

$$K_{rh} = F_n h^2 k_j + \frac{k_u d^2}{1 + \frac{2.6 k_u d}{EA_s}} \dots\dots\dots (A2.4)$$

where

- h = distance between chord centers (opposed pinion chords) or pinion pitch points (single rack chords)

- k_j = combined vertical stiffness of all jacking system components on one chord
- d = distance between upper and lower guides
- k_u = total lateral stiffness of upper guides with respect to lower guides
- A_s = effective shear area of leg

5 Unit Load Approach

The unit load method described in Appendix 1 can also be used for deriving the stiffness properties of the equivalent leg-to-hull connection by applying unit loads, as described below, to a detailed leg model without the leg-to-hull connection and the other detailed leg model combined with the leg-to-hull connection. The differences in deflections and rotations between these two models can be used to determine the stiffness properties of the equivalent leg-to-hull connection. The following unit load cases should be used:

5.1 Unit Axial Load Case

This case determines the vertical leg-to-hull connection stiffness, K_{vh} , based on the difference in axial deflections between the detailed leg model, Δ , and the combined leg and leg-to-hull connection model, Δ_C , under the unit axial load, F :

$$K_{vh} = F/(\Delta_C - \Delta) \dots \dots \dots (A2.5)$$

5.3 Unit Moment Case

This case determines the rotational connection stiffness, K_{rh} , based on the difference in the end slopes between the detailed model, θ , and the combined leg and leg-to-hull connection model, θ_C , under the unit moment, M :

$$K_{rh} = M/(\theta_C - \theta) \dots \dots \dots (A2.6)$$

Alternatively, the rotational stiffness can also be derived based on the difference in the end deflections between the detailed model, δ , and the combined leg and leg-to-hull connection model, δ_C , under unit moment, M :

$$K_{rh} = ML/(\delta_C - \delta) \dots \dots \dots (A2.7)$$

5.5 Unit Shear Load Case

This case determines the horizontal leg-hull connection stiffness, K_{hh} , in a similar manner, accounting for the rotational stiffness already derived. Normally the horizontal leg-to-hull connection stiffness may be assumed infinite.



APPENDIX 3 Equivalent Hydrodynamic Coefficients of Lattice Legs (1 February 2014)

1 Introduction

The hydrodynamic properties of a lattice leg in the “equivalent model” can be represented by an equivalent drag coefficient C_{De} , an equivalent mass coefficient C_{Me} , and an equivalent diameter D_e . The following Subsections can be used to determine these equivalent hydrodynamic parameters.

3 Equivalent Diameter

The equivalent diameter, D_e , of a lattice leg shown in Appendix 3, Figure 1, can be determined as:

$$D_e = \sqrt{\sum (D_i^2 \ell_i)} / s \dots\dots\dots (A3.1)$$

where

- D_e = equivalent diameter of the lattice leg
- D_i = reference diameter of member i
- ℓ_i = reference length of member i (node to node)
- s = height of one bay, or part of bay being considered

5 Equivalent Drag Coefficient

The equivalent drag coefficient, C_{De} , of the lattice leg can be determined as:

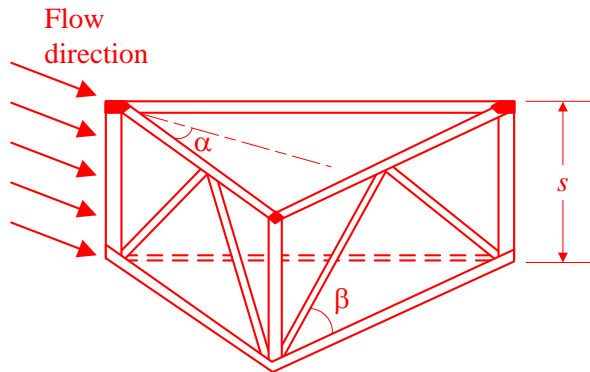
$$C_{De} = \sum C_{Dei} \dots\dots\dots (A3.2)$$

where

- C_{Dei} = equivalent drag coefficient of each individual member
- $= [\sin^2 \beta_i + \cos^2 \beta_i \sin^2 \alpha_i]^{3/2} C_{Di} \frac{D_i \ell_i}{D_e s}$
- C_{Di} = drag coefficient of an individual member i , related to reference dimension D_i
- α_i = angle between flow direction and member axis projected onto a horizontal plane (see Appendix 3, Figure 1 below)
- β_i = angle defining the member inclination from the horizontal plane (see Appendix 3, Figure 1 below)

Note: “ Σ ” indicates summation over all members in one leg bay.

FIGURE 1
One Bay of Lattice Leg (1 February 2014)



For a split tube chord as shown in Appendix 3, Figure 2, the drag coefficient C_{Di} related to the reference dimension D_i , may be taken as:

$$C_{Di} = \begin{cases} C_{D0} & ; 0^\circ < \theta \leq 20^\circ \\ C_{D0} + (C_{D1}W/D_i - C_{D0})\sin^2[\theta - 20^\circ]9/7 & ; 20^\circ < \theta \leq 90^\circ \end{cases} \dots\dots\dots (A3.3)$$

where

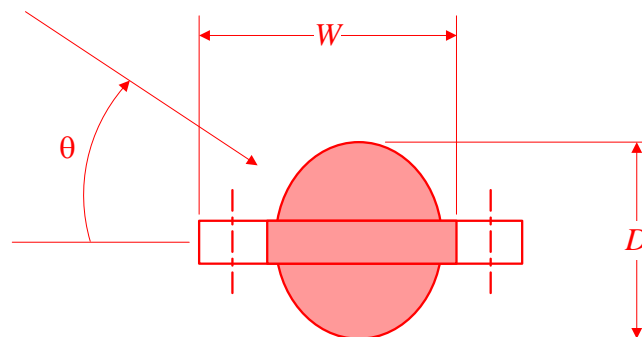
θ = angle in degrees, Appendix 3, Figure 2

C_{D0} = drag coefficient for a tubular

C_{D1} = drag coefficient for flow normal to the rack ($\theta = 90^\circ$), related to projected diameter, W

$$= \begin{cases} 1.8 & ; W/D_i < 1.2 \\ 1.4 + 1/3(W/D_i) & ; 1.2 < W/D_i < 1.8 \\ 2.0 & ; 1.8 < W/D_i \end{cases} \dots\dots\dots (A3.4)$$

FIGURE 2
Split-Tube Chord Section (1 February 2014)



For a triangular chord as shown in Appendix 3, Figure 3, the drag coefficient, C_{Di} , related to the reference dimension $D_i = D$, the back plate width, may be taken as:

$$C_{Di} = C_{Dpr}(\theta) \cdot D_{pr}(\theta)/D_i \dots\dots\dots (A3.5)$$

where the drag coefficient related to the projected diameter, C_{Dpr} , is determined from equation below with linear interpolation applicable for intermediate headings:

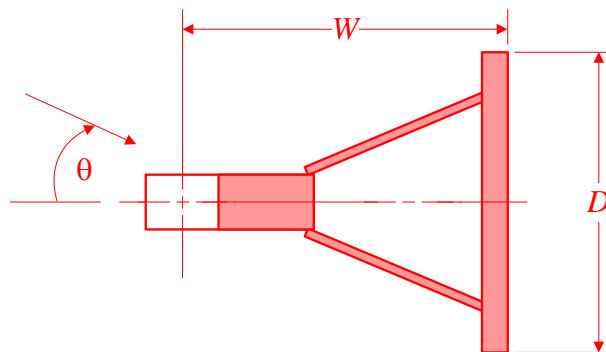
$$C_{Dpr} = \begin{cases} 1.70 & ; \theta = 0^\circ \\ 1.95 & ; \theta = 90^\circ \\ 1.40 & ; \theta = 105^\circ \\ 1.65 & ; \theta = 180^\circ - \theta_o \\ 2.00 & ; \theta = 180^\circ \end{cases} \dots\dots\dots (A3.6)$$

The projected diameter, D_{pr} , may be determined from:

$$D_{pr} = \begin{cases} D \cos(\theta) & ; 0 < \theta < \theta_o \\ W \sin(\theta) + 0.5D |\cos(\theta)| & ; \theta_o < \theta < (180 - \theta_o) \\ D |\cos(\theta)| & ; (180 - \theta_o) < \theta < 180 \end{cases} \dots\dots\dots (A3.7)$$

The angle θ_o , where half the rackplate is hidden, $\theta_o = \tan^{-1}[D/(2W)]$.

FIGURE 3
Triangular Chord Section (1 February 2014)



7 Equivalent Mass Coefficient

The equivalent inertia coefficient, C_{Me} , of the lattice leg may normally be taken as 2.0 and used in conjunction with the effective diameter D_e . For a more accurate model C_{Me} may be determined as:

$$C_{Me} = \sum C_{Mei} \dots\dots\dots (A3.8)$$

where

$$C_{Mei} = [1 + (\sin^2 \beta_i + \cos^2 \beta_i \sin^2 \alpha_i)(C_{Mi} - 1)] \frac{A_i \ell_i}{A_e s}$$

C_{mi} = inertia coefficient of individual member i , related to reference dimension D_i

A_e = equivalent area of leg per unit height = $(\sum A_i \ell_i)/s$

A_i = equivalent area of element = $\pi D_i^2/4$

Note: For dynamic modeling the added mass coefficient may be determined as $C_{Ai} = C_{Mi} - 1$ for a single member or $C_{Ae} = C_{Me} - 1$ for the equivalent model, which is to be used in conjunction with A_e as defined above.

For both split tube and triangular chord, the inertia coefficient $C_M = 2.0$, related to the equivalent volume per unit length of member, may be applied for all heading angles.

9 Current Associated with Waves

The current velocity is to include components due to tidal current, storm surge current and wind driven current. In lieu of defensible alternative methods, the vertical distribution of current velocity in still water and its modification in the presence of waves, as shown in Appendix 3, Figure 4 below, are recommended, where:

$$V_c = V_t + V_s + V_w [(h - z)/h] \quad \text{for } z \leq h$$

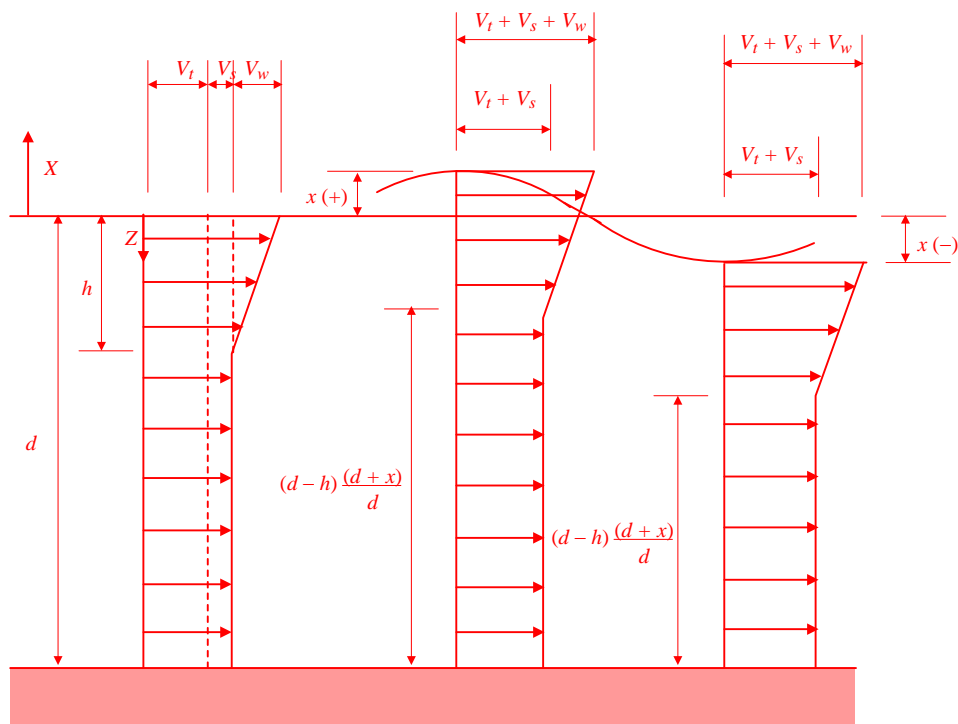
$$V_c = V_t + V_s \quad \text{for } z > h$$

where

- V_c = current velocity, m/s (ft/s)
- V_t = component of tidal current velocity in the direction of the wind, m/s (ft/s)
- V_s = component of storm surge current, m (ft)
- V_w = wind driven current velocity, m/s (ft/s)
- h = reference depth for wind driven current, m (ft). (in the absence of other data, h may be taken as 5 m (16.4 ft)
- z = distance below still water level under consideration, m (ft)
- d = still water depth, m (ft)

In the presence of waves, the current velocity profile is to be modified, as shown Appendix 3, Figure 4, such that the current velocity at the instantaneous free surface is a constant.

FIGURE 4
Current Velocity Profile (1 February 2014)





APPENDIX 4 References (1 February 2014)

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