GUIDANCE NOTES ON

SELECTING DESIGN WAVE BY LONG TERM STOCHASTIC METHOD

OCTOBER 2016

American Bureau of Shipping
Incorporated by Act of Legislature of
the State of New York 1862

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Foreword

The purpose of these Guidance Notes is to supplement the determination of design wave by long term stochastic approach for the ABS *Rules for Building and Classing Mobile Offshore Drilling Units (MODU Rules)*. These Guidance Notes provide users with step-by-step procedures for selecting design wave using long term stochastic method for non-ship type offshore structures. This methodology has been implemented in the ABS Eagle Offshore Structural Assessment Program (OSAP).

These Guidance Notes become effective on the first day of the month of publication.

Users are advised to check periodically on the ABS website www.eagle.org to verify that this version of these Guidance Notes is the most current.

We welcome your feedback. Comments or suggestions can be sent electronically by email to rsd@eagle.org.

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SECTION 1 Introduction

1 General

The design wave calculation by the short term stochastic method is provided in the ABS *Rules for Building and Classing Mobile Offshore Drilling Units (MODU Rules)*. The ABS *Rules for Building and Classing Floating Production Installations (FPI Rules)* require the installation's hull strength and fatigue assessment in the site-specific environmental conditions, considering both 100-year return period environmental events and wave scatter diagram data of wave height/period joint occurrence distributions. As a supplement to select the design wave from the site specific environmental conditions, these Guidance Notes provide the detailed procedures to determine the design wave by the long term stochastic method for non-ship type offshore structures.

3 Features

Information related to the long term stochastic method includes: loading conditions, load cases, dominant load parameters, response RAOs, waves, wave spectra, sea state, wave scatter diagram, design wave, etc.

The primary features of these Guidance Notes include:

- Wave spectrum and wave characteristics for site specific environment
- Wave data for long term design wave analysis
- Methodology for determining long term design wave

5 Application

These Guidance Notes describe procedures to select the design waves based on the long term stochastic approach. The procedures can be used for

- Response analysis – selecting design waves for non-ship type offshore structures.
- Assistance in non-ship type offshore structure design.

These Guidance Notes should be used in association with ABS Rules and Guides for non-ship type offshore structure analysis, such as the *MODU Rules* and *FPI Rules*.

SECTION 2 Waves

1 General

For offshore structures, the most dominant source of dynamic loads is waves. During the service life of an offshore unit, it will experience a large number of cyclic loads due to waves, from very small wavelets to possibly giant waves. A practical way to describe these unceasingly changing waves is to divide them into various categories (sea states), and use short term wave statistics to depict each sea state and long term wave statistics, usually in the form of a wave scatter diagram and wave direction rosette, to delineate the rate at which a sea state occurs.

In a similar way, there are two levels in the description of wave directionality (i.e., wave directional spectrum or wave spreading for short-term, and wave rosette for long term, respectively).

There are numerous texts that present information on ocean waves and the statistically based parameters that are used to define sea states. The key concepts resulting from the application of these theoretical developments are the characterization of a sea state as spectra comprised of numerous individual wave components, and the use of spectra moments to establish sea state defining parameters such as significant wave height and peak or zero crossing periods. For more details on this subject, refer to [5].

3 Wave Spectra (Short-term Wave Statistics)

3.1 Unidirectional Spectra

A wave spectrum describes the energy distribution among wave components of different frequencies of a sea state. Wave spectra can be obtained directly from measured data. However, various mathematical formulae of wave spectra have been available based on analysis of measured data, such as ISSC Wave Spectrum, Bretschneider Spectrum (or Pierson-Moskowitz (P-M) spectrum), JONSWAP spectrum and Ochi's six-parameter spectrum, etc. These spectrum formulae are suitable for different sea states.

A fully-developed sea is a sea state that will not change if wind duration or fetch is further increased (for a fixed wind speed). The Bretschneider spectrum is applicable to fully-developed seas. For most of the ships and offshore structures in ABS's classification, either the Bretschneider spectrum for open ocean areas with fully-developed seas, or the JONSWAP spectrum for fetch-limited regions is used, respectively. For example, the Bretschneider wave spectrum is usually employed to describe tropical storm waves, such as those generated by hurricanes in the Gulf of Mexico or typhoons in the South China Sea. The JONSWAP wave spectrum is used to describe winter storm waves of the North Sea. In some cases, it can also be adjusted to represent waves in Offshore Eastern Canada and swells, such as those in West Africa and Offshore Brazil. A suitable wave spectrum should be chosen based on a partially or fully developed sea state for selecting design waves. In general, the Bretschneider spectrum has a greater frequency bandwidth than the JONSWAP spectrum. Therefore, the selection of a spectrum should be based on the frequency characteristics of the wave environment.

The above-described two spectra are single-modal spectra, which are usually used to represent pure wind waves or swell-only cases. When wind waves co-exist with swells (i.e., there are multi-modes in the spectrum), no single-modal spectrum can match the spectral shape very well. In this case, recourse can be made to the use of the Ochi-Hubble 6-Parameter Spectrum or other wave spectrum.

3.3 Directional Spectra (Wave Spreading)

3.3.1 Long-crested Waves

This is a simple case where the observed wave pattern at a fixed point neglects different directions of wave components. It is equivalent to assuming that all wave components travel in the same direction. These waves are called ‘long-crested’ since the wave motion is two-dimensional and the wave crests are parallel. Waves produced by swell are almost long-crested in many situations since the crests of the wave become nearly parallel as the observation point recedes from the storm area which produced the waves.

3.3.2 Short-crested Waves

If the observation station is inside the storm area, different waves will come from different directions, and the combined wave system will be short-crested waves. The spreading of wave directions should be taken into account to describe the short-crested waves.

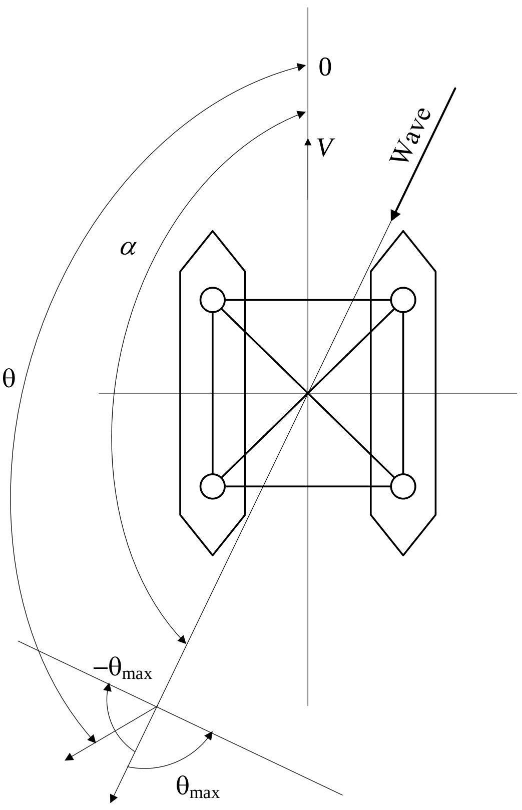
3.3.3 Wave Spreading

Considering the wave spreading, the wave energy spectrum can be obtained by integrating the spreading wave spectrum over the range of directions from $-\theta_{max}$ to $+\theta_{max}$ (θ_{max} can be typically taken as 90°). The general expression for wave spreading is given by:

$$S_\eta(\omega) = \int_{-\theta_{max}}^{\theta_{max}} S_\eta(\omega,\theta)d(\theta - \alpha)$$

where α denotes the predominant wave direction and θ is the wave spreading angle, as shown in Section 2, Figure 1.

FIGURE 1
Definition of Spreading Angles



In general, directional short-crested wave spectra $S(\omega,\theta)$ may be expressed in terms of the uni-directional wave spectra:

$$S(\omega,\theta) = S(\omega)D(\omega,\theta) = S(\omega)D(\theta)$$

Where the latter equality represents a simplification often used in practice. $D(\omega,\theta)$ and $D(\theta)$ are spreading functions and fulfils the requirement:

$$\int_{\theta} D(\omega,\theta)d\theta = 1$$

A common cosine spreading function used for the wave spectrum is:

$$D(\theta) = \frac{\Gamma(1+n/2)}{\sqrt{\pi}\,\Gamma(1/2+n/2)}\cos^n(\theta - \alpha)$$

where

Γ = Gamma function

$|\theta - \alpha| \leq \frac{\pi}{2}$

n = wave spreading parameter, which is a positive integer. Typical values for wind sea are $n = 2$ to $n = 4$. If used for swell waves, $n \geq 6$ is more appropriate.

3.5 Wave Spectral Formulation

The shape of a spectrum supplies useful information about the characteristics of the ocean wave system to which it corresponds. There exist many wave spectral formulations (e.g., Bretschneider spectrum, Pierson-Moskowitz spectrum, ISSC spectrum, ITTC spectrum, JONSWAP spectrum, Ochi-Hubble 6-parameter spectrum, etc.).

3.5.1 Bretschneider or Two-Parameter Pierson-Moskowitz Spectrum

The Bretschneider spectrum or two-parameter Pierson-Moskowitz spectrum, also known as ISSC spectrum (representing by significant wave height and mean period), or ITTC spectrum (representing by significant wave height and one of energy period, peak period, mean period and zero-crossing period) is the spectrum recommended for open-ocean wave conditions (e.g., the Atlantic Ocean).

$$S_\eta(\omega) = \frac{5}{16}\frac{H_s^2\omega_p^4}{\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right] \quad \text{in m}^2\text{/(rad/s) (ft}^2\text{/(rad/s))}$$

or

$$S_\eta(\omega) = \frac{1}{4\pi}\frac{H_s^2}{\omega^5}\left(\frac{2\pi}{T_z}\right)^4\exp\left[-\frac{1}{\pi}\left(\frac{2\pi}{T_z}\right)^4\omega^{-4}\right] \quad \text{in m}^2\text{/(rad/s) (ft}^2\text{/(rad/s))}$$

where

ω_p = $2\pi/T_p$ modal (peak) frequency corresponding to the highest peak of the spectrum, in rad/s

H_s = significant wave height, in m (ft)

ω = circular frequency of the wave, in rad/s

T_z = average zero up-crossing period of the wave, in seconds

3.5.2 JONSWAP Spectrum

The JONSWAP spectrum is derived from the Joint North Sea Wave Project (JONSWAP) and constitutes a modification to the Pierson-Moskowitz spectrum to account for the regions that have geographical boundaries that limit the fetch in the wave generating area (e.g., the North Sea).

$$S_\eta(\omega) = \frac{5}{16}\frac{H_s^2\omega_p^4}{\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^a\left(1 - 0.287\ln\gamma\right) \quad \text{in m}^2\text{/(rad/s) (ft}^2\text{/(rad/s))}$$

where

a = $\exp\left[-\frac{\left(\omega-\omega_p\right)^2}{2\sigma^2\omega_p^2}\right]$

σ = $\begin{cases} 0.07 & \text{when } \omega \le \omega_p \\ 0.09 & \text{when } \omega > \omega_p \end{cases}$

ω = circular frequency of the wave, in rad/s

γ = peakedness parameter, typically 1 to 7

ω_p = $2\pi/T_p$ modal (peak) frequency corresponding to the highest peak of the spectrum, in rad/s

Here, the factor (1 – 0.287ln γ) limits its practical application, because for γ =32.6, the spectral value from above formula becomes zero. For the peakedness larger than 7, it is recommended that an adjustment to the formula has to be made. The formula of the JONSWAP spectrum can be then given by:

$$S_\eta(\omega) = \frac{\alpha g^2}{\omega^5}\exp\left[-\frac{5}{4}\left(\frac{\omega_p}{\omega}\right)^4\right]\gamma^a \quad \text{in m}^2\text{/(rad/s) (ft}^2\text{/(rad/s))}$$

where

γ = peakedness parameter, representing the ratio of the maximum spectral density to that of the corresponding Pierson-Moskowitz spectrum. This means that for γ = 1 the JONSWAP spectrum defaults to the Pierson-Moskowitz spectrum

g = gravitational acceleration = 9.8 m/s^2 (32.2 ft/s^2)

α = parameter to be determined as a function of the significant wave height, through the expression provided in the formula of H_s below, since the integral is a function of α

H_s = $4\sqrt{\int_0^\infty S_\eta(\omega)d\omega}$

3.5.3 Gaussian-Swell Spectrum

The design sea state may come from intensification of the local wind seas (waves) and/or swell propagating with different directions. In general, both are statistically independent. The wind seas are often characterized with the Bretschneider or the JONSWAP spectrum while the Gaussian distribution function can be used to describe swells. The spectral formulation for the swell can be represented by the Gaussian-Swell spectrum:

$$S_\eta(\omega) = \frac{(H_s/4)^2}{2\pi\delta\sqrt{2\pi}}\exp\left[-\frac{(\omega-\omega_p)^2}{2(2\pi\delta)^2}\right] \quad \text{in m}^2\text{/(rad/s) (ft}^2\text{/(rad/s))}$$

where

H_s = significant wave height, in m (ft)

δ = peakedness parameter for Gaussian spectral width

ω_p = $2\pi/T_p$ modal (peak) frequency corresponding to the highest peak of the spectrum, in rad/s

3.5.4 Ochi-Hubble 6-Parameter Spectrum

The Ochi-Hubble 6-Parameter spectrum covers shapes of wave spectra associated with the growth and decay of a storm, including swells. As may be seen in some wave records, the variability in the form of spectra can be great. Multi-modal spectra are common, and a single-modal Bretschneider form may not match the shape of such spectrum in an accurate manner. In order to cover a variety of shapes of wave spectra associated with the growth and decay of a storm, including the existence of swell, the following 6-parameter spectrum was developed by Ochi and Hubble:

$$S_\eta(\omega) = \frac{1}{4}\sum_{j=1}^{2}\frac{\left(\frac{4\lambda_j+1}{4}\omega_{pj}^4\right)^{\lambda_j}}{\Gamma(\lambda_j)}\times\frac{H_{sj}^2}{\omega^{4\lambda_j+1}}\times\exp\left[-\frac{4\lambda_j+1}{4}\left(\frac{\omega_{pj}}{\omega}\right)^4\right] \quad \text{in m}^2/(\text{rad/s}) \ (\text{ft}^2/(\text{rad/s}))$$

where j = 1, 2 stands for lower (swell part) and higher (wind seas part) frequency components. The six parameters, H_{s1}, H_{s2}, ω_{p1}, ω_{p2}, λ_1, λ_2, are determined numerically to minimize the difference between theoretical and observed spectra.

Note that the modal frequency of the first component, ω_{p1}, must be less than that of the second, ω_{p2}. The significant wave height of the first component, H_{s1}, should normally be greater than that of the second, H_{s2}, since most of the wave energy tends to be associated with the lower frequency component. The Ochi-Bubble spectrum makes no allowance for the directionality of the swell and wind components of the sea states. In reality the separate components frequently come from different directions. Because of this it is more appropriate to model a two peaked sea state using two separate single peak wave spectra, one for the swell components and one for the local wind generated component.

3.5.5 Torsethaugen Spectrum

The Torsethaugen spectrum is a double peaked spectrum best suited to North Sea conditions. It represents sea states that include both a remotely generated swell and local wind-generated waves. Input parameters to the Torsethaugen Spectrum are significant wave height H_s and peak period T_p. The Torsethaugen spectrum is given in Appendix 2. Full details of the Torsethaugen spectrum can be found in the Torsethaugen and Haver paper [6].

3.5.6 Owner's Specified Spectrum

Site specific measured wave data may be submitted for review. The owner defined spectrum can be specified by giving a table of values for $S_\eta(f)$, where $S_\eta(f)$ is the spectral energy as a function of frequency f.

$$S_\eta(\omega) = \frac{1}{2\pi}S_\eta(f)$$

where

ω = circular frequency of the wave, in rad/s

f = wave frequency, in Hz

3.7 Wave Scatter Diagram and Rosette (Long-Term Wave Statistics)

Long-term descriptions of the wave environment in the form of "wave scatter diagram" and "rosette" are required for long-term statistical analysis of design waves.

3.7.1 Wave Scatter Diagram

A wave scatter diagram consists of a table of the probabilities of occurrence of various "sea states". Each cell in the table contains information on three data items, namely (1) the significant wave height, H_s, (2) the characteristic wave period, T, and (3) the fraction of the total time or probability of occurrence for the sea state defined by spectrum with parameters H_s and T. The characteristic wave period usually can be given as peak period, average period or zero up-crossing periods. Attention should be paid to which characteristic wave period is specified in a wave scatter diagram so that it will be consistent with the wave period in the wave spectrum formulation.

3.7.2 Wave Rosette

A wave rosette (also called long-term wave directionality) describes the probability of each heading angle (the main wave direction) at a site. Directional convention should be noticed in using the rosette (e.g., for NOAA wave data, index 1 represents wave coming from true north and as the index increases, the wave direction changes clockwise). Directionality has significant effects on structural response. It is recommended that a realistic wave rosette be used. In case the wave rosette is not available, it is reasonable to assume equal probability of all heading angles in open ocean conditions. However, for a moored offshore structure, the waves may have strong directional characteristics that should be accounted for.

SECTION 3 Wave Data for Long Term Design Wave

1 General

A non-ship type offshore unit is designed for a specific site with unique wave statistics, water depth and geotechnical characteristics. The site specific wave environment ordinarily produces the dominant environmentally induced effects and should be taken into account for the long term design wave analysis. For site-specific considerations, it should be noted that special emphasis may need to be given to the directionality of waves because of the mooring system, the recognition of 'short-crestedness' (wave energy spreading) effects, and interactions between dominant wave directions and other environmental actions (e.g., persistent ocean current or winds may alter the presumed wave induced 'weathervaning' behavior).

For the long term wave data, clients are required to submit long term wave data for site specific analysis. The requirement for wave data is specified in the *FPI Rules*. Generally, the wave data may include a wave scatter diagram with a joint probability of significant wave height and characteristic wave periods, the associated wave spectrum, wave directionality, etc.

3 Wave Data for Long Term Design Wave Analysis

The long-term wave data consists of a wave scatter diagram recorded at a certain location over a long period of time. In general, the wave scatter diagram provides the probability or number of occurrence of sea states in a specified ocean area. For each single sea state, a wave scatter diagram is stored together with its associated directional probability distribution (wave rosette). A minimum return period of 100 years is typically required. A minimum return period of 50 years may be specially considered if it is accepted by the coastal state, as specified in Section 3-2-3 of the *FPI Rules*.

Generally, 100 year return period wave data is characterized by a significant wave height with a range of associated characteristic periods. The wave peak period, the wave mean period and the zero up crossing wave period are commonly used to present the wave characteristic periods. The mean period T_ℓ is defined as $2\pi(m_0/m_1)$, and zero up crossing period T_z is defined as $2\pi\sqrt{m_0 / m_2}$, where m_0, m_1, and m_2 are zero, first and second spectral moment of waves, respectively.

5 Load Cases and Dominant Load Parameters

The long term design wave method does not require the development of Load Cases to be investigated in the wave loads. However, the concept of Load Cases is used in these Guidance Notes to represent the sea conditions and headings, which typically produce the most critical response of the non-ship type offshore structure.

The term, Dominant Load parameter (DLP) refers to a global load or motion effect of the offshore unit (the bending moments, shear forces or unit motions) that may yield the maximum structural response for critical structural members. The instantaneous response of the offshore structure can be judged by one of the several Dominant Load Parameters. These parameters are to be maximized to establish the design waves for the further offshore unit analysis.

The common DLPs used in the non-ship type offshore unit but not limited to these are as follows:

- Split Force between Pontoons, (*SF*), refer to 3-2-A2/3.3 of ABS *MODU Rules*.
- Twisting Pitch Moment about Transverse Horizontal Axis, (*TPM*), refer to 3-2-A2/3.5 of ABS *MODU Rules*.
- Vertical Wave Bending Moment, (*VBM*), refer to 3-2-A2/3.11 of ABS *MODU Rules*.
- Longitudinal Shear Force, (*LSF*), refer to 3-2-A2/3.7 of ABS *MODU Rules*.
- Horizontal Bending Moment, (*HBM*).
- Transverse Bending Moment, (*TBM*).
- Vertical accelerations (V_{acc}).
- Longitudinal and Transverse acceleration (L_{acc}, T_{acc}).
- Roll angle (Φ).

Other DLPs that may be deemed critical can also be considered in the long term design wave analysis. For semi-submersible related DLPs and motion and load RAOs details, refer to 3-3-A2 of the *MODU Rules.*

7 Unit Motion and Wave Load Response Amplitude Operators (RAOs)

Computations of the wave-induced motions and loads should be carried out using appropriate, proven methods. Preference should be given to the application of sea-keeping analysis codes utilizing three dimensional potential flow-based diffraction-radiation theories. All six degrees-of-freedom rigid body motions of the offshore unit should be accounted for, and effects of water depth should be considered. These codes, based on linear wave and motion amplitude assumptions, make use of boundary element methods with constant or higher order sink-source panels over the entire wetted surface of the hull on which the hydrodynamic pressures are computed.

Offshore unit motion RAOs and wave load RAOs are to be calculated for each selected Load Case. These RAOs use to determine the calculation of extreme values. The RAOs should represent the pertinent range of wave headings (α), in increments not exceeding 15 degrees.

It is important that a sufficiently broad range of wave frequencies are considered based on the site-specific wave conditions. The recommended range increment refers to 3-2-A2 of ABS *MODU Rules.*

The worst wave frequency-heading (ω,α) combination is to be determined from an examination of the RAOs for each Load Case. Only the heading $\alpha_{\max}$ and the wave frequency ω_e at which the RAO of the selected load cases is a maximum need to be used in further analysis. In general, it may be expected that *VBM* and *SF* will be maximum in head and beam seas, while maximum L_{acc} and T_{acc} are realized in head and beam seas, respectively. Precise headings at which these are maximum, can be determined from the RAO analysis output.

In addition, RAOs for the other load components are to be analyzed.

SECTION 4 Methodology

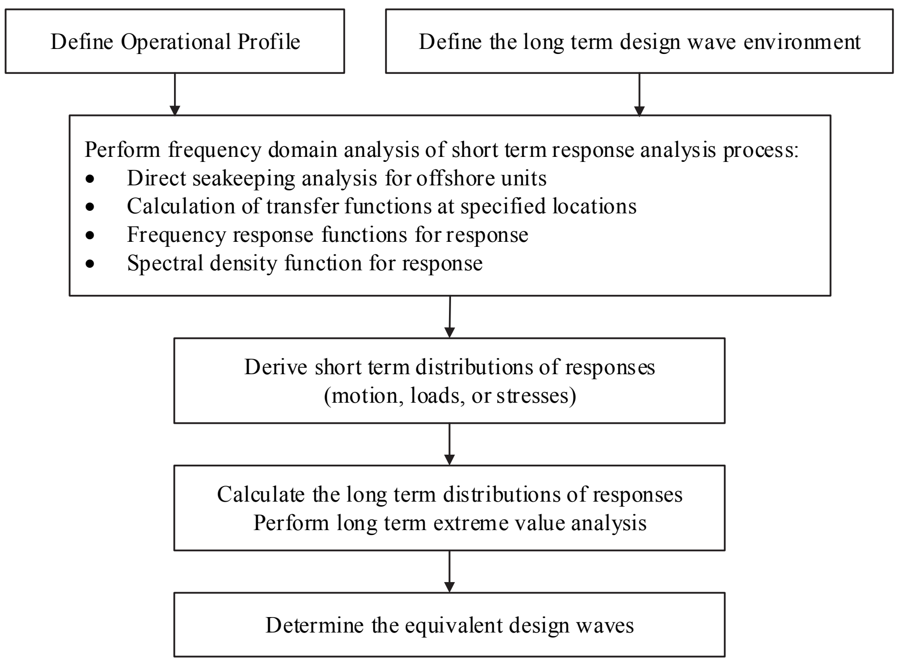
1 General

The long term stochastic method for selecting design wave is a systematic approach to develop a rational, first principle approach whereby the dynamic loads and the wave environment are explicitly considered. The essence of the long term design wave method is to model the long term random sea state process as several short term stationary Gaussian process, each defined by the wave spectral density function. The fraction of time that each short-term process acts is specified. A frequency response function relating the wave spectral density and the spectral density of wave load critical components, and including the structural dynamics, is developed. By taking into account the probability for different wave headings and different wave spectra, the long term response distribution of the response is established. The long term extreme value analysis is performed to determine the waves with critical periods and headings (as characterized in 3-2-A2/3 of the *MODU Rules*) that generate a significant amount of hydrodynamic loads are critical to the design of the primary structural elements.

For each characterized load case, the long term response analysis determines the critical wave period that generates the maximum hydrodynamic loads. It further finds an equivalent regular wave height which will generate the same amount of the hydrodynamic loads as calculated in the response analysis. The critical wave period and wave height are noted as “Design Wave”, and to be used for the design of the global strength of the offshore structures.

The process of determining the design waves through the long term response analysis method is summarized in Section 4, Figure 1. This methodology is also implemented in the ABS Eagle Offshore Structural Assessment Program (OSAP) [9]. The long term design waves can be calculated either using the ABS Eagle OSAP or similar applications following the procedures in these Guidance Notes, in which case consultation with ABS is recommended.

FIGURE 1
Long Term Design Wave Analysis Procedure



3 Extreme Values for Long Term Wave Analysis

Extreme value analysis is to be used for each selected load component to determine the maximum values during design life. Preference is given to an extreme value analysis that follows the so-called long-term approach commonly used for offshore structures. The supplementary use of a validated short-term extreme value approach to confirm or test the sensitivity of the long-term based design values is considered. The result of the short-term approach cannot be used to reduce the long-term extreme value. If the short-term result is significantly larger, the long-term extreme value is to be further studied and validated. The environments specified for use in the short-term approach are "response based" (i.e., a 100-year design storm event is one that leads to the maximum responses expected to occur in 100-years). A useful reference to explain concepts and terminology associated with extreme value analysis can be found in [7].

The procedure for calculating the long-term extreme value corresponding to a particular return period across a combined scatter-diagram-heading distribution of sea states is described below:

For each entry in the wave scatter diagram for each heading, the spectral moment of the response spectra can be given by:

$$m_n = \int_0^\infty \omega^n \left|H_i(\omega)\right|^2 S_\eta(\omega)d\omega \qquad n = 0, 1, 2\ldots$$

where $|H_i(\omega)|$ is the RAO of the offshore structural response. The variance (zeroth moment) of a response spectrum can be generalized to include the direction of offshore unit heading relative to predominant wave direction α and wave spreading angle θ as follows:

$$\sigma_i^2(\alpha) = m_0 = \int_{-\pi/2}^{\pi/2}\int_0^\infty \left|H_i(\omega, \alpha - \theta)\right|^2 S_\eta(\omega, \theta)d\omega d\theta$$

$$m_n = \int_{-\pi/2}^{\pi/2}\int_0^\infty \left|H_i(\omega, \alpha - \theta)\right|^2 S_\eta(\omega, \theta)\omega_e^n d\omega d\theta \qquad n = 0, 1, 2\ldots$$

Considering the offshore unit transit voyage, if an offshore unit travels with constant forward speed U, ω_e represents the wave frequency of encounter defined by:

$$\omega_e = \left|\omega - U\frac{\omega^2}{g}\cos\alpha\right|$$

where

g	=	gravitational acceleration
α	=	wave heading

The other spectral moments can be also generalized in a similar manner over the wave spreading angle. The number of positive maxima per unit time for a Gaussian process is given by:

$$\bar{n} = \frac{1}{4\pi}\left(\frac{1+\sqrt{1-\varepsilon^2}}{\sqrt{1-\varepsilon^2}}\right)\sqrt{\frac{m_2}{m_0}}$$

where the bandwidth parameter ε is given by:

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}}$$

For any scatter-diagram-heading contribution, the number of response cycles will be calculated during the design lifetime of the offshore unit. The contribution that any one scatter-diagram-heading contribution makes to the long-term exceedance distribution of the response is then the sum of Gaussian distributions multiplied by the normalized number of response cycles, so that the long-term probability that the response will exceed a particular value $\bar{x}$ is calculated from the equation:

$$\frac{\sum_m \sum_k \bar{n} p_m p_k p_{\bar{x}}(\bar{x})}{\sum_m \sum_k \bar{n} p_m p_k} = \frac{1}{N} = Q$$

where the sum over m and k is over the entire set of scatter diagram and wave heading contributions; $\bar{n}$ depends on each scatter-diagram entry at each heading; p_m is the probability of occurrence from the wave scatter table; and p_k is the weighing factor for heading to waves from the wave rosette in a given site area. The distribution of probability of exceedance $p_{\bar{x}}(\bar{x})$ for wide-banded Gaussian processes given by:

$$p_{\bar{x}}(\bar{x}) = \begin{array}{l} 1 - \dfrac{2}{1+\sqrt{1-\varepsilon^2}}\left[-\dfrac{1}{2}\left(1-\sqrt{1-\varepsilon^2}\right) + \Phi\left(\dfrac{\bar{x}}{\varepsilon\sqrt{m_0}}\right) - \sqrt{1-\varepsilon^2}\exp\left\{-\dfrac{1}{2}\left(\dfrac{\bar{x}}{\sqrt{m_0}}\right)^2\right\}\right. \\ \left.\times\left\{1-\Phi\left(-\dfrac{\sqrt{1-\varepsilon^2}}{\varepsilon}\dfrac{\bar{x}}{\sqrt{m_0}}\right)\right\}\right] \end{array}$$

where

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u}\exp\left(-\frac{u^2}{2}\right)du = \frac{1}{2} + \frac{1}{\sqrt{2\pi}}\int_{0}^{u}\exp\left(-\frac{u^2}{2}\right)du$$

For narrow-banded cases ($\varepsilon = 0$), the exceedance distribution function for the peak values becomes:

$$p_{\bar{x}}(\bar{x}) = \exp\left(-\frac{\bar{x}^2}{2m_0}\right)$$

which is a Rayleigh distribution. Unless specified, the Rayleigh distribution approximation would be good enough in most single-modal spectral situations. However, in the specific site where the environmental condition is described as a combination of swell and wind seas, the wide-banded Gaussian distribution to represent the multi-modal spectra should be appropriately introduced.

To determine the probability level corresponding to the design life time or return period, the total number of response peaks N expected in the design lifetime T (years) is to be calculated from the following formula:

$$N = T \times 365 \times 24 \times 3600 \times \sum_m \sum_k \bar{n} p_m p_k$$

where T is the design return period or total years exposure time to the seas. The long-term extreme values of $\bar{x}$ that make the expression of probability of exceedance equal to Q are those corresponding to the design return period.

The relevant value to be obtained from the above long-term analysis is the long-term extreme value having a 100-year Return Period for site-specific condition, or 10-year Return Period for transit condition, or 1-year Return Period for inspection/repair and tank testing conditions. The return period of 100 years for site is, for example, approximately equivalent to a probability level of $Q = 10^{-8.7}$, assuming an average period to be 7 sec. However, considering the operational considerations of weathervaning offshore units, the long-term probability level of V_{acc}, L_{acc} and roll angle (Φ) may be reduced to $10^{-6.5}$ (equivalent to 1-year return period) in beam or oblique sea conditions

In specific locations, the environmental condition can be given in combination of swell and wind seas with different directionality. In this case, the two response spectra can be added and then the standard deviation of the combined spectrum can be determined by:

$$\sigma_c = \sqrt{m_0} = \sqrt{\sigma_{wave}^2 + \sigma_{swell}^2}$$

This procedure tends to be rational in evaluating the extreme response calculation compared to the simple summation in which the extreme values from the two processes are simply added.

5 Equivalent Design Wave

An equivalent design wave is a regular sinusoidal wave that simulates the extreme value of the Dominant Load Parameter under consideration. The equivalent design wave is characterized by the wave amplitude, wave frequency (or wave length), and wave heading. For each Load Case, an equivalent design wave is determined which simulates the extreme response value of the Dominant Load Component of the Load Case.

The procedure is to be used to determine the equivalent design wave's characterizing parameters. Subsections describe the formulations to establish the magnitude and distribution of the other load components accompanying the extreme value of the Dominant Load Component in a Load Case.

5.1 Equivalent Wave Amplitude

The amplitude of the equivalent design wave is to be determined by dividing the extreme value of a Dominant Load Parameter under consideration by the RAO value of that Dominant Load Parameter occurring at the wave frequency and wave heading corresponding to the maximum amplitude of the RAO.

The Equivalent Wave Amplitude (EWA) for the *j*-th Dominant Load Parameter is given by:

$$a_w = \frac{MPEV_j}{Max.RAO_j}$$

where

a_w	=	equivalent wave amplitude of the *j*-th Dominant Load Parameter, see Figure 2
$MPEV_j$	=	Most Probable Extreme Value of the *j*- Dominant Load Parameter at a probability level equivalent to the design Return Period (e.g., 100-years for site and 10-years for transit), see Section 4/3
$Max.\ RAO_j$	=	maximum amplitude of the *j*-th Dominant Load Parameter's RAO

5.3 Equivalent Wave Frequency, Length and Direction

The equivalent wave frequency and length for each Dominant Load Parameter are determined from the lifetime maximum value of the Dominant Load Parameter's RAO for each considered heading angle. When the RAO reaches maximum, the corresponding wave frequency and wave direction are denoted as equivalent wave frequency ω and equivalent wave direction. The wavelength of the equivalent wave system can be determined from deep water approximation by:

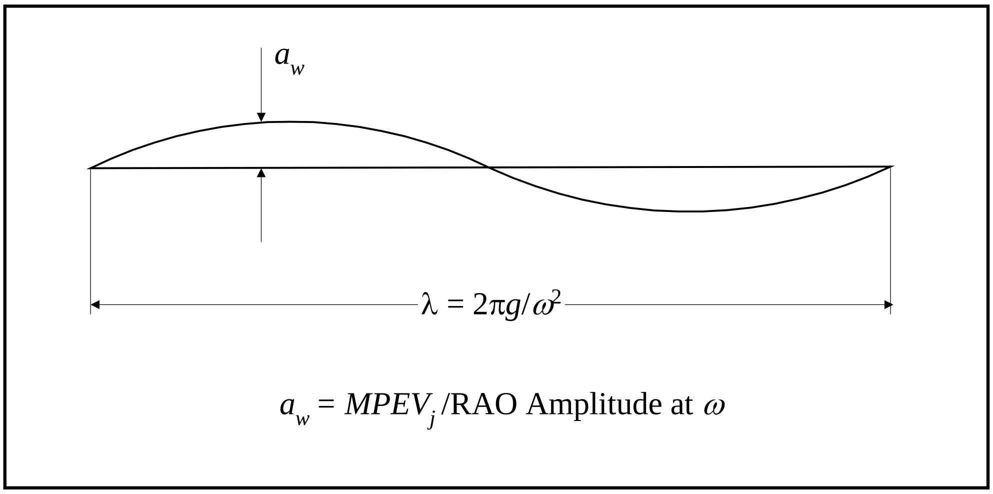
$$\lambda = (2\pi g)/\omega^2$$

where

λ	=	wavelength
g	=	gravitational acceleration = 9.8 m/s^2 (32.2 ft/s^2)
ω	=	equivalent wave frequency

For finite or shallow water depth, if necessary, the equivalent wave length can be calculated by the corresponding dispersion relation, which determines wave length for given values of the water depth and frequency.

FIGURE 2
Determination of Equivalent Wave Amplitude



5.5 Phase Angle and Wave Crest Position

With the wavelength, amplitude and direction from above, the wave crest position is calculated with respect to the LCG of the hull by:

$$X = (\lambda \in)/(-360 \cos \beta)$$

where

X = wave crest position with respect to the LCG for which the Dominant Load Parameter is at its extreme value

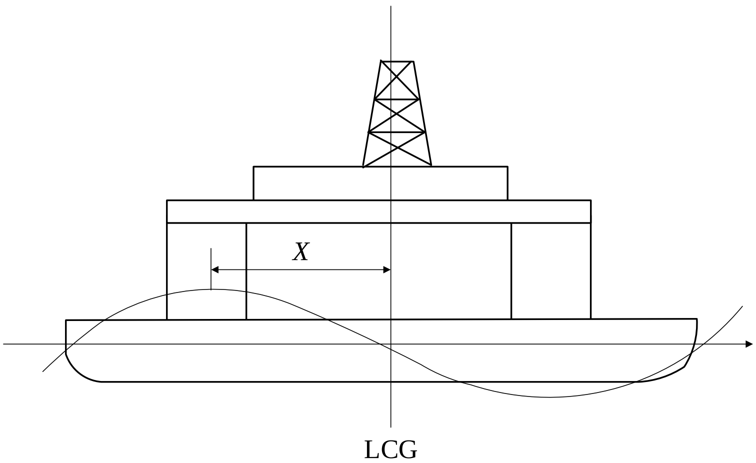
λ = wave length

$\in$ = phase angle of Dominant Load Parameter in degrees

β = wave heading

Section 4, Figure 3 illustrates the crest position X.

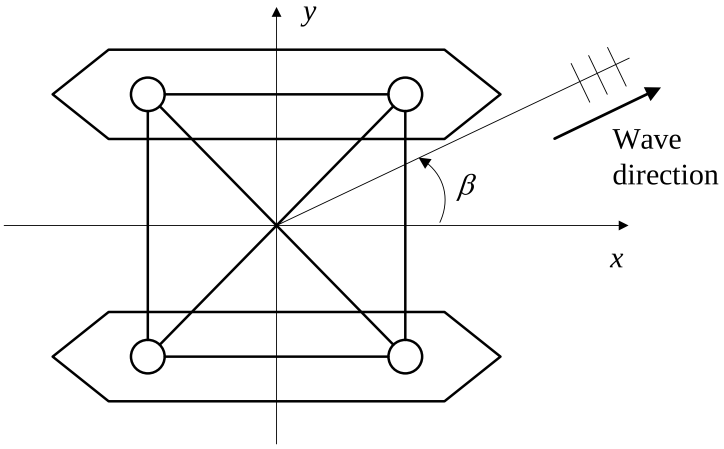
FIGURE 3
Equivalent Wave Length and Crest Position



$X = (\lambda \in)/(-360 \cos\beta)$

The definition of wave heading is illustrated in Section 4 Figure 4.

FIGURE 4
Definition of Wave Heading



It should be noted that X is undefined in beam seas (β = 90° or 270°). Instead the wave crest position from the centerline of the offshore unit in the y (transverse) direction is given by:

$$Y \quad = \quad (\lambda \in)/(-360 \sin \beta)$$

APPENDIX 1 References

1. Rules for Building and Classing Floating Production Installations, ABS
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3. Guide for Spectral-Based Fatigue Analysis for Floating Production, Storage and Offloading (FPSO) Installations, ABS
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5. Hsu, Teng H. "Applied Offshore Structural Engineering: Practical Design methods, Formulas and Data", Gulf Publishing Co, 1984
6. Knut Torsethaugen, Sverre Haver, "Simplified Double Peak Spectral Model for Ocean Waves", Proc. Of 14th International Offshore and Polar Engineering Conference, May 2004, p76-84
7. M. K. Ochi, "Wave Statistics for the Design of Ships and Ocean Structures", SNAME Transactions, Vol. 86, 1978, pp. 47-76.
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APPENDIX 2 Torsethaugen Spectrum

1 General

The Torsethaugen spectrum is a bimodal spectrum developed based on measured spectra for Norwegian waters (Haltenbanken and Statfjord) (Torsethaugen, 1993, 1994, 1996; Torsethaugen and Haver, 2004). It is defined as a sum of wind sea and swell by:

$$S_\eta(f) = \sum_{j=1}^{2} S_{\eta j}(f_{nj}, H_{sj}, T_{pj}, \gamma_j, N_j, M_j) = \sum_{j=1}^{2} E_j S_{\eta j}(f_{nj})$$

where

$S_{\eta j}$	=	JONSWAP spectrum
j	=	1 for the primary sea system
	=	2 for the secondary sea system
H_{sj}	=	significant wave height (meters)
T_{pj}	=	peak period (seconds)

γ_j, N_j, M_j (j = 1, 2) are the spectra shape and normalization parameters for the primary and secondary peak, respectively.

The distinction between wind dominated and swell dominated sea states is defined by the fully developed sea where:

$$T_f = a_f H_s^{1/3}$$

If $T_p < T_f$, the wind sea dominated the spectra peak, and $T_p > T_f$ it is swell dominated spectra peak. The factor a_f depends on fetch length:

$$a_f = \begin{cases} 6.6(sm^{1/3}) & \text{for fetch} = 370\,\text{km} \\ 5.3(sm^{1/3}) & \text{for fetch} = 100\,\text{km} \end{cases}$$

The other parameters in the spectrum are given:

$$f_{nj} = f \cdot T_{pj}$$

$$E_j = \frac{1}{16} H_{sj}^2 T_{pj}$$

$$S_{nj} = G_0 A_{\gamma j} \Gamma_{Sj} \gamma_{Fj}$$

where f is wave frequency (Hz).

Define non-dimensional period scale as:

$$\varepsilon_{lu} = \frac{T_f - T_p}{T_f - T_{lu}}$$

where

$$T_{lu} = \begin{cases} 2\sqrt{H_s} & \text{if } T_p \le T_f \\ 25 & \text{if } T_p > T_f \end{cases}$$

3 General Spectrum Form

$$\Gamma_{Sj} = f_{nj}^{-N} \exp\left[-\frac{N}{M} f_{nj}^{-M}\right]$$

$$G_0 = \left[\frac{1}{M}\left(\frac{N}{M}\right)^{-\frac{N-1}{M}} \Gamma\left(\frac{N-1}{M}\right)\right]^{-1}$$

$$\gamma_{F1} = \gamma \exp\left[-\frac{1}{2\sigma^2}\left(f_{n1} - 1\right)^2\right]$$

$$\gamma_{F2} = 1$$

$$\sigma = \begin{cases} 0.07 & f_{nj} < 1 \\ 0.09 & f_{nj} \ge 1 \end{cases}$$

A_γ can be approximated as:

$$A_\gamma\gamma - 1 = 4.1(N - 2M^{0.28} + 5.3)^{(0.96 - 1.45M^{0.1})}[\ln \gamma]^{f_2}$$

$$f_2 = (2.2M^{-3.3} + 0.57)N^{0.53 - 0.58M^{0.37}} + 0.94 - 1.04M^{-1.9}$$

For $M = 4$ and $\gamma \neq 1$, it can be simplified as:

$$A_\gamma\gamma - 1 = 4.1(N + 2.35)^{-0.71}[\ln \gamma]^{0.87 + 0.59N^{-0.45}}$$

which gives for $N = 4$:

$$A_\gamma\gamma - 1 = 1.1[\ln \gamma]^{1.19}$$

and for $N = 5$:

$$A_\gamma\gamma - 1 = 1.0[\ln \gamma]^{1.16}$$

Use common parameters:

$$N = 0.5\sqrt{H_s} + 3.2$$

$$T_f = 6.6\, H_s^{1/3}$$

3.1 Wind Dominated Sea ($T_p \le T_f$)

3.1.1 Primary Peak

$$H_{s1} = H_{sw} = r_{pw}H_s$$

$$T_{p1} = T_{pw} = T_p$$

$$\gamma = 35[1 + 3.5\exp(-H_s)]\left[\frac{2\pi}{g}\frac{H_{sw}}{T_p^2}\right]^{0.857}$$

$$M = 4$$

3.1.2 Secondary Peak

$$H_{s2} = H_{ssw} = \sqrt{1 - r_{pw}^2}\ H_s$$

$$T_{p2} = T_{pw} = T_f + 2.0$$

$$\gamma = 1$$

$$M = 4$$

The parameter r_{pw} is defined by:

$$r_{pw} = 0.7 + 0.3\exp[-(2\varepsilon_{lu})^2]$$

3.3 Swell Dominated Sea ($T_p > T_f$)

3.3.1 Primary Peak

$$H_{s1} = H_{ssw} = r_{ps}H_s$$

$$T_{p1} = T_{pw} = T_p$$

$$\gamma = 35[1 + 3.5\exp(-H_s)]\left[\frac{2\pi}{g}\frac{H_s}{T_f^2}\right]^{0.857}(1 + 6\varepsilon_{lu})$$

$$M = 4$$

3.3.2 Secondary Peak

$$H_{s2} = H_{sw} = \sqrt{1 - r_{ps}^2}\ H_s$$

$$T_{p2} = T_{pw} = \max\left(2.5, \left[\frac{16 s_4 \cdot 0.4^N}{G_0 H_{sw}^2}\right]^{-\frac{1}{N-1}}\right)$$

$$s_4 = \max\left(0.01, 0.08 \cdot \left[1 - \exp\left(-\frac{1}{3}H_S\right)\right]\right)$$

$$\gamma = 1$$

$$M = 4\left[1 - 0.7\exp\left(-\frac{1}{3}H_s\right)\right]$$

$$r_{ps} = 0.6 + 0.4\exp\left[-\left(\frac{\varepsilon_{lu}}{0.3}\right)^2\right]$$

5 Simplified Spectrum Form

For the simplified version of the spectrum, the spectral parameters $N = 4$ and $M = 4$ are used for all sea states. The spectrum follows:

$$\Gamma_{Sj} = f_{nj}^{-4}\exp\left[-f_{nj}^{-4}\right] \qquad j = 1, 2$$

$$G_0 = 3.26$$

$$\gamma_{F1} = \gamma\exp\left[-\frac{(f_{n1} - 1)^2}{2\sigma^2}\right]$$

$$\gamma_{F2} = 1$$

$$\sigma = \begin{cases} 0.07 & f_{nj} < 1 \\ 0.09 & f_{nj} \geq 1 \end{cases}$$

$$A_{\gamma 1}\gamma - 1 = 1.1[\ln \gamma]^{1.19}$$

$$A_{\gamma 2} = 1$$

$$T_f = 6.6\,H_s^{1/3}$$

5.1 Wind Dominated Sea ($T_p \leq T_f$)

5.1.1 Primary Peak

$$H_{s1} = H_{sw} = r_{pw}H_s$$

$$T_{p1} = T_{pw} = T_p$$

$$\gamma = 35\left[\frac{2\pi}{g}\frac{H_{sw}}{T_p^2}\right]^{0.857}$$

5.1.2 Secondary Peak

$$H_{s2} = H_{ssw} = \sqrt{1 - r_{ps}^2}\; H_s$$

$$T_{p2} = T_{psw} = T_f + 2.0$$

$$\gamma = 1$$

The parameter r_{pw} is defined by:

$$r_{pw} = 0.7 + 0.3\exp[-(2\varepsilon_{lu})^2]$$

5.3 Swell Dominated Sea ($T_p > T_f$)

5.3.1 Primary Peak

$$H_{s1} = H_{ssw} = r_{ps}H_s$$

$$T_{p1} = T_{psw} = T_p$$

$$\gamma = 35\left[\frac{2\pi}{g}\frac{H_s}{T_f^2}\right]^{0.857}(1 + 6\varepsilon_{lu})$$

5.3.2 Secondary Peak

$$H_{s2} = H_{sw} = \sqrt{1 - r_{ps}^2}\; H_s$$

$$T_{p2} = T_{pw} = 6.6\,H_{sw}^{1/3}$$

$$\gamma = 1$$

$$r_{ps} = 0.6 + 0.4\exp\left[-\left(\frac{\varepsilon_{lu}}{0.3}\right)^2\right]$$