Excerpt from ABS Rules Developed for Gears

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Foreword

Purpose
This booklet is an excerpt from the ABS Rules. It has been created to support equipment manufacturers in identifying the appropriate ABS Rules for their products.

Application
Equipment manufacturers seeking to have their products approved for marine and offshore applications under ABS classification or certification will find this booklet a practical reference for determining which sections of the ABS Rules are applicable to their specific equipment types.

This excerpt does not replace or supersede the complete ABS Rules. Users are responsible for consulting the full text of the applicable Rules and ensuring compliance with the most current edition, including any published corrigenda and notices.

Additional Information
For questions regarding the applicability of specific Rules to your equipment. Please contact your local ABS office.

The ABS Rules are available in their complete form at American Bureau of Shipping (ABS) Eagle.org.

1 General (2024)

Gears having a rated power of 100 kW (135 hp) and over intended for propulsion and for auxiliary services essential for propulsion, maneuvering and safety (see 4-1-1/1.3) of the vessel are to be designed, constructed, certified and installed in accordance with the provisions of this section.

Gears for steering mechanism of certified thrusters (ref 4-3-5), regardless of power rating, are to be designed, constructed, certified and installed in accordance with the provisions of this section.

Gears not included in the above listed applications are not required to comply with the provisions of this Section, but are to be designed, constructed and equipped in accordance with good commercial and marine practice. Acceptance of such gears will be based on the following:

For gears having a rated power of less than 100 kW, acceptance will be based on verification of the manufacturer's affidavit and gear nameplate data, as well as performance testing after installation to the satisfaction of the Surveyor.

Gears having a rated power of 100 kW (135 hp) and over, and intended for services considered not essential for propulsion, maneuvering and safety, are not required to be designed, constructed and certified by ABS in accordance with the requirements of this section. They are to be designed to a recognized standard or evidence of satisfactory service history for the intended or similar application is to be provided. Their acceptance will be based on installation and testing to the satisfaction of the Surveyor.

Piping systems of gears, in particular, lubricating oil and hydraulic oil, are addressed in 4-6-5/5, 4-6-6/9 and 4-6-7.

1.1 Objective (2024)

1.1.1 Goals (2024)

The gears covered in this section are to be designed, constructed, operated, and maintained to:

Goal No.	Goals
PROP 1	provide sufficient thrust/power to move or maneuver the vessel when required
PROP 3	*provide sufficient power for going astern...to secure proper control and bring the ship to rest in all normal circumstances.*
PROP 4	provide means to maneuver the vessel
SAFE 1.1	minimize danger to persons on board, the vessel, and surrounding equipment/installations from hazards associated with machinery and systems

Goals in the cross-referenced Rules/Regulations are also to be met.

1.1.2 Functional Requirements (2024)

In order to achieve the above-stated goals, the design, construction, and maintenance of the gears are to be in accordance with the following functional requirements:

Functional Requirement No.	Functional Requirements
Propulsion. Maneuvering, Station Keeping (PROP)	
PROP-FR1	Torque transmission parts are to withstand the design load not only by the steady torque, but also considering the effect from additional stress such as by vibration, shaft alignment and bending moments from gears
PROP-FR2	Load-bearing parts are to be designed to resist deleterious deflection and prevent fatigue failure during the anticipated design life.
PROP-FR3	Gear teeth are to be designed to withstand the contact stress without progressive pitting or scuffing.
PROP-FR4	Gear teeth roots are to be designed to withstand the bending stress without causing tooth root fillet cracking or fracture
PROP-FR5	Provide adequate lubrication to prevent friction as well as wear and tear such that the performance and lifespan of the bearings/gears can be maintained
Safety of Personnel (SAFE)	
SAFE-FR1	Provide access for inspection of gear teeth contacts and bearings clearance for successful performance and operation.

The functional requirements in the cross-referenced Rules/Regulations are also to be met.

1.1.3 Compliance (**2024**)

A vessel is considered to comply with the goals and functional requirements within the scope of classification when the applicable prescriptive requirements are complied with or when an alternative arrangement has been approved. Refer to Part 1D, Chapter 2.

1.3 Definitions

For the purpose of this section the following definitions apply:

1.3.1 Gears

The term *Gears* as used in this section covers external and internal involute spur and helical cylindrical gears having parallel axis as well as bevel gears used for either main propulsion or auxiliary services.

1.3.2 Rated Power

The *Rated Power* of a gear is the maximum transmitted power at which the gear is designed to operate continuously at its rated speed.

1.3.3 Rated Torque

The *Rated Torque* is defined by the rated power and speed and is the torque used in the gear rating calculations.

1.3.4 Gear Rating

The *Gear Rating* is the rating for which the gear is designed in order to carry its rated torque.

1.3.5 Intermittent Duty

Gear units intended for intermittent duty, that is at a higher rating than the rated power defined in 4-3-1/1.3.2 (the rated power at which the gear is designed to operate continuously at its rated speed), are to meet the requirements in 4-3-1/5.17 in addition to all applicable requirements contained in Section 4-3-1. When gear units rated for intermittent duty are installed on a vessel, the vessel's classification and operating profile are to reflect the intermittent rating. Such ratings would normally be considered for only non-commercial services or for commercial service with limited operating time in the intermittent duty service as defined by the mission profile.

1.5 Plans and Particulars to be Submitted

1.5.1 Gear Construction

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Details of gear casings

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Clutch control system including emergency clutching procedure

1.5.3 Data (2020)

Transmitted rated power for each gear

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Bearing lengths and diameters

Teeth data as per 4-3-1-A3

Mass of rotating parts

Balancing data

Spline data

Shrink allowance for rims and hubs

Type of coupling between prime mover and reduction gears

Type and viscosity of lubricating oil recommended by manufacturer

1.5.4 Material Specifications (2020)

The following typical properties of gear materials are to be submitted:

Range of chemical composition,

Physical properties at room temperature,

Endurance limits for pitting resistance, contact stress and tooth root bending stress,

Heat treatment of gears, coupling elements, shafts, quill shafts, and gear cases.

1.5.5 Hardening Procedure **(2020)**

The hardening procedure for surface hardened gears is to be submitted for review. The submittal is to include materials, details for the procedure itself, quality assurance procedures and testing procedures. Surface hardness depth and core hardness (and their shape) are to be determined from sectioned test samples. These test samples are to be of sufficient size to provide for determination of core hardness and are to be of the same material and heat treated as the gears that they represent.

Non Destructive Examination

The non destructive examination procedures are to include surface hardness, roughness of critical fillet. Forgings are to be tested in accordance with Part 2, Chapter 3. Internal soundness verification for gears.

1.5.6 Calculations and Analyses **(2020)**

Bearing life calculations

Tooth coupling and spline connection calculations

Shafts calculation

3 Materials

3.1 Material Specifications and Test Requirements **(2020)**

3.1.1 Material Specifications **(2020)**

Material for gears and gear units is to conform to specifications approved in connection with the design in each case. Specifications other than those given in Part 2, Chapter 3 conforming to national standard or to Manufacturer standard, are to be submitted for review and are to be approved in connection with the designs. Minimum material properties are to comply with 3.3.1.

Copies of material specifications and purchase orders are to be submitted to the Surveyor for information. The Surveyor will inspect and test material manufactured to other specifications than those given in Part 2, Chapter 3 of the ABS *Rules for Materials and Welding (Part 2)*, provided that such specifications are approved in connection with the designs and that they are clearly indicated on purchase orders.

3.1.2 Material Certification **(2022)**

For gear units used for steering of azimuthing thrusters, see 3.1.3.

For gear units whose power is in excess of 375 kW or more, materials for the following components materials are to be tested in the presence of and inspected by the Surveyor. The test results are to meet the specifications in Part 2, Chapter 3 or that approved in connection with the design.

- Forgings for gears, shafting, couplings, coupling fitted bolts.
- Forgings for through hardened, induction hardened, and nitrided gears.
- Forgings for carburized gears. See 2-3-7/3.9.3(g).
- Castings or hot rolled bars approved for use in place of any of the above forgings. (refer to Section and of the ABS *Rules for Materials and Welding (Part 2)*).
- Plates used for fabricating gears, which transmit torque (refer to Section of the ABS *Rules for Materials and Welding (Part 2)* or the plates can be manufactured to a recognized standard).

Material for gear units of 375 kW (500 hp) or less, including shafting, gears, couplings, and coupling bolts will be accepted on the basis of the manufacturer's certified material test reports and a satisfactory surface inspection and hardness check witnessed by the Surveyor.

Materials for power takeoff gears and couplings that are:

- For transmission of power to drive auxiliaries that are for use in port only (e.g., cargo oil pump), and
- Declutchable from propulsion shafting and not essential for propulsion

may be accepted on the basis of the manufacturer's certified material test reports and a satisfactory surface inspection and hardness check witnessed by the Surveyor, regardless of power rating.

Non fitted coupling bolts manufactured to a recognized bolt standard do not require material certification, they are to be traceable to material certificate by the manufacturer.

3.1.3 Gears for Steering of Azimuthing Thrusters

Gears for steering of azimuthing thrusters are to have their material traceable to manufacturer certificates except for output pinion and relevant shaft for which materials are to be tested in the presence of the Surveyor.

For small components of less than 50 kg weight, of mass produced gear units:

- For castings destructive examination of a sample for each batch for castings to prove casting process is to be carried out.
- For forging or rolled bars refer to ABS Rules for Material and Welding and

3.3 Alternative Material Minimum Requirements **(2020)**

Materials for torque transmitting components other than grades listed in Part 2 Chapter 3 are to comply with the following minimum properties

3.3.1 Elongation

Material for torque transmitting components of gear units is to show a minimum elongation at core as follows:

- Case hardened steel 8%
- Surface hardened steel 10%
- Nitriding steels 10%
- Through hardened steels 12%
- For planet carriers and connecting flanges for planetary gears 10%

3.3.2 Cleanliness

Steel for gears is to conform to cleanliness equivalent to a grade MQ ISO 6336-5.

3.3.3 Heat treatment

Heat treatment procedure and relevant controls are to comply with ABS Rules for Material and Welding 2-3-7.

5 Design

5.1 Gear Tooth **(2024)**

Gear teeth mesh capacity is to be calculated taking into consideration the pitting resistance and bending strength of the teeth. The teeth are to be designed for a load which can be transmitted without causing progressive pitting, and without causing tooth root fillet cracking or fracture. The calculations are to be carried out in accordance with Appendix A1.

Where carburized case hardened teeth are shot peened at tooth root fillet to increase bending strength, an increase in the *allowable bending stress number* (ref Appendix 1) of 10% may be credited provided appropriate tests are carried out to demonstrate adequacy of the shot peening process or the gears have adequate service experience. Where cleaning after shot peening is carried out, an increase in the *allowable bending stress number* above 10% may be accepted provided this is demonstrated by appropriate fatigue testing or the gear design has adequate service experience. Gear teeth recalling shot peening are to specify areas to be peened and areas to be masked, peening intensity and location of intensity verification, media type and size for peening.

5.1.1 Finish **(2024)**

The gear teeth surface finish is not to be rougher than 1.05 μm (41 μin.) arithmetic or centerline average.

Commentary:

For gears having a rated power below 3728 kW (5000 hp), a surface rougher than 1.05 um (41 uin.) arithmetic or centerline average may be accepted, provided the manufacturer can demonstrate satisfactory service experience with similar designs including lubricant.

End of Commentary

5.3 Bearings

Bearings of gear units are to be so designed and arranged that their design lubrication rate is maintained in service under working conditions.

5.3.1 Journal Bearings **(2024)**

For journal bearings, the maximum bearing pressures, minimum oil film thickness, maximum predicted internal bearing temperature and the maximum static unit load are to be in accordance with an applicable standard such as ISO 12130-1 (plain tilting plain thrust bearings), ISO 12131-1 (plain thrust pad bearings), ISO 12167-1 (plain journal bearings with drainage grooves), ISO 12168-1 (plain journal bearings without drainage grooves), ANSI/ AGMA 6032-A94, Table 8, etc.

Commentary:

Consideration will be given to bearing pressures exceeding the limits in the standards listed in 5.3.1 and 5.3.2 provided the manufacturer can demonstrate satisfactory service experience with similar designs.

End of Commentary

5.3.2 Roller Bearings **(2024)**

The minimum L10 bearing life is not to be less than 20,000 hours for ahead drives and 5,000 hours for astern. Shorter life may be considered in conjunction with an approved bearing inspection and replacement program reflecting the actual calculated bearing life. See 4-3-5/5.9 for application to thrusters. Calculations are to be in accordance with an applicable standard such as ISO 76, ISO 281 (rolling bearings, for static and dynamic ratings,respectively).

5.3.3 Shaft Alignment Analysis **(2020)**

For gear-shafts which are directly connected to the propulsion shafting, the load on the gear shaft bearings is to be evaluated by taking into consideration the loads resulting from the shafting alignment condition. For generic approval in the absence of an alignment calculation, a multiplication factor of 1.1 will be considered for output shaft bearing load.

5.5 Gear Cases **(2024)**

Gear cases are to be of substantial construction in order to minimize elastic deflections and maintain accurate mounting of the gears.

They are to be designed to withstand without deleterious deflection: the tooth forces generated by the gear elements, thrust bearing arrangement, line shaft alignment, prime mover(s), clutches, couplings and accessories, under all modes of operation.

Additionally, the inertial effects of the gears within the case, due to 1 **g** horizontal and 2 **g** vertical dynamic forces of the ship in a seaway, are to be considered.

Calculations, in accordance with the manufacturer's code of practice, substantiating these Rule requirements are to be submitted.

Commentary:

For gear case designs for which the manufacturer can provide a satisfactory service history, submission of calculations may be waived.

End of Commentary

5.5.1 Oil Sump (2020)

Where forced lubrication systems are installed, the position of suction for lubricating oil pump within the gear unit is to be such as to provide adequate immersion when the vessel is inclined, as per maximum inclination specified in 4-1-1/9 TABLE 7.

5.7 Access for Inspection (2024)

The construction of gear cases is to be such that sufficient number of access is provided for adequate inspection of gears, checking of gear teeth contacts and measurement of thrust bearing clearance.

Commentary:

Alternative methods such as use of special viewing devices may be considered.

End of Commentary

5.9 Calculation of Shafts for Gears

5.9.1 General (2024)

The minimum diameter of shafts for gears is to be determined by the following equations:

$$d = k.\sqrt[6]{(bT)^2 + (mM)^2}$$

$$b = 0.073 + \frac{n}{Y}$$

$$m = \frac{c_1}{c_2 + Y}$$

where (in SI (MKS and US units), respectively):

d	=	shaft diameter at section under consideration; mm (in.)
Y	=	yield strength (see 2-1-1/13.3 of the ABS *Rules for Materials and Welding (Part 2)*; N/mm^2 (kgf/mm^2, psi)
T	=	torsional moment at rated speed; N-m (kgf-cm, lbf-in) (See also 4-3-1/5.9.7 to account for effect of torsional vibrations, where applicable.)
M	=	bending moment at section under consideration; N-m (kgf-cm, lbf-in)

k , n , c_1 and c_2 are constants given in the following table:

	SI units	MKS units	US units
k	5.25	2.42	0.10
n	191.7	19.5	27800
c_1	1186	121	172000
c_2	413.7	42.2	60000

5.9.2 Shaft Diameter in way of Keyways or Splines (**2024**)

The minimum diameter of the shaft in way of keyways or splines is to be increased not less than 10% of the minimum diameter calculated in accordance with 5.9.1.

5.9.3 Maximum Torsional Moment (**2020**)

In determining the required size of the gears, couplings and gear shafting, the maximum torsional moment is the sum of:

i. Maximum steady torque of the operating profile (including astern power where applicable), when it exceeds the transmitted rated torque.
ii. Alternating torques per the torsional vibration calculations.

For generic approval following amplification factor can be considered: (a) For direct diesel engine drives: 1.2 (b) For all other drives, including diesel engine drives with elastic couplings: 1.0

5.9.4 Fatigue Analysis (**2020**)

Where the design of shafts includes geometric discontinuities (e.g., shoulder fillet with radius less than 0.08d), a fatigue analysis is to be submitted by the designer in accordance with ANSI/AGMA 6101 or DIN 743. The safety factor for high cycle fatigue is not to be less than 1.5

5.9.5 Quill Shafts

In the specific case of quill shafts, subjected to small stress raisers and no bending moments, the least diameter is to be determined by the following equation:

SI units	MKS units	US units
$d = 5.25.\sqrt[3]{bT}$	$d = 2.42.\sqrt[3]{bT}$	$d = 0.1.\sqrt[3]{bT}$
$b = 0.053 + \frac{187.8}{Y}$	$b = 0.053 + \frac{19.1}{Y}$	$b = 0.053 + \frac{27200}{Y}$

5.9.6 Shaft Couplings

For shaft couplings, coupling bolts, flexible coupling and clutches, see 4-3-2/5.19. For keys, see 4-3-2/5.7.

5.9.7 Vibration *(1 July 2006)*

The designer or builder is to evaluate:

i. The shafting system for different modes of vibrations (torsional, axial, lateral) and their coupled effect, as appropriate,
ii. The diameter of shafts considering maximum total torque (steady and vibratory torque),
iii. The gears for gear chatter and harmful torsional vibrations stresses. See also .

5.9.8 Shrink Fitted Pinions and Wheels (**2020**)

For pinions and wheels shrink-fitted on shafts, preloading and stress calculations and fitting instructions are to be submitted for review. The torsional holding capacity is to be at least 2.8 times the transmitted mean torque plus vibratory torque due to torsional vibration. For calculation purpose, to take account of torsional vibratory torque, the following factors are to be applied to the transmitted torque, unless the actual measured vibratory torque is higher, in which case the actual vibratory torque is to be used.

- For direct diesel engine drives: 1.2
- For all other drives, including diesel engine drives with elastic couplings: 1.0.

The preload stress based on the maximum available interference fit or maximum pull-up length is not to exceed 90% of the minimum specified yield strength.

The following friction coefficients are to be used:

- Oil injection method of fit: 0.13
- Dry method of fit: 0.18.

Shrink fit couplings are to be designed to avoid the risk of fretting corrosion. Where the length of contact within the coupling is more than 1.5 times the diameter of the shaft, or the shaft is subject to high bending moment, the shaft diameter is to be increased by 10% in way of the contact surface with the shoulder fillet, with a minimum radius of 0.2 times the shaft's smaller diameter.

5.9.9 Shrink Fitted Wheel Rim

For shrink-fitted wheel rims, preloading and stress calculations and fitting instructions are to be submitted for consideration. The preloading stress based on the maximum interference fit or maximum pull-up length is not to exceed 90% of the minimum specified yield strength.

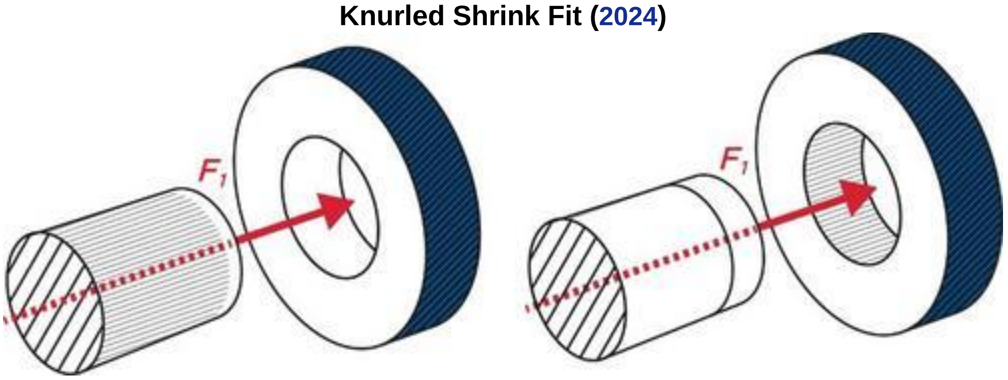
5.9.10 Knurled Shrink Fit (**2020**)

For small gears (pitch diameter less than 400mm) subject to low bending moment (e.g. ring gear of planetary gear units) where it is proposed to shrink a knurled profile into the hub to create a non-demountable coupling, the procedure is to be validated by testing to demonstrate adequate safety against slippage, avoidance of cracks on hub, adequate tolerances after shrinkage.

The difference in hardness between knurled male and hub is to be at least 100 HB

The knurled profile is to be used only to transmit torque. Adequate alignment of the fitted components is to be addressed by the tolerance of the plain cylindrical shoulder.

FIGURE 1
Knurled Shrink Fit (2024)



5.11 Rating of Cylindrical and Bevel Gears

The calculation procedures for the rating of external and internal involute spur and helical cylindrical gears, having parallel axes, and of bevel gears, with regard to surface durability (pitting) and tooth root bending strength, may be as given in Appendix 4-3-1-A1. These procedures are in substantial agreement with ISO 6336 and ISO-DIS 10300 for cylinder and bevel gears respectively.

5.13 Alternative Gear Rating Standards

Consideration will be given to gears that are rated based on any of the following alternative standards. In which case, gear rating calculations and justification of the applied gear design coefficients in accordance with the applicable design standard are to be submitted to ABS for review.

Cylindrical gears	Bevel gears
ANSI/AGMA 6032	AGMA 2003
ISO 6336	ISO 10300
DIN 3990, Part 31	DIN 3991

5.15 Gears with Multiple Prime Mover Inputs

For single helical gears with arrangements utilizing multiple prime mover inputs, and single or multiple outputs, the following analyses for all operating modes are to be conducted:

- All bearing reactions
- Tooth modifications
- Load distributions on the gear teeth
- Contact and tooth root bending stresses

A summary of the results of these analyses for each operating mode is to be submitted for review.

5.17 Rating of Gears for Intermittent Duty

5.17.1 Gears Intended for Intermittent Duty (2020)

When gear units rated for intermittent duty are installed as part of the propulsion system of a vessel, the vessel's classification and operating profile are to reflect the intermittent rating. Such ratings would be considered for only non-commercial services or for commercial service with limited operating time in the intermittent duty service as defined by the mission profile.

Gears intended for propulsion and rated for intermittent duty are to have the conditions associated with the intermittent rating clearly defined and listed as part of the rating.

These conditions include, but are not limited to: the specific intermittent rating (power and rpm) or ratings if there will be multiple intermittent ratings, the limit on the number of hours associated with the particular phase of the rating, limitation of load factor or a combination thereof, load spectrum, the time between overhaul and servicing of the unit, and any other conditions that the manufacturer places on the unit.

5.17.2 Calculations (2020)

Calculations supporting the intermittent rating or ratings are to be submitted by the manufacturer to ABS for review.

Teeth calculations are to be based on ISO 6336-6 *Calculation of local capacity of spur and helical gears – Calculation of service life under variable load*, or an equivalent recognized national standard that uses a fatigue cycle approach.

Bearings for an intermittent duty gear unit are to be calculated with Miner approach as reported in ISO 281 or similar. Finally the following is to be considered: maximum speed of bearings, maximum speed of clutches or clutch reduced capacity at maximum speed, pump speed limits, balance.

7 Piping Systems for Gears

The requirements of piping systems essential for operation of gears for propulsion, maneuvering, electric power generation and vessel's safety are in Section 4-6-5 for diesel engine and gas turbine installations and in Section 4-6-6 for steam turbine installations. Additionally, requirements for hydraulic and pneumatic systems are provided in Section 4-6-7. Specifically, the following references are applicable:

Lubricating oil system:	4-6-5/5 and 4-6-6/9;
Cooling system:	4-6-5/7 and 4-6-6/11;
Hydraulic system:	4-6-7/3 ;
Pneumatic system:	4-6-7/5 ;
Piping system general requirements:	Sections 4-6-1 and 4-6-2.

9 Testing, Inspection and Certification of Gears

9.1 Material Tests

For testing of materials intended for gear construction see 3.1 and 4-3-1/3.3.

9.3 Dynamic Balancing (2020)

Gear and wheel assemblies are to be free from injurious unbalance.

The residual unbalance in each plane is not to exceed the value determined by the following equations:

SI units	*MKS units*	*US units*
$B = 24 \cdot W/N$	$B = 24000 \cdot W/N$	$B = 15.1 \cdot W/N$

where

B	=	maximum allowable residual unbalance; Nmm (gf-mm, oz-in.)
W	=	weight of rotating part; N (kgf, lbf)
N	=	rpm at rated speed

For mass produced gear units whose rated power is below 3000 kW and rated speed below 3500 rpm, maximum unbalance of pinion and wheel assemblies can be determined by calculation taking into consideration machining tolerances of wheels, gear and shafts and relevant coupling: this is not applicable to casted, bolted or welded components or components with non-machined or manual-machined surfaces. A noise test and visual examination at maximum speed of the gear unit will be used as confirmation of adequacy of the gear assemblies: the maximum sound pressure level at 1 m from gearbox at a height between 1.2 m and 1.6 m above gear foundation is not to exceed 100 dB(A).

In all other cases, finished pinions and wheels are to be dynamically balanced as follows:

i. In two planes, when their pitch line velocity exceeds 25 m/s (4920 ft./min).
ii. Where their pitch line velocity does not exceed 25 m/s (4920 ft./min) or where dynamic balance is impracticable due to size, weight, speed or construction of units, the parts may be statically balanced in a single plane.

9.5 Shop Inspection (2023)

Each gear unit that requires to be certified by 4-3-1/1.1 is to be inspected during manufacture by the Surveyor for conformance with the design approved. This is to include but not limited to checks on gear teeth hardness, and surface finish and dimension checks of main load bearing components The accuracy of meshing is to be verified for all meshes and the initial tooth contact pattern is to be checked by the Surveyor. Manufacturer's records of shrink fitting of pinions/gears/wheel rims on to the shafts are to be reviewed for compliance to approved drawings and fitting instructions as required by 1.5.1, 5.9.8, and 5.9.9, as applicable.

Reports on pinions and wheels balancing as per 4-3-1/9.3 are to be made available to the Surveyor for verification.

9.7 Certification of Gears

9.7.1 General (2020)

Each gear required to be certified by 4-3-1/1.1 before being installed is:

i. To have its design approved by ABS; for which purpose, plans and data as required by are to be submitted to ABS for approval;
ii. To be surveyed during its construction, which is to include, but not limited to, material tests as indicated in , meshing accuracy and tooth contact pattern checks as indicated in , gear teeth hardness check, teeth surface roughness, verification of gear backlash, and verification of dynamic balancing or noise test reports, as indicated in .

9.7.2 Approval Under the Type Approval Program

9.7.2(a) *Product design assessment*. **(2020)**

(PDA) Upon application by the manufacturer, each rating of a type of gear may be design assessed as described in 1A-1-A3/5.1 of the ABS *Rules for Conditions of Classification (Part 1A)*. For this purpose, each rating of a gear type is to be approved in accordance with 9.7.1.i. The prototype test, however, is to be conducted in accordance with an approved test schedule and is to be witnessed by the Surveyor. Gear so approved may be applied to ABS for listing on the ABS website as Products Design Assessed. Once listed, and subject to renewal and updating of certificate as required by 1A-1-A3/5.7 of the ABS *Rules for Conditions of Classification (Part 1A)*, gear particulars will not be required to be submitted to ABS each time the gear is proposed for use on board a vessel.

9.7.2(b) *Mass produced gears*.

Manufacturer of mass-produced gears, who operates a quality assurance system in the manufacturing facilities, may apply to ABS for quality assurance assessment described in 1A-1-A3/5.5 (PQA) of the ABS *Rules for Conditions of Classification (Part 1A)*.

Upon satisfactory assessment under 1A-1-A3/5.5 (PQA) of the ABS *Rules for Conditions of Classification (Part 1A)*, gears produced in those facilities will not require a Surveyor's attendance at the tests and inspections indicated in 9.7.1.ii. Such tests and inspections are to be carried out by the manufacturer whose quality control documents will be accepted. Certification of each gear will be based on verification of approval of the design and on continued effectiveness of the quality assurance system. See 1A-1-A3/5.7.1(a) of the ABS *Rules for Conditions of Classification (Part 1A)*.

9.7.2(c) *Non-mass Produced Gears*. **(2024)**

Manufacturer of non-mass produced gears, who operates a quality assurance system in the manufacturing facilities, may apply to ABS for quality assurance assessment described in 1A-1-A3/5.3.1(a) (Manufacturers Procedure) or 1A-1-A3/5.3.1(b) (RQS) of the ABS *Rules for Conditions of Classification (Part 1A)*. Certification to 1A-1-A3/5.5 (PQA) of the ABS *Rules for Conditions of Classification (Part 1A)* may also be considered in accordance with 4-1-1/9 TABLE 2.

9.7.2(d) *Type Approval Program*. **(2020)**

Gear types which have their ratings approved in accordance with 4-3-1/9.7.2(a) and the quality assurance system of their manufacturing and heat treatment facilities approved in accordance with 4-3-1/9.7.2(b) or 4-3-1/9.7.2(c) will be deemed Type Approved and will be eligible for listing on the ABS website as Type Approved.

9.9 Shipboard Trials for Propulsion Gears (2024)

After installation on board a vessel, the gear unit is to be operated in the presence of the Surveyor to demonstrate its ability to function satisfactorily under operating conditions and be free from excessive vibrations at speeds within the operating range. When the propeller is driven through reduction gears, the Surveyor is to ascertain that no gear-tooth chatter occurs throughout the operating range; otherwise a barred speed range as specified in 4-3-2/7.5.3 is to be provided.

For conventional propulsion gear units above 1120 kW (1500 hp), a record of gear-tooth contact is to be made at the trials. To facilitate the survey of extent and uniformity of gear-tooth contact, selected bands of pinion or gear teeth on each meshing are to be coated beforehand with copper or layout dye. See also 7-6-2/1.1.2 of the ABS *Rules for Survey After Construction* for the first annual survey after the vessel enters service.

Post-trial examination of spur and helical type gears is to indicate essentially uniform contact across 90% of the effective face width of the gear teeth excluding end relief.

The gear-tooth examination for spur and helical type gear units of 1120 kW (1500 hp) and below, all epicyclical gear units and bevel type gears will be subject to special consideration. The gear manufacturer's recommendations will be considered.

Appendix 1 Rating of Cylindrical and Bevel Gears

1 Application

The following calculation procedures cover the rating of external and internal involute spur and helical cylindrical gears, having parallel axis, and of bevel gears with regard to surface durability (pitting) and tooth root bending strength.

For normal working pressure angles in excess of 25° or helix angles in excess of 30°, the results obtained from these calculation procedures are to be confirmed by experience data which are to be submitted by the manufacturer.

The influence factors are defined regarding their physical interpretation. Some of the influence factors are determined by the gear geometry or have been established by conventions. These factors are to be calculated in accordance with the equations provided.

Other influence factors, which are approximations, and are indicated as such can be calculated according to alternative methods for which engineering justification is to be provided for verification.

3 Symbols and Units

The main symbols used are listed below. Symbols specifically introduced in connection with the definition of influence factors are described in the appropriate sections.

Units of calculations are given in the sequence of SI units (MKS units, and US units.)

a	center distance	mm (in.)
b	common facewidth	mm (in.)
b_1, b_2	facewidth of pinion, wheel	mm (in.)
b_{eH}	effective facewidth	mm (in.)
b_s	web thickness	mm (in.)
b_B	facewidth of one helix on a double helical gear	mm (in.)
d_1, d_2	reference diameter of pinion, wheel	mm (in.)
d_{a1}, d_{a2}	tip diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{b1}, d_{b2}	base diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{f1}, d_{f2}	root diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{i1}, d_{i2}	rim inside diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)

d_{sh}	external diameter of shaft	mm (in.)
d_{shi}	internal diameter of hollow shaft	mm (in.)
d_{w1}, d_{w2}	working pitch diameter of pinion, wheel	mm (in.)
f_{fa1}, f_{fa2}	profile form deviation of pinion, wheel	mm (in.)
f_{pb1}, f_{pb2}	transverse base pitch deviation of pinion, wheel	mm (in.)
h_1, h_2	tooth depth of pinion, wheel	mm (in.)
h_{a1}, h_{a2}	addendum of pinion, wheel	mm (in.)
h_{a01}, h_{a02}	addendum of tool of pinion, wheel	mm (in.)
h_{f1}, h_{f2}	dedendum of pinion, wheel	mm (in.)
h_{fp1}, h_{fp2}	dedendum of basic rack of pinion, wheel	mm (in.)
h_{F1}, h_{F2}	bending moment arm for tooth root bending stress for application of load at the outer point of single tooth pair contact for pinion, wheel	mm (in.)
ℓ	bearing span	mm (in.)
ℓ_b	length of contact	mm (in.)
m_n	normal module	mm (in.)
m_{na}	outer normal module	mm (in.)
m_t	transverse module	mm (in.)
n_1, n_2	rotational speed of pinion, wheel	rpm
p_d	outer diametral pitch	mm^{-1} (in^{-1})
p_{r01}, p_{r02}	protuberance of tool for pinion, wheel	mm (in.)
q_1, q_2	machining allowance of pinion, wheel	mm (in.)
s_{Fn1}, s_{Fn2}	tooth root chord in the critical section of pinion, wheel	mm (in.)
s	distance between mid-plane of pinion and the middle of the bearing span	mm (in.)
u	gear ratio	---
v	tangential speed at reference diameter	m/s (m/s, ft/min)
x_1, x_2	addendum modification coefficient of pinion, wheel	---
x_{sm}	tooth thickness modification coefficient (midface)	---
z_1, z_2	number of teeth of pinion, wheel	---
z_{n1}, z_{n2}	virtual number of teeth of pinion, wheel	---
B	total facewidth, of double helical gear including gap width	mm (in.)
F_{bt}	nominal tangential load on base cylinder in the transverse section	N (kgf, lbf)

F_t	nominal transverse tangential load at reference cylinder	N (kgf, lbf)
P	transmitted rated power	kW (mhp, hp)
Q	ISO grade of accuracy	---
R	cone distance	mm (in.)
R_m	middle cone distance	mm (in.)
R_{zf1}, R_{zf2}	flank roughness of pinion, wheel	μm (μin.)
R_{zr1}, R_{zr2}	fillet roughness of pinion. Wheel	μm (μin.)
T_1, T_2	nominal torque of pinion, wheel	N-m (kgf-m, lbf-ft.)
U	minimum ultimate tensile strength of core (applicable to through hardened, normalized and cast gears only)	N/mm^2 (kgf/mm^2, lbf/in^2)
$\alpha_{en1}, \alpha_{en2}$	form-factor pressure angle: pressure angle at the outer point of single pair tooth contact for pinion, wheel	degrees
$\alpha_{Fen1}, \alpha_{Fen2}$	load direction angle: relevant to direction of application of load at the outer point of single pair tooth contact of pinion, wheel	degrees
α_n	normal pressure angle at reference cylinder	degrees
α_t	transverse pressure angle at reference cylinder	degrees
α_{vt}	transverse pressure angle of virtual cylindrical gear	degrees
α_{wt}	transverse pressure angle at working pitch cylinder	degrees
β	helix angle at reference cylinder	degrees
β_b	helix angle at base cylinder	degrees
β_{vb}	helix angle at base circle	degrees
δ_1, δ_2	reference cone angle of pinion, wheel	degrees
δ_{a1}, δ_{a2}	tip angle of pinion, wheel	degrees
ϵ_α	transverse contact ratio	---
ϵ_β	overlap ratio	---
ϵ_γ	total contact ratio	---
ρ_{a01}, ρ_{a02}	tip radius of tool of pinion, wheel	mm (in.)
ρ_c	radius of curvature at pitch surface	mm (in.)
ρ_{fp}	root radius of basic rack	mm (in.)
ρ_{F1}, ρ_{F2}	root fillet radius at the critical section of pinion, wheel	mm (in.)
Subscripts	1 = pinion; 2 = wheel; 0 = tool	

5 Geometrical Definitions

For internal gearing $z_2, a, d_2, d_{a2}, d_{b2}, d_{w2}$ and u are negative.

The pinion is defined as the gear with the smaller number of teeth. Therefore the absolute value of the gear ratio, defined as follows, is always greater or equal to the unity:

$$u = z_2/z_1 = d_{w2}/d_{w1} = d_2/d_1$$

In the equation of surface durability, b is the common facewidth on the pitch diameter.

In the equation of tooth root bending stress, $b_1 \mathrm{or} b_2$ are the facewidths at the respective tooth roots. In any case, $b_1 \mathrm{and} b_2$ are not to be taken as greater than b by more than one normal module m_n on either side.

The common facewidth b may be used also in the equation of teeth root bending stress if significant crowning or end relief have been applied.

Additional geometrical definitions are given in the following expressions.

$$\tan\alpha_t = \tan\alpha_n/\cos\beta$$

$$\tan\beta_b = \tan\beta \cdot \cos\alpha_t$$

$$d_{1,2} = z_{1,2} \cdot m_n/\cos\beta$$

$$d_{b1,2} = d_{1,2} \cdot \cos\alpha_t = d_{w1,2} \cdot \cos\alpha_{tw}$$

$$a = 0.5(d_{w1} + d_{w2})$$

$$z_{n1,2} = z_{1,2}/(\cos^2\beta_b \cdot \cos\beta)$$

$$m_t = m_m/\cos\beta$$

$$\mathrm{inv}\alpha = \tan\alpha - \pi \cdot \alpha/180, \alpha \mathrm{in degrees}$$

$$\mathrm{inv}\alpha_{wt} = \mathrm{inv}\alpha_t + 2 \cdot \tan\alpha_n \cdot \frac{(x_1 + x_2)}{(z_1 + z_2)}$$

$$\alpha_{wt} = \arccos\left(\frac{d_{b1} + d_{b2}}{2 \cdot a}\right), \alpha_{wt} \mathrm{in degrees}$$

$$x_1 + x_2 = \frac{(z_1 + z_2) \cdot (inv\alpha_{wt} - \mathrm{inv}\alpha_t)}{2 \cdot \tan\alpha_n}$$

$$x_1 = \frac{h_{a0}}{m_n} - \frac{d_1 - d_{f1}}{2 \cdot m_n}, x_2 = \frac{h_{a0}}{m_n} - \frac{d_2 - d_{f2}}{2 \cdot m_n}$$

$$\epsilon_\alpha = \frac{0.5 \cdot \sqrt{d_{a1}^2 - d_{b1}^2} \pm 0.5 \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - a \cdot \sin\alpha_{wt}}{\pi \cdot m_n \cdot \frac{\cos\alpha_t}{\cos\beta}}$$

(The positive sign is to be used for external gears; the negative sign for internal gears.)

$$\epsilon_\beta = \frac{b \cdot \sin\beta}{\pi \cdot m_n}$$

(For double helix, b is to be taken as the width of one helix.)

$$\epsilon_\gamma = \epsilon_a + \epsilon_\beta$$

$$\rho_c = \frac{a \cdot \sin\alpha_{wt} \cdot u}{\pi \cdot m_n}$$

$$v = d_{1,2} \cdot n_{1,2}/19099[SI\ and\ MKS\ units]$$

$$v = d_{1,2} \cdot n_{1,2}/3.82\ [\text{USunits}]$$

7 Bevel Gear Conversion and Specific Formulas

Conversion of bevel gears to virtual (equivalent) cylindrical gears is based on the bevel gear midsection.

Index v refers to the virtual (equivalent) cylindrical gear.

Index m refers to the midsection of bevel gear.

δ_1, δ_2	=	pitch angle pinion, wheel
δ_{a1}, δ_{a2}	=	face angle pinion, wheel
$\sum$	=	shaft angle
β_m	=	mean spiral angle
$d_{e1,2}$	=	outer pitch diameter pinion, wheel
$d_{m1,2}$	=	mean pitch diameter pinion, wheel
$d_{v1,2}$	=	reference diameter of virtual cylindrical gear pinion, wheel
$R_{e1,2}$	=	outer cone distance pinion, wheel
R_m	=	mean cone distance

Number of teeth of virtual cylindrical gear:

$$z_{v1} = \frac{z_{v1}}{\cos\delta_1}$$

$$z_{v2} = \frac{z_{v2}}{\cos\delta_2}$$

- For $\sum = 90^\circ$:

$$z_{v1} = z_1 \frac{\sqrt{u^2+1}}{u}$$

$$z_{v2} = z_2 \frac{\sqrt{u^2+1}}{u}$$

Gear ratio of virtual cylindrical gear

$$u_v = \frac{z_{v2}}{z_{v1}}$$

- For $\sum = 90^\circ$:

$$u_v = u^2$$

Geometrical definitions:

$$\delta_1 + \delta_1 = \Sigma$$

$$\mathbf{tan}\alpha_{vt} = \frac{\mathbf{tan}\alpha_n}{\cos\beta_m}$$

$$\mathbf{tan}\beta_{bm} = \mathbf{tan}\beta_m \cdot \cos\alpha_{vt}$$

$$\beta_{vb} = \mathbf{arcsin}\left(\sin\beta_m \cdot \cos\alpha_n\right)$$

$$R_e = \frac{d_{e1,2}}{2 \cdot \sin\delta_{1,2}}$$

$$R_m = R_e - \frac{b}{2}, b \leq \frac{R}{3}$$

Reference diameter of pinion, wheel refers to the midsection of the bevel gear:

$$d_{m1} = d_{e1} - b \cdot \sin\delta_1$$

$$d_{m2} = d_{e2} - b \cdot \sin\delta_2$$

Modules:

Outer transverse module:

$$m_{et} = \frac{d_{e2}}{z_2} = \frac{d_{e1}}{z_1} = \frac{25.4}{p_d}$$

Outer normal module:

$$m_{na} = m_t \cdot \cos\beta_m$$

Mean normal module:

$$m_{mn} = m_{mt} \cdot \cos\beta_m$$

$$m_{mn} = m_{et} \cdot \frac{R_m}{R_e} \cdot \cos\beta_m$$

$$m_{mn} = \frac{d_{m1}}{z_1} \cdot \cos\beta_m$$

$$m_{mn} = \frac{d_{m2}}{z_2} \cdot \cos\beta_m$$

Base pitch:

$$p_{btm} = \frac{\pi \cdot m_{mn} \cdot \cos\alpha_{vt}}{\cos\beta_m}$$

Reference diameter of pinion, wheel refers to the virtual (equivalent) cylindrical gear:

$$d_{v1} = \frac{d_{m1}}{\cos\delta_1}$$

$$d_{v2} = \frac{d_{m2}}{\cos\delta_2}$$

Base diameter of pinion, wheel

$$d_{vb1} = d_{v1} \cdot \cos\alpha_{vt}$$

$$d_{vb2} = d_{v2} \cdot \cos\alpha_{vt}$$

Center distance of virtual cylindrical gear:

$$a_{v=}0.5 \cdot (d_{v1} + d_{v2})$$

Transverse pressure angle of virtual cylindrical gear:

$$\alpha_{vt} = \arccos\left(\frac{d_{vb1} + d_{vb2}}{2 \cdot a_v}\right), \alpha_{vt}\text{in degrees}$$

Mean Addendum:

For gears with constant addendum *Zyklo-Palloid(Klingelnberg)*:

$$h_{am1} = m_{mn} \cdot (1 + x_{hm1})$$

$$h_{am2} = m_{mn} \cdot (1 + x_{hm2})$$

For gears with variable addendum *(Gleason):*

$$h_{am1} = h_{ae1} - \frac{b}{2} \cdot \tan(\delta_{a1} - \delta_1)$$

$$h_{am2} = h_{ae2} - \frac{b}{2} \cdot \tan(\delta_{a2} - \delta_2)$$

where h_{ae} is the outer addendum.

Profile shift coefficients:

$$x_{hm1} = \frac{h_{am1} - h_{am2}}{2 \cdot m_{mn}}$$

$$x_{hm2} = \frac{h_{am2} - h_{am1}}{2 \cdot m_{mn}}$$

Mean Dedendum:

For gears with constant dedendum *Zyklo-Palloid (Klingelnberg):*

$$h_{fp} = (1.25 \ldots 1.30) \cdot m_{mn}$$

$$\rho_{fp} = (0.2 \ldots 0.3) \cdot m_{mn}$$

For gears with variable dedendum (*Gleason):*

$$h_{fm1} = h_{fe1} - \frac{b}{2} \cdot \tan(\delta_{a1} - \delta_1)$$

$$h_{fm2} = h_{fe2} - \frac{b}{2} \cdot \tan(\delta_{a2} - \delta_2)$$

$$h_{f1} = h_{fm1} + x_{hm1} \cdot m_{mn}$$

$$h_{f2} = h_{fm2} + x_{hm2} \cdot m_{mn}$$

where h_{fe} is the outer dedendum and h_{fm} is the mean dedendum.

Tip diameter of pinion, wheel:

$$d_{va1} = d_{v1} + 2 \cdot h_{am1}$$

$$d_{va2} = d_{v2} + 2 \cdot h_{am2}$$

Transverse contact ratio:

$$\epsilon_{va} = \frac{0.5 \cdot \sqrt{d_{va1}^2 - d_{vb1}^2} + 0.5 \cdot \sqrt{d_{va2}^2 - d_{vb2}^2} - a_v \cdot \sin a_{vt}}{\pi \cdot m_{mn} \cdot \frac{\cos a_{vt}}{\cos \beta_m}}$$

Overlap ratio:

$$\epsilon_{v\beta} = \frac{b \cdot \sin \beta_m}{\pi \cdot m_{mn}}$$

Modified contact ratio:

$$\epsilon_{v\gamma} = \sqrt{\epsilon_{va}^2 + \epsilon_{v\beta}^2}$$

Tangential speed at midsection:

$$v_{mt} = \frac{d_{m1,2} \cdot n_{1,2}}{19098} \; m/s \; [SI \; and \; MKS \; units]$$

$$v_{mt} = \frac{d_{m1,2} \cdot n_{1,2}}{3.82} \; ft/min \; [US \; units]$$

Radius of curvature (normal section):

$$\rho_{vc} = \frac{a_v \cdot \sin \alpha_{vt}}{\cos \beta_{bm}} \cdot \frac{u_v}{(1 + u_v)^2}$$

Length of the middle line of contact:

$$\ell_{bm} = \frac{b \cdot \epsilon_{v\alpha}}{\cos\beta_{vb}} \cdot \frac{\sqrt{\epsilon_{v\gamma}^2 - [(2-\epsilon_{v\alpha}) \cdot (1-\epsilon_{v\beta})]^2}}{\epsilon_{v\gamma}^2} \text{ for } \epsilon_{v\beta} < 1$$

$$\ell_{bm} = \frac{b \cdot \epsilon_{v\alpha}}{\epsilon_{v\gamma} \cdot \cos\beta_{vb}} \text{ for } \epsilon_{v\beta} \geq 1$$

9 Nominal Tangential Load, F_t , F_{mt}

The nominal tangential load, F_t or F_{mt} , tangential to the reference cylinder and perpendicular to the relevant axial plane, is calculated directly from the rated power transmitted by the gear by means of the following equations:

	SI units	*MKS units*	*US units*
	$T_{1,2} = 9549 \cdot P/n_{1,2}$ N-m	$T_{1,2} = 716.2 \cdot P/n_{1,2}$ kgf-m	$T_{1,2} = 5252 \cdot P/n_{1,2}$ lbf-ft
Cylindrical gears	$F_t = \frac{2000T_{1,2}}{d_{1,2}} = \frac{19.1P \times 10^6}{n_{1,2}d_{1,2}}$ N	$F_t = \frac{2000T_{1,2}}{d_{1,2}} = \frac{1.4325P \times 10^6}{n_{1,2}d_{1,2}}$ kgf	$F_t = \frac{24T_{1,2}}{d_{1,2}} = \frac{126.05P \times 10^3}{n_{1,2}d_{1,2}}$ lbf
Bevel gears	$F_{mt} = \frac{2000T_{1,2}}{d_{m1,2}} = \frac{19.1P \times 10^6}{n_{1,2}d_{m1,2}}$ N	$F_{mt} = \frac{2000T_{1,2}}{d_{m1,2}} = \frac{1.4325P \times 10^6}{n_{1,2}d_{m1,2}}$ kgf	$F_{mt} = \frac{24T_{1,2}}{d_{m1,2}} = \frac{126.05P \times 10^3}{n_{1,2}d_{m1,2}}$ lbf

Where the vessel on which the gear unit is being used, is receiving an *Ice Class* notation, see Part 6, Chapter 1.

11 Application Factor, K_A

The application factor, K_A , accounts for dynamic overloads from sources external to the gearing.

The application factor, K_A , for gears designed for infinite life is defined as the ratio between the maximum repetitive cyclic torque applied to the gear set and the nominal rated torque.

The factor mainly depends on:

- Characteristics of driving and driven machines;
- Ratio of masses;
- Type of couplings;
- Operating conditions as e.g. overspeeds, changes in propeller load conditions.

When operating near a critical speed of the drive system, a careful analysis of these conditions is to be made.

The application factor, K_A , is to be determined by measurements or by appropriate system analysis. Where a value determined in such a way cannot be provided, the following values are to be used:

a. *Main propulsion gears:*

Turbine and electric drive:	1.00
Diesel engine with hydraulic or electromagnetic slip coupling:	1.00

Diesel engine with high elasticity coupling:	1.30
Diesel engine with other couplings:	1.50

b. *Auxiliary gears:*

Electric motor, diesel engine with hydraulic or electromagnetic slip coupling:	1.00
Diesel engine with high elasticity coupling:	1.20
Diesel engine with other couplings:	1.40

13 Load Sharing Factor, $K\gamma$

The load sharing factor K_γ accounts for the maldistribution of load in multiple path transmissions as e.g. dual tandem, epicyclical, double helix.

The load sharing factor K_γ is defined as the ratio between the maximum load through an actual path and the evenly shared load. The factor mainly depends on accuracy and flexibility of the branches.

The load sharing factor K_γ is to be determined by measurements or by appropriate system analysis. Where a value determined in such a way cannot be provided, the following values are to be used:

a)	*Epicyclical gears:*	
	Up to 3 planetary gears:	1.00
	4 planetary gears:	1.20
	5 planetary gears:	1.30
	6 planetary gears and over:	1.40
b)	*Other gear arrangements including bevel gears:*	1.00

15 Dynamic Factor, K_v

The dynamic factor K_v , accounts for internally generated dynamic loads due to vibrations of pinion and wheel against each other.

The dynamic factor, K_v , is defined as the ratio between the maximum load which dynamically acts on the tooth flanks and the maximum externally applied load ($F_t \cdot K_A \cdot K_\gamma$).

The factor mainly depends on:

- Transmission errors (depending on pitch and profile errors) ;
- Masses of pinion and wheel;
- Gear mesh stiffness variation as the gear teeth pass through the meshing cycle;
- Transmitted load including application factor;
- Pitch line velocity;
- Dynamic unbalance of gears and shaft;
- Shaft and bearing stiffnesses;
- Damping characteristics of the gear system.

The dynamic factor, K_v , is to be advised by the manufacturer as supported by his measurements, analysis or experience data or is to be determined as per 4-3-1-A1/15.1, except that where vz_1 /100 is 3 m/s (590 ft/min.) or above, K_v may be obtained from 4-3-1-A1/15.3.

15.1 Determination of K_v - Simplified Method

Where all of the following four conditions are satisfied, K_v may be determined in accordance with 4-3-1-A1/15.1.

a. Steel gears of heavy rims sections.

b. Values of F_t/b are in accordance with the following table:

SI units	MKS units	US units
> 150 N/mm	> 15 kgf/mm	> 856 lbf/in.

c. z_1 < 50

d. Running speeds in the subcritical range are in accordance with the following table:

	($v \cdot z_1$) / 100	
	SI & MKS units	US units
Helical gears	< 14 m/s	< 2756 ft/min
Spur gears	< 10 m/s	< 1968 ft/min
All types of gears	< 3 m/s	< 590 ft/min

For gears other than specified above, the single resonance method as per 4-3-1-A1/15.3 below may be applied. The methods of calculations are as follows:

	SI and MKS units	US units
For helical gears of overlap ratio ϵ_β ≥ unity	$K_v = K_{vh} = 1 + K_1 \cdot v \cdot z_1/100$	$K_v = K_{vh} = 1 + K_1 \cdot v \cdot z_1/19685$
For helical gears of overlap ratio ε_β < unity	K_v is obtained by means of linear interpolation: $$K_v = K_{vs} - \epsilon_\beta(K_{vs} - K_{vh})$$	
For spur gears	$K_v = K_{vs} = 1 + K_1 \cdot v \cdot z_1/100$	$K_v = K_{vs} = 1 + K_1 \cdot v \cdot z_1/19685$
For bevel gears	In the above conditions (b, c and d) and in the above formulas: • the real z_1 is to be used instead of the virtual (equivalent) z_{v1} ; • v is to be substituted by v_{mt} ; (tangential speed at midsection); and • F_t is to be substituted by F_{mt} .	

	SI and MKS units	US units
For all gears	K_1 values are specified in table below.	

	K_1 for different values of Q (ISO Grades of accuracy)					
	3	4	5	6	7	8
Spur gears	0.0220	0.0300	0.0430	0.0620	0.0920	0.1250
Helical gears	0.0125	0.0165	0.0230	0.0330	0.0480	0.0700
Q is to be according to ISO 1328. In case of mating gears with different grades of accuracy the grade corresponding to the lower accuracy is to be used.						

15.3 Determination of K_v - Single Resonance Method

For single stage gears, the dynamic factor K_v may be determined from 4-3-1-A1/15.3.3 through 4-3-1-A1/15.3.6 for the ratio N in 4-3-1-A1/15.3.1 and using the factors given in 4-3-1-A1/15.3.2.

15.3.1 Resonance Ratio, N

$$N = \frac{n_1}{n_{E1}}$$

$$n_{E1} = \frac{30 \times 10^3}{\pi \cdot z_1} \cdot \sqrt{\frac{C\gamma}{m_{red}}} \mathrm{rpm} [SI\ units]$$

$$n_{E1} = \frac{93.947 \times 10^3}{\pi \cdot z_1} \cdot \sqrt{\frac{C\gamma}{m_{red}}} \mathrm{rpm} [MKS\ units]$$

$$n_{E1} = \frac{589.474 \times 10^3}{\pi \cdot z_1} \cdot \sqrt{\frac{C\gamma}{m_{red}}} \mathrm{rpm} [US\ units]$$

where*:*

m_{red}	=	relative reduced mass of the gear pair, per unit facewidth referred to the line of action:	
m_{red}	=	$\frac{\pi}{8} \cdot \left(\frac{d_{m1}}{d_{b1}}\right)^2 \cdot \left[\frac{d_{m1}^2}{\left(1/\left(1-q_1^4\right) \cdot \rho_1\right) + \left(1/\left(1-q_2^4\right) \cdot \rho_2 \cdot u^2\right)}\right]$	kg/mm(lb/in.)
m_{red}	=	$\frac{J_1 \cdot J_2}{p \cdot J_1 \cdot \left(\frac{d_{b2}}{2}\right)^2 + J_2 \cdot \left(\frac{d_{b1}}{2}\right)^2}$	kg/mm(lb/in)
p	=	for planetary gears, number of planets	

$d_{m1,m2}$	=	$\dfrac{d_{a1,a2} + d_{f1,f2}}{2}$	mm(in.)
$q_1 = \dfrac{d_{i1}}{d_{m1}}$; $q_2 = \dfrac{d_{i2}}{d_{m2}}$			for reference, see 4-3-1-A1/23.29 FIGURE 5
J_1	=	moment of inertia per unit facewidth for pinion:	
J_1	=	$\dfrac{\pi \cdot \rho_1 \cdot d_{b1}^4}{32}$	kg-mm^2 /mm,(lb-in^2 /in.)
J_2	=	moment of inertia per unit facewidth for wheel:	
J_2	=	$\dfrac{\pi \cdot \rho_2 \cdot d_{b2}^4}{32}$	kg-mm^2 /mm,(lb-in^2/in.)
$\rho_{1,2}$	=	density of pinion, wheel materials	
ρ	=	density of steel material:	
	=	7.83×10^{-6}	kg/mm^3 [SI and MKS units]
	=	2.83×10^{-1}	lb/in^3 [US units]

Bevel gears:

For bevel gears, the real z_1 (not the equivalent) is to be inserted in the above formulas. Determination of m_{red} is to be as follows:

$$m_{1,2}^* = \frac{\pi}{8} \cdot \rho_{1,2} \cdot \left[\frac{d_{m1,2}^2}{\cos^2 a_n}\right] kg/mm\ (lb/in.)$$

$$m_{red} = \frac{m_1^* \cdot m_2^*}{m_1^* + m_2^*}\ kg/mm\ (lb/in.)$$

Mesh stiffness per unit facewidth, C_γ :

$$C_\gamma = \frac{20}{0.85} \cdot B_b \mathrm{N/mm}\text{-}\mu\ \mathrm{m}[SI\ units]$$

$$C_\gamma = \frac{2.039}{0.85} \cdot B_b \mathrm{kgf/mm}\text{-}\mu\ \mathrm{m}[MKS\ units]$$

$$C_\gamma = \frac{2.901}{0.85} \cdot B_b \mathrm{lbf/in}\text{-}\mu\ \mathrm{in}[US\ units]$$

Tooth stiffness of one pair of teeth per unit facewidth (single stiffness), c' :

$$c' = \frac{14}{0.85} \cdot B_b \mathrm{N/mm}\text{-}\mu\ \mathrm{m}[SI\ units]$$

$$c' = \frac{1.428}{0.85} \cdot B_b \mathrm{kgf/mm}\text{-}\mu\ \mathrm{m}[MKS\ units]$$

$$c' = \frac{2.031}{0.85} \cdot B_b \text{lbf/in-}\mu \text{ in}[US\ units]$$

For combination of different materials for pinion and wheel, c' is to be multiplied by ξ,

where:

ξ	=	$\frac{E}{E_{st}}$
E	=	$\frac{2E_1E_2}{E_1+E_2}$ where values of E_1 and E_2 are to be obtained from 4-3-1-A1/23.29 TABLE 1
E_{st}	=	Young's modulus of steel; see 4-3-1-A1/23.29 TABLE 1

Overall facewidth, B_b :

$$B_b = \frac{b_{eH}}{b}$$

where: b_{eH} = effective facewidth, mm (in.)

Note: Higher values than B_b = 0.85 are not to be used.

Cylindrical gears:

Mesh stiffness per unit facewidth, C_γ *:*

$$C_\gamma = c' \cdot (0.75 \cdot \epsilon_a + 0.25) N/mm - \mu m (kgf/mm - \mu m, lbf/in - \mu in)$$

Tooth stiffness of one pair of teeth per unit facewidth (single stiffness), c' :

$$c' = 0.8 \cdot \frac{\cos\beta}{q'} C_{BS} \cdot C_R \text{N/mm-}\mu \text{ m}[SI\ units]$$

$$c' = 8.158 \cdot 10^{-2} \cdot \frac{\cos\beta}{q'} \cdot C_{BS} \cdot C_R \text{kgf/mm-}\mu \text{ m}[MKS\ units]$$

$$c' = 11.603 \cdot 10^{-2} \cdot \frac{\cos\beta}{q'} \cdot C_{BS} \cdot C_R \text{lbf/in-}\mu \text{ in}[US\ units]$$

For combination of different materials for pinion and wheel, c' is to be multiplied by ξ, where

ξ	=	$\frac{E}{E_{st}}$
E	=	$\frac{2 \cdot E_1 \cdot E_2}{E_1+E_2}$ where values of E_1 and E_2 are to be obtained from 4-3-1-A1/23.29 TABLE 1

E_{st} = Young's modulus of steel; see 4-3-1-A1/23.29 TABLE 1

q' = $$0.04723 + \frac{0.15551}{z_{n1}} + \frac{0.25791}{z_{n2}} - 0.00635 \cdot x_1 - \frac{0.11654 \cdot x_1}{z_{n1}} - 0.00193 \cdot x_2 - \frac{0.24188 \cdot x_2}{z_{n2}} + 0.00529 \cdot x_1^2 + 0.00182 \cdot x_2^2$$

z_{n1} = $$\frac{z_1}{\cos^2\beta_b \cdot \cos\beta}$$

z_{n2} = $$\frac{z_2}{\cos^2\beta_b \cdot \cos\beta}$$

Note: For internal gears, use $z_{n2}(= \infty)$ equal infinite and x_2 = 0.

$$C_{BS} = \left[1 + 0.5 \cdot \left(1.2 - \frac{h_{fP}}{m_n}\right)\right] \cdot [1 - 0.02(20 - a_n)]$$

When the pinion basic rack dedendum is different from that of the wheel, the arithmetic mean of C_{BS1} for a gear pair conjugate to the pinion basic rack and C_{BS2} for a gear pair conjugate to the basic rack of the wheel:

$$C_{BS} = 0.5 \cdot (C_{BS1} + C_{BS2})$$

Gear blank factor, C_R :

$$C_R = 1 + \frac{\ln(b_s/b)}{5 \cdot e^{(s_R/5 \cdot m_n)}}$$

when:

1) b_s/b < 0.2 , use 0.2 for b_s/b

2) b_s/b > 1.2 , use 1.2 for b_s/b

3) s_R/m_n < 1 , use 1.0 for s_R/m_n

For b_s and s_R see 4-3-1-A1/23.29 FIGURE 3

15.3.2 Factors B_p, B_j , and B_k

a. Values for f_{pt} and y_a according to ISO grades of accuracy Q :

Q [1]		3	4 [3]	5	6	7	8	9	10	11 [4]	12
f_{pt}	µm	3	6	12	25	45	70	100	150	201	282
	µin	118	236	472	984	1772	2756	3937	5906	7913	11102
y_a (2)	µm	0	0.5	1.5	4	7	15	25	40	55	75
	µin	0	19.7	59.1	157	276	591	984	1575	2165	2953

Notes:

1 ISO grades of accuracy according to ISO 1328. In case of mating gears with different grades of accuracy the grade corresponding to the lower accuracy is to be used.

2 For specific determination of y_a, see b) through d) below.

3 Hardened nitrided.

4 Tempered normalized.

b. Determination of y_a for structural steels, through hardened steels and nodular cast iron (perlite, bainite):

SI units	MKS units	US units
$y_a = \frac{160}{\sigma_{H\lim}} \cdot f_{pb}$	$y_a = \frac{16.315}{\sigma_{H\lim}} \cdot f_{pb}$	$y_a = \frac{2.331 \times 10^4}{\sigma_{H\lim}} \cdot f_{pb}$
For v ≤ 5 m/s : no restriction	For v ≤ 5 m/s : no restriction	For v ≤ 984 ft/min : no restriction
For 5 m/s < v ≤ 10 m/s : $y_{a\max} = \frac{12800}{\sigma_{H\lim}}$ and $f_{pb\max}$ = 80 µm	For 5 m/s < v ≤ 10 m/s : $y_{a\max} = \frac{1.305 \times 10^3}{\sigma_{H\lim}}$ and $f_{pb\max}$ = 80 µm	For 984 ft/min < v ≤ 1968 ft/min : $y_{a\max} = \frac{7.341 \times 10^7}{\sigma_{H\lim}}$ and $f_{pb\max}$ = 3150 µin
For v > 10 m/s : $y_{a\max} = \frac{6400}{\sigma_{H\lim}}$ and $f_{pb\max}$ = 40 µm	For v > 10 m/s : $y_{a\max} = \frac{652.618}{\sigma_{H\lim}}$ and $f_{pb\max}$ = 40 µm	For v > 1968 ft/min : $y_{a\max} = \frac{3.67 \times 10^7}{\sigma_{H\lim}}$ and $f_{pb\max}$ = 1575 µin

c. Determination of y_a for gray cast iron and nodular cast iron (ferritic):

SI and MKS units	US units
$y_a = 0.275 \cdot f_{pb}$ µm	$y_a = 0.275 \cdot f_{pb}$ µin
For v ≤ 5 m/s : no restriction	For v ≤ 984 ft/min : no restriction

SI and MKS units	US units
For 5 m/s < v ≤ 10 m/s: $y_{a\max}$ = 22 and $f_{pb\max}$ = 80 μm	For 984 ft/min < v ≤ 1968 ft/min : $y_{a\max}$ = 866 and $f_{pb\max}$ = 3150 μin
For v > 10 m/s : $y_{a\max}$ = 11 and $f_{pb\max}$ = 40 μm	For v > 1968 ft/min $y_{a\max}$ = 433 and $f_{pb\max}$ = 1575 μin

d. Determination of y_a for case hardened, nitrided or nitrocarburized steels:

SI and MKS units	US units
$y_a = 0.075 \cdot f_{pb}$ μm	$y_a = 0.075 \cdot f_{pb}$ μin
For all velocities but with the restriction: $y_{a\max}$ = 3 and $f_{pb\max}$ = 40 μm	For all velocities but with the restriction: $y_{a\max}$ = 118 and $f_{pb\max}$ = 1575 μin

When the material of pinion differs from that of the wheel, y_{a1} for pinion and y_{a2} for wheel are to be determined separately. The mean value:

$$y_a = 0.5(y_{a1} + y_{a2})$$

is to be used for the calculation.

For bevel gears, f_{pt} is substituted for f_{pb} when determining y_a in *b)*, *c)* and *d)*.

e. Determination of factors B_p, B_f, B_k

$$B_p = \frac{c' \cdot f_{pbeff} \cdot b}{F_t \cdot K_A \cdot K_\gamma}$$

$$B_f = \frac{c' \cdot f_{feff} \cdot b}{F_t \cdot K_A \cdot K_\gamma}$$

where:

f_{pbeff}	=	$f_{pb} - y_a$	μm (μin)
f_{feff}	=	$f_{fa} - y_f$	μm (μin)
f_{pb}	=	$f_{pt} \cdot \cos\alpha_t$	μm (μin)

y_f can be determined in the same way as y_a when the profile deviation f_{fa} is used instead of the base pitch deviation f_{pb} .

$$B_k = \left|1 - \frac{C_a}{C_{eff}}\right|$$

where:

$$C_a = \frac{1}{18} \cdot \left(\frac{\sigma_{H\lim}}{97} - 18.45\right)^2 + 1.5\mathrm{SIunits}[\mu\mathrm{m}]$$

$$C_a = \frac{1}{18} \cdot \left(\frac{\sigma_{H\lim}}{9.891} - 18.45\right)^2 + 1.5\mathrm{MKSunits}[\mu\mathrm{m}]$$

$$C_a = \frac{1}{0.457} \cdot \left(\frac{\sigma_{H\lim}}{1.407 \times 10^4} - 18.45\right)^2 + 59\mathrm{USunits}[\mu\mathrm{in}]$$

When the materials differ, C_{a1} is to be determined for the pinion material and C_{a1} for the wheel material using the following equations. The average value $C_a = \frac{C_{a1} + C_{a2}}{2}$ is used for the calculation.

$$C_{a1} = \frac{1}{18} \cdot \left(\frac{\sigma_{H\lim1}}{97} - 18.45\right)^2 + 1.5\mathrm{SIunits}[\mu\mathrm{m}]$$

$$C_{a1} = \frac{1}{18} \cdot \left(\frac{\sigma_{H\lim1}}{9.891} - 18.45\right)^2 + 1.5\mathrm{MKSunits}[\mu\mathrm{m}]$$

$$C_{a1} = \frac{1}{0.457} \cdot \left(\frac{\sigma_{H\lim1}}{1.407 \times 10^4} - 18.45\right)^2 + 59\mathrm{USunits}[\mu\mathrm{in}]$$

$$C_{a2} = \frac{1}{18} \cdot \left(\frac{\sigma_{H\lim2}}{97} - 18.45\right)^2 + 1.5\mathrm{SIunits}[\mu\mathrm{m}]$$

$$C_{a2} = \frac{1}{18} \cdot \left(\frac{\sigma_{H\lim2}}{9.891} - 18.45\right)^2 + 1.5\mathrm{MKSunits}[\mu\mathrm{m}]$$

$$C_{a2} = \frac{1}{0.457} \cdot \left(\frac{\sigma_{H\lim2}}{1.407 \times 10^4} - 18.45\right)^2 + 59\mathrm{USunits}[\mu\mathrm{in}]$$

For cylindrical gears:

$$C_{eff} = \frac{F_t \cdot K_A \cdot K_\gamma}{b \cdot c'}\ \mu m\ (\mu in)$$

For bevel gears:

$$C_{eff} = \frac{F_{mbt} \cdot K_A}{b_{eH} \cdot c'}\ \mu m\ (\mu in)$$

15.3.3 Dynamic Factor, K_v , in the Subcritical Range

Cylindrical gears: (N ≤ 0.85)

$$C_{v1} = 0.32$$

$$C_{v2} = 0.34 \text{for} \epsilon_\gamma \leq 2$$

$$C_{v2} = \frac{0.57}{\epsilon_\gamma - 0.3} \text{for} \epsilon_\gamma > 2$$

$$C_{v3} = 0.23 \text{for} \epsilon_\gamma \leq 2$$

$$C_{v3} = \frac{0.096}{\epsilon_\gamma - 1.56} \text{for} \epsilon_\gamma > 2$$

$$K = (C_{v1} \cdot B_p) + (C_{v2} \cdot B_f) + (C_{v3} \cdot B_k)$$

Bevel gears: (N ≤ 0.75)

For bevel gears, $\epsilon_{v\gamma}$ are to be substituted for ϵ_γ .

$$K = \frac{b \cdot f_{peff} \cdot c'}{F_{mt} \cdot K_A} \cdot c_{v1,2} + c_{v3}$$

$$f_{peff} = f_{pt-y_p} \text{with} y_p \approx y_a$$

$$c_{v1,2} = c_{v1} + c_{v2}$$

$$K_v = (N \cdot K) + 1$$

15.3.4 Dynamic Factor, K_v , in the Main Resonance Range

$$C_{v4} = 0.90 \text{for} \epsilon_\gamma \leq 2$$

$$C_{v4} = \frac{0.57 - 0.05 \cdot \epsilon_\gamma}{\epsilon_\gamma - 1.44} \text{for} \epsilon_\gamma > 2$$

Cylindrical gears: (0.85 < N ≤ 1.15)

$$K_{v(N=1.15)} = (C_{v1} \cdot B_p) + (C_{v2} \cdot B_f) + (C_{v4} \cdot B_k) + 1$$

Bevel gears: (0.75 < N ≤ 1.25)

For bevel gears, $\epsilon_{v\gamma}$ are to be substituted for ϵ_γ .

$$K_{v(N=1.25)} = \frac{b \cdot f_{peff} \cdot c'}{F_{mt} \cdot K_A} \cdot c_{v1,2} + c_{v4} + 1$$

For $C_{v1}, C_{v2}, C_{v1,2},$ and $f_{peff},$ see 4-3-1-A1/15.3.3 above.

15.3.5 Dynamic Factor, K_v , in the Supercritical Range *(N ≥ 1.5)*

$$C_{v5} = 0.47$$

$$C_{v6} = 0.47 \text{for} \epsilon_\gamma \leq 2$$

$$C_{v6} = \frac{0.12}{\epsilon_\gamma - 1.74} \text{for} \epsilon_\gamma > 2$$

$$C_{v7} = 0.75 \text{for} 1.0 < \epsilon_\gamma \leq 1.5$$

$$C_{v7} = 0.125 \cdot \sin[\pi \cdot (\epsilon_\gamma - 2)] + 0.875 \text{for} 1.5 < \epsilon_\gamma \leq 2.5$$

$$C_{v7} = 1.0 \text{for} \epsilon_\gamma > 2.5$$

Cylindrical gears:

$$K_{v(N=1.5)} = (C_{v5} \cdot B_p) + (C_{v6} \cdot B_f) + C_{v7}$$

Bevel gears:

For bevel gears, $\epsilon_{v\gamma}$ are to be substituted for ϵ_γ .

$$K_{v(N=1.5)} = \frac{b \cdot f_{peff} \cdot c'}{F_{mt} \cdot K_A} \cdot c_{v5,6} + c_{v7}$$

$$c_{v5,6} = c_{v5} + c_{v6}$$

For f_{peff} , see 4-3-1-A1/15.3.3 above.

15.3.6 Dynamic Factor, K_v , in the Intermediate Range

Cylindrical gears:

In this range, the dynamic factor is determined by linear interpolation between K_v at *N* = 1.15 as specified in 4-3-1-A1/15.3.4 and K_v at N = 1.5 as specified in 4-3-1-A1/15.3.5.

$$K_v = K_{v(N=1.5)} + \frac{K_{v(N=1.15)} - K_{v(N=1.5)}}{0.35} \cdot (1.5 - N)$$

Bevel gears:

In this range, the dynamic factor is determined by linear interpolation between K_v at N = 1.25 as specified in 4-3-1-A1/15.3.4 and K_v at N = 1.5 as specified in 4-3-1-A1/15.3.5.

$$K_v = K_{v(N=1.5)} + \frac{K_{v(N=1.25)} - K_{v(N=1.5)}}{0.25} \cdot (1.5 - N)$$

17 Face Load Distribution Factors, $K_{H\beta}$ and $K_{F\beta}$

The face load distribution factor, $K_{H\beta}$ for contact stress, $K_{F\beta}$ for tooth root bending stress, accounts for the effects of non-uniform distribution of load across the facewidth.

$K_{H\beta}$ is defined as follows:

$$K_{H\beta} = \frac{\mathrm{maximum load per unit facewidth}}{\mathrm{mean load per unit facewidth}}$$

$K_{F\beta}$ is defined as follows:

$$K_{F\beta} = \frac{\mathrm{maximum bending stress at tooth root per unit facewidth}}{\mathrm{mean bending stress at tooth root per unit facewidth}}$$

Note:

The mean bending stress at tooth root relates to the considered facewidth b_1 or b_2

$K_{F\beta}$ can be expressed as a function of the factor $K_{H\beta}$.

The factors $K_{H\beta}$ and $K_{F\beta}$ mainly depend on:

- Gear tooth manufacturing accuracy;
- Errors in mounting due to bore errors;
- Bearing clearances;
- Wheel and pinion shaft alignment errors;
- Elastic deflections of gear elements, shafts, bearings, housing and foundations which support the gear elements;
- Thermal expansion and distortion due to operating temperature;
- Compensating design elements (tooth crowning, end relief, etc.).

These factors can be obtained from 4-3-1-A1/17.3 and 4-3-1-A1/17.5, using the factors in 4-3-1-A1/17.1.

17.1 Factors Used for the Determination of $K_{H\beta}$

17.1.1 Helix Deviation F_β

The helix deviation, F_β , is to be determined by the designer or by the following equations and tables:

SI and MKS units, µm	US units, µin.
$F_\beta = 0.1\sqrt{d_{geom}} + 0.63\sqrt{b_{geom}} + 4.2$ for ISO Grade of accuracy Q =5	$F_\beta = 19.8\sqrt{d_{geom}} + 125.0\sqrt{b_{geom}} + 165.4$ for ISO Grade of accuracy Q =5

SI and MKS units, µm	US units, µin.
For other accuracy grades, multiply F_β by the following formula: $2^{0.5(Q-5)}$ where 0≤ Q ≤ 12	

SI and MKS units		US units	
Reference diameter d mm	Corresponding geometric mean diameter d_{geom} mm	Reference diameter d in.	Corresponding geometric mean diameter d_{geom} in.
5 ≤ *d* ≤ 20	10.00	0.2 ≤ *d* ≤ 0.79	0.3937
20 < *d* ≤ 50	31.62	0.79 ≤ *d* ≤ 2.0	1.245
50 < *d* ≤ 125	79.06	2.0 ≤ *d* ≤ 4.92	3.112
125 < *d* ≤ 280	187.1	4.92 ≤ *d* ≤ 11.0	7.365
280 < *d* ≤ 560	396.0	11.0 ≤ *d* ≤ 22.0	15.59
560 < *d* ≤ 1000	748.3	22.0 ≤ *d* ≤ 39.37	29.46
1000 < *d* ≤ 1600	1265	39.37 ≤ *d* ≤ 62.99	49.80
1600 < *d* ≤ 2500	2000	62.99 ≤ *d* ≤ 98.43	78.74
2500 < *d* ≤ 4000	3162	98.43 ≤ *d* ≤ 157.5	124.5
4000 < *d* ≤ 6000	4899	157.5 ≤ *d* ≤ 236.2	192.9
6000 < *d* ≤ 8000	6928	236.2 ≤ *d* ≤ 315.0	272.8
8000 < *d* ≤ 10000	8944	315.0 ≤ *d* ≤ 393.70	352.1

SI and MKS units		US units	
Facewidth b mm	Corresponding geometric mean facewidth b_{geom} mm	Facewidth b in.	Corresponding geometric mean facewidth b_{geom} in.
4 < *b* ≤ 10	6.325	0.16 < *b* ≤ 0.39	0.2490
10 < *b* ≤ 20	14.14	0.39 < *b* ≤ 0.79	0.5568
20 < *b* ≤ 40	28.28	0.79 < *b* ≤ 1.6	1.114
40 < *b* ≤ 80	56.57	1.6 < *b* ≤ 3.15	2.227
80 < *b* ≤ 160	113.1	3.15 < *b* ≤ 6.30	4.454
160 < *b* ≤ 250	200.0	6.30 < *b* ≤ 9.84	7.874
250 < *b* ≤ 400	316.2	9.84 < *b* ≤ 15.7	12.45
400 < *b* ≤ 650	509.9	15.7 < *b* ≤ 25.6	20.07
650 < *b* ≤ 1000	806.2	25.6 < *b* ≤ 39.37	31.74

Rounding Rules			
For resulting F_β	F_β < 5µm	5 µm ≤ F_β ≤ 10 µm	F_β > 10 µm (µin)
	Round to the nearest 0.1 µm value or integer number	Round to the nearest 0.5 µm value or integer number	Round to the nearest integer number

17.1.2 Mesh Alignment f_{ma} [μm (μin)] **(2024)**

Generally: $f_{ma} = 1.0 \cdot F_\beta$

For gear pairs with well-designed end relief: $f_{ma} = 0.7 \cdot F_\beta$

For gear pairs with provision for adjustment (lapping or running-in under light load, adjustment bearings or appropriate helix modification) and gear pairs suitably crowned: $f_{ma} = 0.5 \cdot F_\beta$

For helix deviation due to manufacturing inaccuracies: $f_{ma} = 0.5 \cdot F_\beta$

In all other cases, the following is to used: $f_{ma} = 1.0 \cdot F_\beta$

17.1.3 Initial Equivalent Misalignment $F_{\beta\chi}$ [μm (μin)]

$F_{\beta\chi}$ is the absolute value of the sum of manufacturing deviations and pinion and shaft deflections, measured in the plane of action.

$$F_{\beta\chi} = 1.33 \cdot f_{sh} + f_{ma}$$

$$f_{sh} = f_{sh0} \cdot \frac{F_t \cdot K_A \cdot K_\gamma \cdot K_v}{b}$$

$$f_{sh0min} = 0.005\mu \text{ m-mm/N}[MKS\ units]$$

$$f_{sh0min} = 0.049\mu \text{ m-mm/kgf}[MKS\ units]$$

$$f_{sh0min} = 0.03445\mu \text{ in-in/lbf}[US\ units]$$

	SI units [μm-mm/N]	**MKS units** [μm-mm/kgf]	**US units** [μin-in/lbf]
For spur and helical gears without crowning or end relief	$f_{sh0} = 0.023 \cdot \gamma$	$f_{sh0} = 0.22555 \cdot \gamma$	$f_{sh0} = 0.15858 \cdot \gamma$
For spur and helical gears without crowning but with end relief	$f_{sh0} = 0.016 \cdot \gamma$	$f_{sh0} = 0.15691 \cdot \gamma$	$f_{sh0} = 0.11032 \cdot \gamma$
For spur and helical gears with crowning	$f_{sh0} = 0.012 \cdot \gamma$	$f_{sh0} = 0.11768 \cdot \gamma$	$f_{sh0} = 0.08274 \cdot \gamma$
For spur and helical gears with crowning and end relief	$f_{sh0} = 0.010 \cdot \gamma$	$f_{sh0} = 0.09807 \cdot \gamma$	$f_{sh0} = 0.06895 \cdot \gamma$

$$\gamma = \left[\left| 1 + K' \cdot \frac{\ell \cdot s}{d_1^2} \cdot \left(\frac{d_1}{d_{sh}} \right)^4 - 0.3 \right| + 0.3 \right] \cdot \left(\frac{b}{d_1} \right)^2$$ for spur and single helical gear

$$\gamma = 2 \cdot \left[\left| 1.5 + K' \cdot \frac{\ell \cdot s}{d_1^2} \cdot \left(\frac{d_1}{d_{sh}} \right)^4 - 0.3 \right| + 0.3 \right] \cdot \left(\frac{b_B}{d_1} \right)^2$$ for double helical gears

where $b_B = b/2$ is the width of one helix.

The constant K' makes allowances for the position of the pinion in relation to the torqued end. It can be taken from 4-3-1-A1/23.29 TABLE 6.

17.1.4 Determination of y_β and χ_β for Structural Steels,Through Hardened Steels and Nodular Cast Iron(Perlite, Bainite)

SI units	MKS units	US units
$y_\beta = \frac{320}{\sigma_{H\lim}} \cdot F_{\beta\chi} \mu\text{m}$	$y_\beta = \frac{32.63}{\sigma_{H\lim}} \cdot F_{\beta\chi} \mu\text{m}$	$y_\beta = \frac{4.662 \cdot 10^4}{\sigma_{H\lim}} \cdot F_{\beta\chi} \mu\text{in}$
$\chi_\beta = 1 - \frac{320}{\sigma_{H\lim}}$ with $y_\beta \le F_{\beta\chi}; \chi_\beta \ge 0$	$\chi_\beta = 1 - \frac{32.63}{\sigma_{H\lim}}$ with $y_\beta \le F_{\beta\chi}; \chi_\beta \ge 0$	$\chi_\beta = 1 - \frac{4.662 \cdot 10^4}{\sigma_{H\lim}}$ with $y_\beta \le F_{\beta\chi}; \chi_\beta \ge 0$
For v ≤ 5 m/s no restriction	For v ≤ 5 m/s no restriction	For v ≤ 984 ft/min no restriction
For 5 m/s < v ≤ 10 m/s : $y_{\beta\max} = \frac{25600}{\sigma_{H\lim}}$ corresponding to $F_{\beta\chi}$ = 80 µm	For 5 m/s < v ≤ 10 m/s : $y_{\beta\max} = \frac{2.61 \cdot 10^3}{\sigma_{H\lim}}$ corresponding to $F_{\beta\chi}$ = 80 µm	For 984 ft/min < v ≤ 1968 ft/min : $y_{\beta\max} = \frac{14.682 \cdot 10^7}{\sigma_{H\lim}}$ corresponding to $F_{\beta\chi}$ = 3150 µin
For v > 10 m/s : $y_{\beta\max} = \frac{12800}{\sigma_{H\lim}}$ corresponding to $F_{\beta\chi}$ = 40 µm	For v > 10 m/s : $y_{\beta\max} = \frac{1.305 \cdot 10^3}{\sigma_{H\lim}}$ corresponding to $F_{\beta\chi}$ = 40 µm	For v > 1968 ft/min : $y_{\beta\max} = \frac{7.341 \cdot 10^7}{\sigma_{H\lim}}$ corresponding to $F_{\beta\chi}$ = 1575 µin

For $\sigma_{H\lim}$, see 4-3-1-A1/23.29 TABLE 3 below.

17.1.5 Determination of y_β and χ_β for Gray Cast Iron and Nodular Cast Iron (Ferritic):

SI & MKS units, µm	US units, µin
$Y_\beta = 0.55 \cdot F_{\beta\chi}$	$Y_\beta = 0.55 \cdot F_{\beta\chi}$
χ_β = 0.45	χ_β = 0.45
For v ≤ 5 m/s, no restriction	For v ≤ 984 ft/min, no restriction
For 5 m/s < v ≤ 10 m/s: $y_{\beta\max}$ = 45 corresponding to $F_{\beta\chi}$ = 80 µm	For 984 ft/min < v ≤ 1968 ft/min: $y_{\beta\max}$ = 1771 corresponding to $F_{\beta\chi}$ = 3150 µin
For v > 10 m/s $y_{\beta\max}$ = 22 corresponding to $F_{\beta\chi}$ = 40 µm	For v > 1968 ft/min $y_{\beta\max}$ = 866 corresponding to $F_{\beta\chi}$ = 1575 µin

17.1.6 Determination of y_β and χ_β for Case Hardened, Nitrided or Nitrocarburized Steels :

SI & MKS units	US units
$Y_\beta = 0.15 \cdot F_{\beta\chi}$ µm	$Y_\beta = 0.15 \cdot F_{\beta\chi}$ µin
χ_β = 0.85	χ_β = 0.85
For all velocities but with the restriction: $y_{\beta\max}$ = 6 corresponding to $F_{\beta\chi}$ = 40 µm	For all velocities but with the restriction: $y_{\beta\max}$ = 236 corresponding to $F_{\beta\chi}$ = 1575 µin.

When the material of the pinion differs from that of the wheel, $y_{\beta 1}$ and $\chi_{\beta 1}$ for pinion, and $y_{\beta 2}$ and $\chi_{\beta 2}$ for wheel are to be determined separately. The mean of either value:

$$y_\beta = 0.5 \cdot (y_{\beta 1} + y_{\beta 2})$$

$$\chi_\beta = 0.5 \cdot (\chi_{\beta 1} + \chi_{\beta 2})$$

is to be used for the calculation.

17.3 Face Load Distribution Factor for Contact Stress $K_{H\beta}$

17.3.1 $K_{H\beta}$ for Helical and Spur Gears

$$K_{H\beta} = 1 + \frac{b \cdot F_{\beta y} \cdot C_\gamma}{2 \cdot F_t \cdot K_A \cdot K_\gamma \cdot K_v} < 2, \text{for } \frac{b \cdot F_{\beta y} \cdot C_\gamma}{2 \cdot F_t \cdot K_A \cdot K_\gamma \cdot K_v} < 1$$

$$K_{H\beta} = \sqrt{\frac{2 \cdot b \cdot F_{\beta y} \cdot C_\gamma}{F_t \cdot K_A \cdot K_\gamma \cdot K_v}} \geq 2 \text{for } \frac{b \cdot F_{\beta y} \cdot C_\gamma}{2 \cdot F_t \cdot K_A \cdot K_\gamma \cdot K_v} \geq 1$$

where:

$F_{\beta y} = F_{\beta\chi} - y_\beta$ or $F_{\beta y} = F_{\beta\chi} \cdot \chi_\beta$

Calculated values of $K_{H\beta}$ ≥ 2 are to be reduced by improvement accuracy and helix deviation.

17.3.2 $K_{H\beta}$ for Bevel Gears

$$K_{H\beta} = 1.5 \cdot \frac{0.85}{B_b} \cdot K_{H\beta be}$$

The bearing factor, $K_{H\beta be}$, represents the influence of the bearing arrangement on the faceload distribution, is given in the following table:

Mounting conditions of pinion and wheel		
Both members straddle mounted	One member straddle mounted	Neither member straddle mounted
1.10	1.25	1.50
Based on optimum tooth contact pattern under maximum operating load as evidence by results of a deflection test on the gears in their mountings.		

17.5 Face Load Distribution Factor for Tooth Root Bending Stress $K_{F\beta}$

17.5.1 In Case the Hardest Contact is at the End of the Facewidth $K_{F\beta}$ is Given by the Following Equations

$$K_{F\beta} = (K_{H\beta})^N$$

$$N = \frac{(b/h)^2}{1 + (b/h) + (b/h)^2} = \frac{1}{1 + (h/b) + (h/b)^2}$$

(b/h) = (facewidth/tooth depth), the lesser of b_1/h_1 or b_2/h_2 . For double helical gears, the facewidth of only one helix is to be used, i.e. $b_B = b/2$ is to be substituted for b in the equation for N .

17.5.2 In Case of Gears Where the Ends of the Facewidth are Lightly Loaded or Unloaded (End Relief or Crowning)

$$K_{F\beta} = K_{H\beta}$$

17.5.3 Bevel Gears:

$$K_{F\beta} = \frac{K_{H\beta}}{K_{FO}}$$

$$K_{FO} = 0.211 \cdot \left(\frac{r_{eo}}{R_m}\right)^q + 0.789 \mathrm{for spiral bevel gears.}$$

$$q = \frac{0.279}{\log\left(\sin\beta_m\right)}$$

where:

K_{FO}	=	1 for straight or zero bevel gears.
r_{eo}	=	cutter radius, mm (in.)
R_m	=	mean cone distance, mm (in.)

Limitations of K_{FO} :

If K_{FO} < unity, use K_{FO} = unity

If K_{FO} > 1.15, use K_{FO} = 1.15

19 Transverse Load Distribution Factors, $K_{H\alpha}$ and $K_{F\alpha}$

The transverse load distribution factors, $K_{H\alpha}$ for contact stress and $K_{F\alpha}$ for tooth root bending stress, account for the effects of pitch and profile errors on the transversal load distribution between two or more pairs of teeth in mesh.

The factors $K_{H\alpha}$ and $K_{F\alpha}$ mainly depend on:

- Total mesh stiffness
- Total tangential load $F_t, K_A, K_\gamma, K_v, K_{H\beta}$
- Base pitch error
- Tip relief
- Running-in allowances

The load distribution factors, $K_{H\alpha}$ and $K_{F\alpha}$ are to be advised by the manufacturer as supported by his measurements, analysis or experience data or are to be determined as follows.

19.1 Determination of $K_{H\alpha}$ for Contact Stress $K_{F\alpha}$ for Tooth Root Bending Stress

$$K_{H\alpha} = K_{F\alpha} = 0.9 + 0.4 \cdot \sqrt{\frac{2 \cdot (\epsilon_\gamma - 1)}{\epsilon_\gamma} \cdot \frac{C_\gamma \cdot (f_{pbe} - y_a) \cdot b}{F_{tH}}} \mathrm{for}\, \epsilon_\gamma > 2$$

$$K_{H\alpha} = K_{F\alpha} = \frac{\epsilon_\gamma}{2} \cdot \left[0.9 + 0.4 \cdot \frac{C_\gamma \cdot (f_{pbe} - y_a) \cdot b}{F_{tH}}\right] \text{ for} \epsilon_\gamma \leq 2$$

Cylindrical gears:

$$F_{tH} = F_t \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{H\beta} \mathrm{N(kgf, lbf)}$$

$$f_{pbe} = f_{pb} \cdot \cos\alpha_t$$

Bevel Gears:

$$F_{mtH} = F_{mt} \cdot K_A \cdot K_v \cdot K_{H\beta} \mathrm{N(kgf, lbf)}$$

For bevel gears, $f_{pt}, \epsilon_{v\gamma}, F_{mtH}, F_{mt}$ and α_{vt} (equivalent) are to be substituted for $f_{pbe}, \epsilon_\gamma, F_{tH}, F_t$ and α_t in the above formulas.

19.3 Limitations of $K_{H\alpha}$ and $K_{F\alpha}$

19.3.1 $K_{H\alpha}$

When $K_{H\alpha}$ < 1 use 1.0 for $K_{H\alpha}$

Cylindrical gears:

When $K_{H\alpha} > \frac{\epsilon_\gamma}{\epsilon_\alpha \cdot Z_\epsilon^2}$, use $\frac{\epsilon_\gamma}{\epsilon_\alpha \cdot Z_\epsilon^2}$ for $K_{H\alpha}$;

$Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3} \cdot (1 - \epsilon_\beta) + \frac{\epsilon_\beta}{\epsilon_\alpha}}$, contact ratio factor (pitting) for helical gears for ϵ_β < 1

$Z_\epsilon = \sqrt{\frac{1}{\epsilon_\alpha}}$, contact ratio factor (pitting) for helical gears for ϵ_β ≥ 1

$Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3}}$, contact ratio factor (pitting) for spur gears

Bevel gears:

When $K_{H\alpha} > \frac{\epsilon_{v\gamma}}{\epsilon_{v\alpha} \cdot Z_{LS}^2}$, use $\frac{\epsilon_{v\gamma}}{\epsilon_{v\alpha} \cdot Z_{LS}^2}$ for $K_{H\alpha}$;

For the calculation of Z_{LS} , see 4-3-1-A1/21.13.

19.3.2 $K_{F\alpha}$

When $K_{F\alpha}$ < 1, use 1.0 for $K_{F\alpha}$.

When $K_{F\alpha} > \frac{\epsilon_\gamma}{\epsilon_\alpha \cdot Y_\epsilon}$, use $\frac{\epsilon_\gamma}{\epsilon_\alpha \cdot Y_\epsilon}$ for $K_{F\alpha}$;

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_\alpha}$$, contact ratio factor for ϵ_β = 0

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_\alpha} - \left(\frac{0.75}{\epsilon_\alpha} - 0.375\right) \cdot \epsilon_\beta$$, contact ratio factor for 0 < ϵ_β < 1

$$Y_\epsilon = 0.625$$, contact ratio factor for ϵ_β ≥ 1

or:

$$Y_\epsilon = 0.25 + \frac{0.75 \cdot \cos^2 \beta_b}{\epsilon_\alpha}$$, for cylinderical gears only

For bevel gears, $\epsilon_{v\gamma}, \epsilon_{v\beta},$ and $\epsilon_{v\alpha}$, (equivalent) are to be substituted for $\epsilon_\gamma, \epsilon_\beta,$ and ϵ_α in the above formulas.

21 Surface Durability

The criterion for surface durability is based on the Hertzian pressure on the operating pitch point or at the inner point of single pair contact. The contact stress σ_H is not to exceed the permissible contact stress σ_{HP} .

21.1 Contact Stress

$$\sigma_{H1} = \sigma_{HO1} \cdot \sqrt{K_A \cdot K_\gamma \cdot K_v \cdot K_{H\alpha} \cdot K_{H\beta}} \leq \sigma_{HP1}$$

$$\sigma_{H2} = \sigma_{HO2} \cdot \sqrt{K_A \cdot K_\gamma \cdot K_v \cdot K_{H\alpha} \cdot K_{H\beta}} \leq \sigma_{HP2}$$

Cylindrical gears:

$\sigma_{HO1,2}$ = basic value of contact stress for pinion and wheel

$$\sigma_{HO1} = Z_B \cdot Z_H \cdot Z_E \cdot Z_\epsilon \cdot Z_\beta \cdot \sqrt{\frac{F_t}{d_1 \cdot b} \cdot \frac{u+1}{u}}$$ for pinion

$$\sigma_{HO2} = Z_D \cdot Z_H \cdot Z_E \cdot Z_\epsilon \cdot Z_\beta \cdot \sqrt{\frac{F_t}{d_1 \cdot b} \cdot \frac{u+1}{u}}$$ for wheel

where

Z_B	=	single pair mesh factor for pinion, see 4-3-1-A1/21.5 below
Z_D	=	single pair mesh factor for wheel, see 4-3-1-A1/21.5 below
Z_H	=	zone factor, see 4-3-1-A1/21.7 below
Z_E	=	elasticity factor, see 4-3-1-A1/21.9 below
Z_ϵ	=	contact ratio factor (pitting), see 4-3-1-A1/21.11 below
Z_β	=	helix angle factor see 4-3-1-A1/21.17 below
F_t	=	nominal transverse tangential load, see 4-3-1-A1/9 of this Appendix.

Gear ratio u for external gears is positive, for internal gears u is negative

Regarding factors $K_A, K_\gamma, K_v, K_{H\alpha}$ and $K_{H\beta}$ see 4-3-1-A1/11, 4-3-1-A1/13, 4-3-1-A1/15, 4-3-1-A1/19 and 4-3-1-A1/17 of this Appendix.

Bevel gears:

σ_{HO1} = basic value of contact stress for pinion

$$\sigma_{HO1} = Z_{M-B} \cdot Z_H \cdot Z_E \cdot Z_{LS} \cdot Z_\beta \cdot Z_K \cdot \sqrt{\frac{F_{mt}}{d_{v1} \cdot \ell_{bm}} \cdot \frac{u_v + 1}{u_v}}$$

For the shaft angle $\Sigma = \delta_1 + \delta_2$ = 90° the following applies:

$$\sigma_{HO1} = Z_{M-B} \cdot Z_H \cdot Z_E \cdot Z_{LS} \cdot Z_\beta \cdot Z_K \cdot \sqrt{\frac{F_{mt}}{d_{m1} \cdot \ell_{bm}} \cdot \frac{\sqrt{u^2 + 1}}{u}}$$

where

Z_{M-B}	=	mid-zone factor, see 4-3-1-A1/21.5 below
Z_H	=	zone factor, see 4-3-1-A1/21.7 below
Z_E	=	elasticity factor, see 4-3-1-A1/21.9 below
Z_{LS}	=	load sharing factor, see 4-3-1-A1/21.13 below
Z_β	=	helix angle factor, see 4-3-1-A1/21.17 below
Z_K	=	bevel gear factor (flank), see 4-3-1-A1/21.15 below
F_{mt}	=	nominal transverse tangential load, see 4-3-1-A1/9 of this appendix.
d_{m1}	=	mean pitch diameter of pinion of bevel gear
d_{v1}	=	reference diameter of pinion of virtual (equivalent) cylindrical gear
ℓ_{bm}	=	length of middle line of contact
u_v	=	gear ratio of virtual (equivalent) cylindrical gear
u	=	gear ratio of bevel gear

21.3 Permissible Contact Stress

The permissible contact stress σ_{HP} is to be evaluated separately for pinion and wheel.

$$\sigma_{HP} = \frac{\sigma_{H\lim}}{S_H} \cdot Z_N \cdot Z_L \cdot Z_V \cdot Z_R \cdot Z_W \cdot Z_X \mathrm{N/mm^2} (kgf/mm^2,\ psi)$$

where:

$\sigma_{H\lim}$	=	endurance limit for contact stress, see 4-3-1-A1/21.19 below
Z_N	=	life factor for contact stress, see 4-3-1-A1/21.21 below
Z_L	=	lubrication factor, see 4-3-1-A1/21.23 below
Z_V	=	speed factor, see 4-3-1-A1/21.23 below
Z_R	=	roughness factor, see 4-3-1-A1/21.23 below
Z_W	=	hardness ratio factor, see 4-3-1-A1/21.25 below
Z_X	=	size factor for contact stress, see 4-3-1-A1/21.27 below
S_H	=	safety factor for contact stress, see 4-3-1-A1/21.29 below

For shrink-fitted wheel rims σ_{HP} is to be at least K_S times the mean contact stress σ_H , where

K_S = safety factor available for induced contact stresses, and is to be calculated as follows:

$$1 + \frac{\delta_{\max} \cdot 2.2 \times 10^5}{Y} \cdot \frac{d_{ri}}{d_{w2}^2} \cdot \frac{0.25 m_n}{\rho_{F2}}$$ [SI units]

$$1 + \frac{\delta_{\max} \cdot 2.243 \times 10^4}{Y} \cdot \frac{d_{ri}}{d_{w2}^2} \cdot \frac{0.25 m_n}{\rho_{F2}}$$ [MKS units]

K_S =

$$1 + \frac{\delta_{\max} \cdot 3.194 \times 10^7}{Y} \cdot \frac{d_{ri}}{d_{w2}^2} \cdot \frac{0.25 m_n}{\rho_{F2}}$$ [US units]

where

$\delta_{\max}$	=	maximum available interference fit or maximum pull-up length; mm (mm, in.)
d_{ri}	=	inner diameter of wheel rim; mm (in.)
d_{w2}	=	working pitch diameter of wheel; mm (in.)
m_n	=	normal module; mm (in.)
Y	=	yield strength of wheel rim material is to be as follows:

- minimum specified yield strength for through hardened (quenched and tempered) steels
- 500 N/mm^2 (51 kgf/mm^2, 72520 psi) for case hardened, nitrided steels

21.5 Single Pair Mesh Factors, Z_B , Z_D and Mid-zone Factor Z_{M-B}

The single pair mesh factors, Z_B for pinion and Z_D for wheel, account for the influence on contact stresses of the tooth flank curvature at the inner point of single pair contact in relation to Z_H .

The factors transform the contact stresses determined at the pitch point to contact stresses considering the flank curvature at the inner point of single pair contact.

21.5.1 For Cylindrical and Bevel Gears when $\epsilon_\beta = 0$

21.5.1(a) *Cylindrical Gears*

$Z_B = M_1$ or 1 whichever is the larger value

$Z_D = M_2$ or 1 whichever is the larger value

$$M_1 = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{(d_{a1}/d_{b1})^2 - 1} - (2\pi/z_1)\right] \cdot \left[\sqrt{(d_{a2}/d_{b2})^2 - 1} - (\epsilon_\alpha - 1) \cdot (2\pi/z_2)\right]}}$$

$$M_2 = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{(d_{a2}/d_{b2})^2 - 1} - (2\pi/z_2)\right] \cdot \left[\sqrt{(d_{a1}/d_{b1})^2 - 1} - (\epsilon_\alpha - 1) \cdot (2\pi/z_1)\right]}}$$

21.5.1(b) *Bevel Gears*

$$Z_{M-B} = M$$

$$M = \frac{\tan\alpha_{vt}}{\sqrt{\left[\sqrt{(d_{va1}/d_{vb1})^2 - 1} - (2\pi/z_{v1})\right] \cdot \left[\sqrt{(d_{va2}/d_{vb2})^2 - 1} - (\epsilon_{v\alpha} - 1) \cdot (2\pi/z_{v2})\right]}}$$

21.5.2 For Cylindrical and Bevel Gears when $\epsilon_\beta \geq 1$

21.5.2(a) *Cylindrical Gears*

Z_B = Z_D = 1 for cylindrical gears

21.5.2(b) *Bevel Gears*

$$Z_{M-B} = M$$

$$M = \frac{\tan\alpha_{vt}}{\sqrt{\left[\sqrt{(d_{va1}/d_{vb1})^2 - 1} - \epsilon_{v\alpha} \cdot (\pi/z_{v1})\right] \cdot \left[\sqrt{(d_{va2}/d_{vb2})^2 - 1} - \epsilon_{v\alpha} \cdot (\pi/z_{v2})\right]}}$$

21.5.3 For Cylindrical and Bevel Gears when $0 < \epsilon_\beta < 1$

21.5.3(a) *Cylindrical gears*

The values of Z_B , Z_A are determined by linear interpolation between Z_B , Z_A for spur gears and Z_B , Z_A for helical gears having $\epsilon_\beta \geq 1$

Thus:

$Z_B = M_1 - \epsilon_\beta \cdot (M_1 - 1)$ and $Z_B \geq 1$

$Z_D = M_2 - \epsilon_\beta \cdot (M_2 - 1)$ and $Z_D \geq 1$

21.5.3(b) *Bevel gears*

$$Z_{M-B} = M$$

$$M = \frac{\tan\alpha_{vt}}{\sqrt{\left[\sqrt{(d_{va1}/d_{vb1})^2 - 1} - (2 + (\epsilon_{v\alpha} - 2) \cdot \epsilon_{v\beta}) \cdot (\pi/z_{v1})\right] \cdot \left[\sqrt{(d_{va2}/d_{vb2})^2 - 1} - (2\,(\epsilon_{v\alpha} - 1) + (2 - \epsilon_{v\alpha}) \cdot \epsilon_{v\beta}) \cdot (\pi/z_{v2})\right]}}$$

21.7 Zone Factor, Z_H

Cylindrical gears:

The zone factor, Z_H , accounts for the influence on the Hertzian pressure of tooth flank curvature at pitch point and relates the tangential force at the reference cylinder to the normal force at the pitch cylinder.

$$Z_H = \sqrt{\frac{2 \cdot \cos\beta_b \cdot \cos\alpha_{wt}}{\cos^2\alpha_t \cdot \sin\alpha_{wt}}}$$

Bevel gears:

$$Z_H = 2 \cdot \sqrt{\frac{\cos\beta_{vb}}{\sin\,(2 \cdot \alpha_{vt})}}$$

21.9 Elasticity Factor, Z_E

The elasticity factor, Z_E , accounts for the influence of the material properties E (modulus of elasticity) and ν (Poisson's ratio) on the Hertzian pressure.

$$Z_E = \sqrt{\frac{1}{\pi \cdot \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right]}}$$

With Poisson's ratio of 0.3 and same E and ν for pinion and wheel, Z_E may be obtained from the following:

$$Z_E = \sqrt{0.175 \cdot E}$$

with E = Young's modulus of elasticity.

The elasticity factor, Z_E, for steel gears E_{st} = 206000 N / mm^2 (2.101 x 10^4 kgf/mm^2, 2.988 x 10^7 psi) is:

Elasticity factor Z_E		
SI units	MKS units	US units
189.8 N$^{1/2}$ /mm	60.61 kgf$^{1/2}$ /mm	2.286×10^3 lbf $^{1/2}$ /in

For other material combinations refer to 4-3-1-A1/23.29 TABLE 1.

21.11 Contact Ratio Factor (Pitting), Z_ϵ

The contact ratio factor, Z_ϵ, accounts for the influence of the transverse contact ratio and the overlap ratio on the specific surface load of gears.

Spur gears:

$$Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3}}$$

Helical gears:

$$\text{For}\, \epsilon_\beta < 1 : Z_\epsilon = \sqrt{\frac{4 - \epsilon_\alpha}{3} \cdot (1 - \epsilon_\beta) + \frac{\epsilon_\beta}{\epsilon_\alpha}}$$

$$\text{For}\, \epsilon_\beta \geq 1 : Z_\epsilon = \sqrt{\frac{1}{\epsilon_\alpha}}$$

21.13 Bevel Gear Load Sharing Factor, Z_{LS}

The load sharing factor, Z_{LS}, accounts for the load sharing between two or more pair of teeth in contact.

For $\epsilon_{v\gamma} \leq 2$:	Z_{LS} = 1
For $\epsilon_{v\gamma} > 2$ and $\epsilon_{v\beta} > 1$:	$Z_{LS} = \left[1 + 2 \cdot 1 - (2/\epsilon_{v\gamma})^{1.5} \cdot \sqrt{1 - (4/\epsilon_{v\gamma}^2)}\right]^{-0.5}$
For other cases:	The calculation of Z_{LS} can be based upon the method used in ISO 10300-2, Annex A, Load Sharing Factor, Z_{LS} .

21.15 Bevel Gear Factor (Flank), Z_K

The bevel gear factor (flank), Z_K , accounts the difference between bevel and cylindrical loading and adjusts the contact stresses so that the same permissible stresses apply.

Z_K = 0.8

21.17 Helix Angle Factor, Z_β

The helix angle factor; Z_β , accounts for the influence of helix angle on surface durability, allowing for such variables as the distribution of load along the lines of contact. Z_β is dependent only on the helix angle.

Cylindrical gears:

$$Z_\beta = \frac{1}{\sqrt{\cos\beta}}$$

Bevel gears:

$$Z_\beta = \sqrt{\cos\beta_m}$$

21.19 Allowable Stress Number (Contact), $\sigma_{H\lim}$

For a given material, $\sigma_{H\lim}$ is the limit of repeated contact stress that can be sustained without progressive pitting. For most materials, their load cycles may be taken at $50x10^6$, unless otherwise specified.

For this purpose, pitting is defined as follows:

- Not surface hardened gears: pitted area > 2% of total active flank area.
- Surface hardened gears: pitted area > 0.5% of total active flank area, or > 4% of one particular tooth flank area.

The endurance limit mainly depends on:

- Material composition, cleanliness and defects;
- Mechanical properties;
- Residual stresses;
- Hardening process, depth of hardened zone, hardness gradient;
- Material structure (forged, rolled bar, cast).

The $\sigma_{H\lim}$ values correspond to a failure probability of 1% or less. The values of $\sigma_{H\lim}$ are to be determined from 4-3-1-A1/23.29 TABLE 3 or to be advised by the manufacturer together with technical justification for the proposed values.

21.21 Life Factor, Z_N

The life factor, Z_N , accounts for the higher permissible contact stress, including static stress in case a limited life (number of load cycles) is specified.

The factor mainly depends on:

- Material and hardening;
- Number of cycles;

- Influence factors (Z_R, Z_V, Z_L, Z_W, Z_X).

The life factor, Z_N , can be determined from 4-3-1-A1/23.29 TABLE 4**.**

21.23 Influence Factors on Lubrication Film, Z_L , Z_V and Z_R

The lubricant factor, Z_L , accounts for the influence of the type of lubricant and its viscosity.

The speed factor, Z_V , accounts for the influence of the pitch line velocity.

The roughness factor, Z_R , accounts for the influence of the surface roughness on the surface endurance capacity.

The factors mainly depend on:

- Viscosity of lubricant in the contact zone;
- The sum of the instantaneous velocities of the tooth surfaces;
- Load;
- Relative radius of curvature at the pitch point;
- Surface roughness of teeth flanks;
- Hardness of pinion and gear.

Where gear pairs are of different hardness, the factors are to be based on the less hardened material.

21.23.1 Lubricant Factor, Z_L

$$Z_L = C_{ZL} + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (134/v_{40})]^2} \ [\mathrm{SI and MKS units}]$$

$$Z_L = C_{ZL} + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (0.208/v_{40})]^2} \ [\mathrm{US units}]$$

or

$$Z_L = C_{ZL} + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (80/v_{50})]^2} \ [\mathrm{SI and MKS units}]$$

$$Z_L = C_{ZL} + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (0.127/v_{50})]^2} \ [\mathrm{US units}]$$

where $\sigma_{H\lim}$ is the allowable stress number (contact) of the softer material.

a. with $\sigma_{H\lim}$ in the range of:

850 N/mm^2 (87 kgf/mm^2, 1.23×10^5 psi) < $\sigma_{H\lim}$ < 1200 N/mm^2 (122 kgf/mm^2, 1.74×10^5 psi)

$$C_{ZL} = \left(0.08 \cdot \frac{\sigma_{H\lim} - 850}{350}\right) + 0.83[SI\ units]$$

$$C_{ZL} = \left(0.08 \cdot \frac{\sigma_{H\lim} - 87}{35.7}\right) + 0.83\ [\text{MKSunits}]$$

$$C_{ZL} = \left(0.08 \cdot \frac{\sigma_{H\lim} - 1.23 \cdot 10^5}{5.076 \times 10^4}\right) + 0.83\ [\text{USunits}]$$

b. with $\sigma_{H\lim}$ in the range of $\sigma_{H\lim}$ < 850 N/mm^2 (87 kgf/mm^2 , 1.23×10^5 psi), C_{ZL} = 0.83;

c. with $\sigma_{H\lim}$ in the range of $\sigma_{H\lim}$ > 1200 N/mm^2 (122 kgf/mm^2, 1.74×10^5 psi), C_{ZL} = 0.91.

and

ν_{40} = nominal kinematic viscosity of the oil at 40°C (104°F), mm^2 /s; see table below.

ν_{50} = nominal kinematic viscosity of the oil at 50°C (122°F), mm^2 /s; see table below.

ISO lubricant viscosity grade		VG 32(1)	VG 46(1)	VG 68(1)	VG 100	VG 150	VG 220	VG 320
SI/MKS units	average viscosity ν_{40} mm^2/s	32	46	68	100	150	220	320
	average viscosity ν_{50} mm^2/s	21	30	43	61	89	125	180
US units	average viscosity ν_{40} in^2/s	0.0496	0.0713	0.1054	0.1550	0.2325	0.3410	0.4960
	average viscosity ν_{50} in^2/s	0.0326	0.0465	0.0667	0.0945	0.1380	0.1938	0.2790
1) Only for high speed (> 1400 rpm) transmission.								

21.23.2 Speed Factor, Z_V

$$Z_V = C_{Zv} + \frac{2\,(1.0 - C_{Zv})}{\sqrt{0.8 + (32/v)}}\ [SI\ and\ MKS\ units]$$

$$Z_V = C_{Zv} + \frac{2\,(1.0 - C_{Zv})}{\sqrt{0.8 + (6.4 \cdot 10^3/v)}}\ [US\ units]$$

where $\sigma_{H\lim}$ is the allowable stress number (contact) of the softer material.

For bevel gears, v is to be substituted by v_{mt} in the above formula.

a. with $\sigma_{H\lim}$ in the range of:

850 N/mm^2 (87 kgf/mm^2 , 1.23×10^5 psi) < $\sigma_{H\lim}$ < 1200 N/mm^2 (122 kgf/mm^2, 1.74×10^5 psi)

$$C_{Zv} = \left(0.08\ \frac{\sigma_{H\lim} - 850}{350}\right) + 0.85[SI\ units]$$

$$C_{Zv} = \left(0.08\,\frac{\sigma_{H\lim} - 87}{35.7}\right) + 0.85[MKS\ units]$$

$$C_{Zv} = \left(0.08\,\frac{\sigma_{H\lim} - 1.23 \times 10^5}{5.076 \times 10^4}\right) + 0.85[US\ units]$$

b. with $\sigma_{H\lim}$ < 850 N/mm^2 (87 kgf/mm^2 , 1.23×10^5 psi), C_{Zv} = 0.85.

c. with $\sigma_{H\lim}$ > 1200 N/mm^2 (122 kgf/mm^2, 1.74×10^5 psi), C_{Zv} = 0.93.

21.23.3 Roughness Factor, Z_R

$$Z_R = \left(\frac{3}{R_{Z10}}\right)^{C_{ZR}}$$

The peak-to-valley roughness, R_Z , is to be advised by the manufacturer or to be determined as a mean value of R_Z measured on several tooth flanks of the pinion and the gear, as given by the following expression:

$$R_Z = \frac{R_{Zf1} + R_{Zf2}}{2}$$

Where roughness values are not available, roughness of the pinion R_{Zf1} = 6.3 μm (248 μin) and of the wheel R_{Zf2} = 6.3 μm (248 μin) may be used.

R_{Z10} is to be given by:

$$R_{Z10} = R_Z \sqrt[3]{\frac{10}{\rho_{red}}}[SI\ and\ MKS\ units]$$

$$R_{Z10} = R_Z \sqrt[3]{\frac{6.4516 \cdot 10^{-6}}{\rho_{red}}}[US\ units]$$

and the relative radius of curvature is to be given by:

$$\rho_{red} = \frac{\rho_1 \cdot \rho_2}{\rho_1 + \rho_2}$$ for cylindrical gears

$$\rho_{red} = \frac{\rho_{v1} \cdot \rho_{v2}}{\rho_{v1} + \rho_{v2}}$$ for bevel gears

$$\rho_1 = 0.5 \cdot d_{b1} \cdot \tan\alpha_{tw}$$

$$\rho_{v1} = 0.5 \cdot d_{vb1} \cdot \tan\alpha_{tw}$$

$$\rho_2 = 0.5 \cdot d_{b2} \cdot \tan\alpha_{tw}$$

$$\rho_{v2} = 0.5 \cdot d_{vb2} \cdot \tan\alpha_{tw}$$

If the stated roughness is an R_a value, also known as arithmetic average (*AA*) and centerline average (*CLA*), the following approximate relationship may be applied:

$$R_a = CLA = AA = R_{Zf}/6$$

Where R_{Zf} is either R_{Zf1} for pinion or R_{Zf2} for gear and $\sigma_{H\lim}$ is the allowable stress number (contact) of the softer material.

In the range of 850 N/mm^2 ≤ $\sigma_{H\lim}$ ≤ 1200 N/mm^2 (87 kgf/mm^2 ≤ $\sigma_{H\lim}$ ≤ 122 kgf/mm^2 ; 1.23×10^5 psi ≤ $\sigma_{H\lim}$ ≤ 1.74×10^5 psi) , C_{ZR} can be calculated as follows:

$$C_{ZR} = 0.32–2.00 \times 10^{-4} \cdot \sigma_{H\lim}[\text{SIunits}]$$

$$C_{ZR} = 0.32–1.96 \times 10^{-3} \cdot \sigma_{H\lim}[\text{MKSunits}]$$

$$C_{ZR} = 0.32–1.38 \times 10^{-6} \cdot \sigma_{H\lim}[\text{USunits}]$$

If $\sigma_{H\lim}$ < 850 N/mm^2 (87 kgf/mm^2, 1.23×10^5 psi), take C_{ZR} = 0.150

If $\sigma_{H\lim}$ > 1200 N/mm^2 (122 kgf/mm^2 , 1.74×10^5 psi), take C_{ZR} = 0.080

21.25 Hardness Ratio Factor; Z_W

The hardness ratio factor, Z_W , accounts for the increase of surface durability of a soft steel gear when meshing with a surface hardened gear with a smooth surface.

The hardness ratio factor, Z_W , applies to the soft gear only and depends mainly on:

- Hardness of the soft gear;
- Alloying elements of the soft gear;
- Tooth flank roughness of the harder gear.

$$Z_W = 1.2 - \frac{HB - 130}{1700}$$

where:

HB	=	Brinell hardness of the softer material
$HV10$	=	Vickers hardness with F = 98.1 N

For unalloyed steels

$$HB \approx HV10 \approx U/3.6$$ [SI units]

$$HB \approx HV10 \approx U/0.367$$ [MKS units]

$$HB \approx HV10 \approx U/522$$ [US units]

For alloyed steels

$$HB \approx HV10 \approx U/3.4$$ [SI units]

$$HB \approx HV10 \approx U/0.347$$ [MKS units]

$$HB \approx HV10 \approx U/493$$ [US units]

For HB < 130 , Z_W = 1.2 is to be used.

For HB > 470 , Z_W = 1.0 is to be used.

21.27 Size Factor, Z_X

The size factor, Z_X , accounts for the influence of tooth dimensions on permissible contact stress and reflects the non-uniformity of material properties.

The factor mainly depends on:

- Material and heat treatment;
- Tooth and gear dimensions;
- Ratio of case depth to tooth size;
- Ratio of case depth to equivalent radius of curvature.

For through-hardened gears and for surface-hardened gears with minimum required effective case depth including root of 1.14 mm (0.045 in.) relative to tooth size and radius curvature Z_X = 1. When the case depth is relatively shallow, then a smaller value of Z_X is to be chosen.

The size factors, Z_X , are to be obtained from 4-3-1-A1/23.29 TABLE 2.

21.29 Safety Factor for Contact Stress, S_H

Based on the application, the following safety factors for contact stress, S_H , are to be applied:

Main propulsion gears (including PTO):	1.40
Duplicated (or more) independent main propulsion gears (including azimuthing thrusters):	1.25
Main propulsion gears for yachts, single screw:	1.25
Main propulsion gears for yachts, multiple screw:	1.20
Auxiliary gears:	1.15

Note:

For the above purposes, yachts are considered pleasure craft not engaged in trade or carrying passengers, and not intended for charter-service.

23 Tooth Root Bending Strength

The criterion for tooth root bending strength is the permissible limit of local tensile strength in the root fillet. The tooth root stress, σ_F , and the permissible tooth root stress, σ_{FP} , are to be calculated separately for the pinion and the wheel, whereby σ_F is not to exceed the permissible tooth root stress σ_{FP} .

The following formulas apply to gears having a rim thickness greater than 3.5 m and further for all involute basic rack profiles, with or without protuberance, however with the following restrictions:

- The 30° tangents contact the tooth-root curve generated by the basic rack of the tool
- The basic rack of the tool has a root radius ρ_{fp} > 0
- The gear teeth are generated using a rack type tool.

23.1 Tooth Root Bending Stress for Pinion and Wheel

Cylindrical gears:

$$\sigma_{F1,2} = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \cdot Y_\beta \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{Fa} \cdot K_{F\beta} \leq \sigma_{FP1,2} \mathrm{N/mm^2, kgf/mm^2, psi}$$

Bevel gears:

$$\sigma_{F1,2} = \frac{F_{mt}}{b \cdot m_{mn}} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\epsilon \cdot Y_K \cdot Y_{LS} \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{Fa} \cdot K_{F\beta} \leq \sigma_{FP1,2} \mathrm{N/mm^2, kgf/mm^2, psi}$$

where

Y_F, Y_{Fa}	=	tooth form factor, see 4-3-1-A1/23.5 below
Y_S, Y_{Sa}	=	stress correction factor, see 4-3-1-A1/23.7 below
Y_β	=	helix angle factor, see 4-3-1-A1/23.9 below
Y_ϵ	=	contact ratio factor, see 4-3-1-A1/23.11 below
Y_K	=	bevel gear factor, see 4-3-1-A1/23.13 below
Y_{LS}	=	load sharing factor, see 4-3-1-A1/23.15 below

$F_t, F_{mt}, K_A, K_\gamma, K_v, K_{Fa}, K_{F\beta}, b, m_n, m_{mn}$, see 4-3-1-A1/9, 4-3-1-A1/11, 4-3-1-A1/13, 4-3-1-A1/15, 4-3-1-A1/19, 4-3-1-A1/17, 4-3-1-A1/5, and 4-3-1-A1/7 of this appendix respectively.

23.3 Permissible Tooth Root Bending Stress

$$\sigma_{FP1,2} = \frac{\sigma_{FE}}{S_F} \cdot Y_d \cdot Y_N \cdot Y_{\delta relT} \cdot Y_{RrelT} \cdot Y_X \mathrm{N/mm^2, kgf/mm^2, psi}$$

where

σ_{FE}	=	bending endurance limit, see 4-3-1-A1/23.17 below
Y_d	=	design factor, see 4-3-1-A1/23.19 below

Y_N	=	life factor, see 4-3-1-A1/23.21 below
$Y_{\delta relT}$	=	relative notch sensitivity factor, see 4-3-1-A1/23.23 below
Y_{RrelT}	=	relative surface factor, see 4-3-1-A1/23.25 below
Y_X	=	size factor, see 4-3-1-A1/23.27 below
S_F	=	safety factor for tooth root bending stress, see 4-3-1-A1/23.29 below

23.5 Tooth Form Factor, Y_F , Y_{Fa}

The tooth form factors, Y_F and Y_{Fa} represent the influence on nominal bending stress of the tooth form with load applied at the outer point of single pair tooth contact.

The tooth form factors, Y_F and Y_{Fa} are to be determined separately for the pinion and the wheel. In the case of helical gears, the form factors for gearing are to be determined in the normal section, i.e. for the virtual spur gear with virtual number of teeth, z .

Cylindrical gears:

$$Y_F = \frac{6 \cdot (h_F/m_n) \cdot \cos\alpha_{Fen}}{(s_{Fn}/m_n)^2 \cdot \cos\alpha_n}$$

Bevel gears:

$$Y_{Fa} = \frac{6 \cdot (h_{Fa}/m_{mn}) \cdot \cos\alpha_{Fan}}{(s_{Fn}/m_{mn})^2 \cdot \cos\alpha_n}$$

where

h_F, h_{Fa}	=	bending moment arm for tooth root bending stress for application of load at the outer point of single tooth pair contact; mm (in.)
s_{Fn}	=	width of tooth at highest stressed section; mm (in.)
$\alpha_{Fen}, \alpha_{Fan}$	=	normal load pressure angle at tip of tooth; degrees

For determination of h_F, h_{Fa}, s_{Fn} and $\alpha_{Fen}, \alpha_{Fan}$ see 4-3-1-A1/23.5.1, 4-3-1-A1/23.5.2, 4-3-1-A1/23.5.3 below and 4-3-1-A1/23.29 FIGURE 1.

23.5.1 External Gears

Width of tooth, s_{Fn} , at tooth-root normal chord:

$$\frac{s_{Fn}}{m_n} = z_n \cdot \sin\left(\frac{\pi}{3} - \vartheta\right) + \sqrt{3} \cdot \left(\frac{G}{\cos\vartheta} - \frac{\rho_{a0}}{m_n}\right)$$

$$\vartheta = 2 \cdot \frac{G}{z_n} \cdot \tan\vartheta - H$$ degrees; to be solved iteratively

$$G = \frac{\rho_{a0}}{m_n} - \frac{h_{a0}}{m_n} + x$$

$$H = \frac{2}{z_n} \cdot \left(\frac{\pi}{2} - \frac{E}{m_n}\right) - \frac{\pi}{3}$$ [SI and MKS units]

$$H = \frac{2}{z_n} \cdot \left(\frac{\pi}{2} - \frac{E}{25.4 \cdot m_n}\right) - \frac{\pi}{3}$$ [US units]

$$z_n = \frac{z}{\cos^2\beta_b \cdot \cos\beta}$$

$$\beta_b = \arccos\sqrt{1 - (\sin\beta \cdot \cos\alpha_n)^2}$$ degrees

$$E = \frac{\pi}{4} \cdot m_n - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n}$$ [SI and MKS units]

$$E = 25.4 \cdot \left(\frac{\pi}{4} \cdot m_n - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n}\right)$$ [US units]

$$S_{pr} = p_{r0} - q$$

$S_{pr} = 0$ when gears are not undercut (non-protuberance hob)

where:

$E, h_{a0}, \alpha_n, S_{pr}, p_{r0}, q$ and ρ_{a0} are shown in 4-3-1-A1/23.29 FIGURE 2.

h_{a0}	=	addendum of tool; mm (in.)
S_{pr}	=	residual undercut left by protuberance; mm (in.)
p_{r0}	=	protuberance of tool; mm (in.)
q	=	material allowances for finish machining; mm (in.)
ρ_{a0}	=	tip radius of tool; mm (in.)
z_n	=	virtual number of teeth
x	=	addendum modification coefficient
α_{Fen}	=	angle for application of load at the highest point of single tooth contact
α_{en}	=	pressure angle at the highest point of single tooth contact

Bending moment arm h_F :

$$\frac{h_F}{m_n} = \frac{1}{2} \cdot \left[(\cos\gamma_e - \sin\gamma_e \cdot \tan\alpha_{Fen}) \cdot \frac{d_{en}}{m_n} - z_n \cdot \cos\left(\frac{\pi}{3} - \vartheta\right) - \frac{G}{\cos\vartheta} + \frac{\rho_{a0}}{m_n} \right]$$

$$\frac{\rho_F}{m_n} = \frac{\rho_{a0}}{m_n} + \frac{2 \cdot G^2}{\cos\vartheta \cdot (z_n \cos^2\vartheta - 2G)}$$

where

ρ_F = root fillet radius in the critical section at 30° tangent; mm (mm, in.); see 4-3-1-A1/23.29 FIGURE 1.

Normal load pressure angle at tip of tooth, $\alpha_{Fen}, \alpha_{Fan}$:

$$\alpha_{Fen} = \alpha_{en} - \gamma_e \text{degrees}$$

$$\alpha_{en} = \arccos\left(\frac{d_{bn}}{d_{en}}\right) \text{ degrees}$$

$$\gamma_e = \left(\frac{0.5 \cdot \pi + 2 \cdot x \cdot \tan\alpha_n}{z_n} + \text{inv}\alpha_n - \text{inv}\alpha_{en}\right) \cdot \frac{180}{\pi} \text{ degrees}$$

$$d_{an1} = d_{n1} + d_{a1} - d_1 \text{mm (in.)}$$

$$d_{an2} = d_{n2} + d_{a2} - d_2 \text{mm (in.)}$$

$$d_{n1,2} = z_{n1,2} \cdot m_n \text{mm (in.)}$$

$$d_{bn1,2} = d_{n1,2} \cdot \cos\alpha_n \text{mm (in.)}$$

$$d_{en1} = \frac{2 \cdot z_1}{|z_1|} \cdot \sqrt{\left[\sqrt{\left(\frac{d_{an1}}{2}\right)^2 - \left(\frac{d_{bn1}}{2}\right)^2} - \frac{\pi \cdot d_1 \cdot \cos\beta \cdot \cos\alpha_n}{|z_1|} \cdot (\epsilon_{an} - 1)\right]^2 + \left(\frac{d_{bn1}}{2}\right)^2} \text{mm (in.)}$$

$$d_{en2} = \frac{2 \cdot z_2}{|z_2|} \cdot \sqrt{\left[\sqrt{\left(\frac{d_{an2}}{2}\right)^2 - \left(\frac{d_{bn2}}{2}\right)^2} - \frac{\pi \cdot d_2 \cdot \cos\beta \cdot \cos\alpha_n}{|z_2|} \cdot (\epsilon_{an} - 1)\right]^2 + \left(\frac{d_{bn2}}{2}\right)^2} \text{mm (in.)}$$

$$\epsilon_{an} = \frac{\epsilon_a}{\cos^2\beta_b} \text{ degrees}$$

Note: z_1, z_2 are positive for external gears and negative for internal gears.

23.5.2 Internal Gears

The tooth form factor of a special rack can be substituted as an approximate value of the form of an internal gear. The profile of such a rack is to be a version of the basic rack profile, so modified that it would generate the normal profile, including tip and root circles, of an exact counterpart of the internal gear. The tip load angle is $\alpha_{Fen} = \alpha_n$.

Width of tooth, s_{Fn2}, at tooth-root normal chord:

$$\frac{s_{Fn2}}{m_n} = 2 \cdot \left[\frac{\pi}{4} + \tan\alpha_n \cdot \left(\frac{h_{fp2} - \rho_{fp2}}{m_n}\right) + \frac{\rho_{fp2} - S_{pr2}}{m_n \cdot \cos\alpha_n} - \frac{\rho_{fp2}}{m_n} \cdot \cos\frac{\pi}{6}\right]$$

Bending moment arm h_{F2} :

$$\frac{h_{F2}}{m_n} = \frac{d_{en2} - d_{fn2}}{2 \cdot m_n} - \left[\frac{\pi}{4} + \left(\frac{h_{fp2}}{m_n} - \frac{d_{en2} - d_{fn2}}{2 \cdot m_n}\right) \cdot \tan\alpha_n\right] \cdot \tan\alpha_n - \frac{\rho_{fp2}}{m_n} \cdot \left(1 - \sin\frac{\pi}{6}\right)$$

$$d_{fn2} = d_{n2} + d_{f2} - d_2 \text{mm (in.)}$$

$$h_{fp2} = \frac{d_{n2} - d_{fn2}}{2} \text{ mm (in.)}$$

$$\rho_{fp2} = \rho_{F2} = \frac{\rho_{a02}}{2} \text{ mm (in.)}$$

Note: In the case of a full root fillet $\rho_{F2} = \rho_{a02}$ is to be used, or as an approximation:

$$\rho_{a02} = 0.3 \cdot m_n \text{mm (in.)}$$

$$\rho_{fp2} = \rho_{F2} = 0.15 \cdot m_n \text{mm (in.)}$$

$$h_{f2} = (1.25...1.30) \cdot m_n \text{mm (in.)}$$

$$\rho_{fp2} = \frac{c_p}{1 - \sin\alpha_n} = \frac{h_{f2} - h_{Nf2}}{1 - \sin\alpha_n} = \frac{d_{Nf2} - d_{f2}}{2 \cdot (1 - \sin\alpha_n)}$$

where:

d_{f2}	=	root diameter of wheel; mm (in.); see 4-3-1-A1/23.29 FIGURE 1.
d_{Nf2}, h_{Nf2}	=	is the diameter, dedendum of basic rack at which the usable flank and root fillet of the annulus gear meet; mm(in.)
c_p	=	is the bottom clearance between basic rack and mating profile; mm(in.)

Note: The diameters d_{n2} and d_{en2} are to be calculated with the same formulas as for external gears.

23.5.3 Bevel Gears

Width of tooth, s_{Fn}, at tooth-root normal chord:

$$\frac{s_{Fn}}{m_{mn}} = z_{vn} \cdot \sin\left(\frac{\pi}{3} - \vartheta\right) + \sqrt{3} \cdot \left(\frac{G}{\cos\vartheta} - \frac{\rho_{a0}}{m_{mn}}\right)$$

$$\vartheta = 2 \cdot \frac{G}{z_{vn}} \cdot \tan\vartheta - H$$ degrees; to be solved iteratively

$$G = \frac{\rho_{a0}}{m_{mn}} - \frac{h_{a0}}{m_{mn}} + x_{hm}$$

$$H = \frac{2}{z_{vn}} \cdot \left(\frac{\pi}{2} - \frac{E}{m_{mn}}\right) - \frac{\pi}{3}$$ [SI and MKS units]

$$H = \frac{2}{z_{vn}} \cdot \left(\frac{\pi}{2} - \frac{E}{25.4 \cdot m_{mn}}\right) - \frac{\pi}{3}$$ [US units]

$$E = \frac{\pi}{4} \cdot m_{mn} - x_{sm} \cdot m_{mn} - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n} [SI\ and\ MKS\ units]$$

$$E = 25.4 \cdot \left(\frac{\pi}{4} \cdot m_{mn} - x_{sm} \cdot m_{mn} - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n}\right) [\mathrm{USunits}]$$

$$S_{pr} = p_{r0} - q$$

$$S_{pr} = 0$$ when gears are not undercut (non-protuberance hob)

where:

$E, h_{a0}, \alpha_n, S_{pr}, p_{r0}, q$ and ρ_{a0} are shown in 4-3-1-A1/23.29 FIGURE 2.

h_{a0}	=	addendum of tool; mm (in.)
S_{pr}	=	residual undercut left by protuberance; mm (in.)
p_{r0}	=	protuberance of tool; mm (in.)
q	=	material allowances for finish machining; mm (in.)
ρ_{a0}	=	tip radius of tool; mm (in.)
z_{vn}	=	virtual number of teeth
x_{hm}	=	profile shift coefficient
x_{sm}	=	tooth thickness modification coefficient (midface)
α_{Fan}	=	angle for application of load at the highest point of single tooth contact
α_{an}	=	pressure angle at the highest point of single tooth contact

Bending moment arm h_{Fa} :

$$\frac{h_{Fa}}{m_{mn}} = \frac{1}{2} \cdot \left[(\cos\gamma_a - \sin\gamma_a \cdot \tan\alpha_{Fan}) \cdot \frac{d_{van}}{m_{mn}} - z_{vn} \cdot \cos\left(\frac{\pi}{3} - \vartheta\right) - \frac{G}{\cos\vartheta} + \frac{\rho_{a0}}{m_{mn}}\right]$$

$$\frac{\rho_F}{m_{mn}} = \frac{\rho_{a0}}{m_{mn}} + \frac{2 \cdot G^2}{\cos\vartheta \cdot (z_{vn}\cos^2\vartheta - 2 \cdot G)}$$

where

ρ_F = root fillet radius in the critical section at 30° tangent; mm (mm, in.); see 4-3-1-A1/23.29 FIGURE 1.

Normal load pressure angle at tip of tooth $\alpha_{Fan}, \alpha_{an}$

$$\alpha_{Fan} = \alpha_{an} - \gamma_a \text{degrees}$$

$$\alpha_{an} = \arccos\left(\frac{d_{vbn}}{d_{van}}\right) \text{ degrees}$$

$$\gamma_a = \left(\frac{0.5 \cdot \pi + 2 \cdot (x_{hm} \cdot \tan\alpha_n + x_{sm})}{z_{vn}} + \text{inv}\alpha_n - \text{inv}\alpha_{an}\right) \cdot \frac{180}{\pi} \text{ degrees}$$

$$\beta_{bm} = \arccos\sqrt{1 - (\sin\beta_m \cdot \cos\alpha_n)^2} \text{ degrees}$$

(See also 4-3-1-A1/7 of this Appendix.)

$$d_{van1} = d_{vn1} + d_{va1} - d_{v1} \qquad mm\ (in.)$$
$$d_{van2} = d_{vn2} + d_{va2} - d_{v2} \qquad mm\ (in.)$$
$$d_{vn1} = z_{vn1} \cdot m_{mn} \qquad mm\ (in.)$$
$$d_{vn2} = z_{vn2} \cdot m_{mn} \qquad mm\ (in.)$$
$$d_{vbn1} = d_{vn1} \cdot \cos\alpha_n \qquad mm\ (in.)$$
$$d_{vbn2} = d_{vn2} \cdot \cos\alpha_n \qquad mm\ (in.)$$

23.7 Stress Correction Factor, Y_S, Y_{Sa}

The stress correction factors, Y_S and Y_{Sa}, are used to convert the nominal bending stress to the local tooth root stress.

Y_S applies to the load application at the outer point of single tooth pair contact. Y_S is to be determined for pinion and wheel separately.

For notch parameter q_s within a range of (1 ≤ q_s < 8):

$$q_s = \frac{S_{Fn}}{2 \cdot \rho_F}$$

Cylindrical gears:

$$Y_S = (1.2 + 0.13 \cdot L)\, q_s^{\left(\frac{1}{1.21 + (2.3/L)}\right)}$$

Bevel gears:

$$Y_{Sa} = (1.2 + 0.13 \cdot L_a)\, q_s^{\left(\frac{1}{1.21 + (2.3/L_a)}\right)}$$

where:

ρ_F	=	root fillet radius in the critical section at 30° tangent mm (in.)
L	=	s_{Fn}/h_F for cylindrical gears
L_a	=	$s_{Fn}/h_{F\alpha}$ for bevel gears

$h_F, h_{Fa}, s_{Fn}, \rho_F$ see 4-3-1-A1/23.29 FIGURE 3.

23.9 Helix Angle Factor, Y_β

The helix angle factor, Y_β , converts the stress calculated for a point loaded cantilever beam representing the substitute gear tooth to the stress induced by a load along an oblique load line into a cantilever plate which represents a helical gear tooth.

$$Y_\beta = 1 - \epsilon_\beta \cdot \frac{\beta}{120}$$

where:

β	=	reference helix angle in degrees for cylindrical gears.

ϵ_β > 1.0 a value of 1.0 is to be substituted for ϵ_β

β > 30° an angle of 30° is to be substituted for β

23.11 Contact Ratio Factor, Y_ϵ

The contact factor Y_ϵ , covers the conversion from load application at the tooth tip to the load application for bevel gears.

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_\alpha} \text{; for} \epsilon_\beta = 0$$

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_\alpha} - \left(\frac{0.75}{\epsilon_\alpha} - 0.375\right) \cdot \epsilon_\beta \text{; for} 0 < \epsilon_\beta < 1$$

$$Y_\epsilon = 0.625 \text{; for} \epsilon_\beta \geq 1$$

23.13 Bevel Gear Factor, Y_K

The bevel gear factor, Y_K accounts the differences between bevel and cylindrical gears.

$$Y_K = \frac{1}{2} + \frac{b}{4 \cdot \ell'_{bm}} + \frac{\ell'_{bm}}{4 \cdot b}$$

$$\ell'_{bm} = \ell_{bm}.\cos\beta_{vb}$$

23.15 Load Sharing Factor, Y_{LS}

The load sharing factor, Y_{LS} , accounts the differences between two or more pair of teeth for ϵ_γ > 2 .

$$Y_{LS} = Z_{LS}^2 \geq 0.7$$

for Z_{LS} see 4-3-1-A1/21.13 of this appendix.

23.17 Allowable Stress Number (Bending), σ_{FE}

For a given material, σ_{FE} is the limit of repeated tooth root stress that can be sustained. For most materials their stress cycles is to be taken at 3 x 10^6 as the beginning of the endurance limit, unless otherwise specified.

The endurance limit σ_{FE} is defined as the unidirectional pulsating stress with a minimum stress of zero (disregarding residual stresses due to heat treatment). Other conditions such as e.g. alternating stress or prestressing are covered by the design factor Y_d .

The endurance limit mainly depends on:

- Material composition, cleanliness and defects;
- Mechanical properties;
- Residual stress;
- Hardening process, depth of hardened zone, hardness gradient;
- Material structure (forged, rolled bar, cast).

The σ_{FE} values are to correspond to a failure probability of 1% or less. The values of σ_{FE} are to be determined from 4-3-1-A1/23.29 TABLE 3 or to be advised by the manufacturer together with technical justification for the proposed values. For gears treated with controlled shot peening process the value σ_{FE} can be increased by 10%.

23.19 Design Factor, Y_d

The design factor, Y_d , takes into account the influence of load reversing and shrink fit prestressing on the tooth root strength, relative to the tooth root strength with unidirectional load as defined for σ_{FE} .

Y_d	=	0.9 for gears with part load in reversed direction, such as main wheel in reversing gearboxes;
Y_d	=	0.7 for idler gears;
Y_d	=	1.0 for all other cases.

23.21 Life Factor, Y_N

The life factor, Y_N , accounts for the higher permissible tooth root bending stress in case a limited life (number of load cycles) is specified.

The factor mainly depends on:

- Material and hardening
- Number of cycles
- Influence factors ($Y_{\delta relT}$, Y_{RrelT} , Y_X).

The life factor, Y_N , can be determined from 4-3-1-A1/23.29 TABLE 5.

23.23 Relative Notch Sensitivity Factor, $Y_{\delta relT}$

The relative notch sensitivity factor, $Y_{\delta relT}$, indicates the extent to which the theoretically concentrated stress lies above the fatigue endurance limit.

The factor mainly depends on the material and relative stress gradient.

For notch parameter values within the range of 1.5 ≤ q_s < 4, $Y_{\delta relT}$ = 1.0

For q_s < 1.5, $Y_{\delta relT}$ = 0.95

For notch parameter q_s ≥ 4, $Y_{\delta relT}$ can be determined by the methods outlined in ISO 6336-3, Section 11 Sensitivity factors, $Y_\delta, Y_{\delta T}, Y_{\delta K}$ and relative notch sensitivity factors, $Y_{\delta relT}$, $Y_{\delta relK}$.

23.25 Relative Surface Factor, Y_{RrelT}

The relative surface factor, Y_{RrelT} , as given in the following table, takes into account the dependence of the tooth root bending strength on the surface condition in the tooth root fillet, but mainly the dependence on the peak to valley surface roughness.

	R_{zr} < 1 µm	SI & MKS units	US units
	R_{zr} < 39 µin	1 µm ≤ R_{zr} ≤ 40 µm	39 µin ≤ R_{zr} ≤ 1575 µin
Case hardened steels, through - hardened steels U ≥ 800 N/mm ^{2}(82 kgf/mm ^{2}, 1.16×10^{5}psi)	1.120	$1.675 - 0.53 \cdot (R_{zr} + 1)^{0.1}$	$1.675–0.53 \cdot (0.0254 \cdot R_{zr} + 1)^{0.1}$ + 1) $^{0.1}$
Normalized steels U < 800 N/mm^2(82 kgf/mm^2, 1.16×10^{5}psi)	1.070	$5.3 - 4.2 \cdot (R_{zr} + 1)^{0.01}$	$5.3–4.2 \cdot (0.0254R_{zr} + 1)^{0.01}$
Nitrided steels	1.025	$4.3 - 3.26 \cdot (R_{zr} + 1)^{0.005}$	$4.3–3.26 \cdot (0.0254 \cdot R_{zr} + 1)^{0.005}$
R_{zr} = mean peak to-valley roughness of tooth root fillets; µm(µm,µin)			

This method is only applicable where scratches or similar defects deeper than 2 R_{zr} are not present.

If the stated roughness is an R_a value, also known as arithmetic average (*AA*) and centerline average (*CLA*),the following approximate relationship is to be applied:

$$R_a = CLA = AA = R_{zr}/6$$

23.27 Size Factor (Root), Y_X

The size factor (root), Y_X , takes into account the decrease of the strength with increasing size.

The factor mainly depends on:

- Material and heat treatment;
- Tooth and gear dimensions;
- Ratio of case depth to tooth size.

SI and MKS units		
Y_X = 1.00	For m_n ≤ 5	For all cases
Y_X = 1.03 – 0.006 $\cdot m_n$	For 5 < m_n < 30	Normalized and through tempered- hardened steels
Y_X = 0.85	for m_n ≥ 30	
Y_X = 1.05 – 0.010 $\cdot m_n$	for 5 < m_n < 25	Surface hardened steels
Y_X = 0.80	for m_n ≥ 25	
US units		
Y_X = 1.00	for m_n ≤ 0.1968	For all cases
Y_X = 1.03 – 0.1524 $\cdot m_n$	for 0.1968 < m_n < 1.181	Normalized and through tempered- hardened steels
Y_X = 0.85	for m_n ≥ 1.181	
Y_X = 1.05 – 0.254 $\cdot m_n$	for 0.1968 < m_n < 0.9842	Surface hardened steels
Y_X = 0.80	for m_n ≥ 0.9842	

Note: For *Bevel gears* the m_n (normal module) is to be substituted by m_{mn} (normal module at mid-facewidth).

23.29 Safety Factor for Tooth Root Bending Stress, S_F

Based on the application, the following safety factors for tooth root bending stress, S_F , are to be applied:

Main propulsion gears (including PTO):	1.80
Duplicated (or more) independent main propulsion gears (including azimuthing thrusters):	1.60
Main propulsion gears for yachts, single screw:	1.50

Main propulsion gears for yachts, multiple screw:	1.45
Auxiliary gears:	1.40

Note:

For the above purposes, yachts are considered pleasure craft not engaged in trade or carrying passenger, and not intended for charter-service.

TABLE 1

Values of the Elasticity Factor Z_E and Young's Modulus of Elasticity E

(Ref. 4-3-1-A1/21.9)

The value of E for combination of different materials for pinion and wheel is to be calculated by:

$$E = \frac{2 \cdot E_1 \cdot E_2}{E_1 + E_2}$$

SI units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 N/mm²	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 N/mm²	Poisson's ratio ν	Elasticity Factor Z_E $N^{1/2}$/mm
Steel	206000	0.3	Steel	206000	0.3	189.8
			Cast steel	202000		188.9
			Nodular cast iron	173000		181.4
			Cast tin bronze	103000		155.0
			Tin bronze	113000		159.8
			Lamellar graphite cast iron (gray cast iron)	126000 to 118000		165.4 to 162.0
Cast steel	202000		Cast steel	202000		188.0
			Nodular cast iron	173000		180.5
			Lamellar graphite cast iron (gray cast iron)	118000		161.4
Nodular cast iron	173000	0.3	Nodular cast iron	173000	0.3	173.9
			Lamellar graphite cast iron (gray cast iron)	118000		156.6
Lamellar graphite cast iron (gray cast iron)	126000 to 118000		Lamellar graphite cast iron (gray cast iron)	118000		146.0 to 143.7
Steel	206000	0.3	Nylon	7850 (mean value)	0.5	56.4

MKS units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 kgf/mm²	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 kgf/mm²	Poisson's ratio ν	Elasticity Factor Z_E $kgf^{1/2}/mm$
Steel	2.101×10^4	0.3	Steel	2.101×10^4	0.3	60.609
			Cast steel	2.060×10^4		60.321
			Nodular cast iron	1.764×10^4		57.926
			Cast tin bronze	1.050×10^4		49.496
			Tin bronze	1.152×10^4		51.029
			Lamellar graphite cast iron (gray cast iron)	1.285×10^4 to 1.203×10^4		52.817 to 51.731
Cast steel	2.060×10^4		Cast steel	2.060×10^4		60.034
			Nodular cast iron	1.764×10^4		57.639
			Lamellar graphite cast iron (gray cast iron)	1.203×10^4		51.540
Nodular cast iron	1.764×10^4	0.3	Nodular cast iron	1.764×10^4	0.3	55.531
			Lamellar graphite cast iron (gray cast iron)	1.203×10^4		50.007
Lamellar graphite cast iron (gray cast iron)	1.285×10^4 to 1.203×10^4		Lamellar graphite cast iron (gray cast iron)	1.203×10^4		46.622 to 45.888
Steel	2.101×10^4	0.3	Nylon	800.477 (mean value)	0.5	18.010

US units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 psi	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 psi	Poisson's ratio ν	Elasticity factor Z_E $lbf^{1/2}/in.$
Steel	2.988×10^7	0.3	Steel	2.988×10^7	0.3	2.286×10^3
			Cast steel	2.930×10^7		2.275×10^3
			Nodular cast iron	2.509×10^7		2.185×10^3
			Cast tin bronze	1.494×10^7		1.867×10^3
			Tin bronze	1.639×10^7		1.924×10^3
			Lamellar graphite cast iron (gray cast iron)	1.827×10^7 to 1.711×10^7		1.992×10^3 to 1.951×10^3
Cast steel	2.930×10^7		Cast steel	2.930×10^7		2.264×10^3
			Nodular cast iron	2.509×10^7		2.174×10^3

US units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 psi	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 psi	Poisson's ratio ν	Elasticity factor Z_E lbf$^{1/2}$/in.
			Lamellar graphite cast iron (gray cast iron)	1.711×10 ^{7}		1.944×10 ^{3}
Nodular cast iron	2.509×10 ^{7}	0.3	Nodular cast iron	2.509×10 ^{7}	0.3	2.094×10 ^{3}
			Lamellar graphite cast iron (gray cast iron)	1.711×10 ^{7}		1.886×10 ^{3}
Lamellar graphite cast iron (gray cast iron)	1.827×10 ^{7} to 1.711×10 ^{7}		Lamellar graphite cast iron (gray cast iron)	1.711×10 ^{7}		1.758×10 ^{3} to 1.731×10 ^{3}
Steel	2.988×10 ^{7}	0.3	Nylon	1.139×10 ^{6} (mean value)	0.5	679.234

TABLE 2
Size Factor Z_X for Contact Stress - (Ref. 4-3-1-A1/21.27)

SI and MKS units	
Z_X , size factor for contact stress	Material
1.0	For through-hardened pinion treatment All modules (m_n)
1.0 1.05 – 0.005 m_n 0.9	For carburized and induction-hardened pinion heat treatment m_n ≤ 10 m_n < 30 m_n ≥ 30
1.0 1.08 – 0.011 m_n 0.75	For nitrided pinion treatment m_n < 7.5 m_n < 30 m_n ≥ 30
For ***Bevel gears*** the m_n (normal module) is to be substituted by m_{mn} (normal module at mid-facewidth).	
US units	
Z_X , size factor for contact stress	Material
	For through-hardened pinion treatment

US units	
Z_X , size factor for contact stress	Material
1.0	All modules (m_n)
1.0 1.05 – 0.127 m_n 0.9	For carburized and induction-hardened pinion heat treatment m_n ≤ 0.394 m_n < 1.181 m_n ≥ 1.181
1.0 1.08 – 0.279 m_n 0.75	For nitrided pinion treatment m_n < 0.295 m_n < 1.181 m_n ≥ 1.181
For ***Bevel gears*** the m_n (normal module) is to be substituted by m_{mn} (normal module at mid-facewidth).	

TABLE 3
Allowable Stress Number (contact) $\sigma_{H\lim}$ and Allowable Stress Number (bending)

(Ref. 4-3-1-A1/21.19, 4-3-1-A1/23.17)

SI units	$\sigma_{H\lim}$ N/mm²	σ_{FE} N/mm²	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Case hardened (carburized) CrNiMo steels:			
• of ordinary grade;	1500	920	Fig. 9, MQ, Fig. 11, MQ (1)
• of specially approved high quality grade (to be based on review and verification of established testing procedure).			Fig. 9, ME
	1650	1050	Fig. 11, ME (2)
Other case hardened (carburized) steels	1500		Fig. 9, MQ
		840	Fig. 11, MQ (3)
Gas nitrided steels: hardened, tempered and gas nitrided, Surface hardness: 700-850 HV10	1250		Fig. 13a, MQ
		920	Fig. 14a, MQ
Through hardened steels: hardened, tempered and gas nitrided, Surface hardness: 500-650 HV10	1000		Fig. 13b, MQ
		740	Fig. 14b, MQ
Through hardened steels: hardened, tempered or normalized and nitro-carburized, Surface hardness: 450-650 HV10	950		Fig. 13c, ME-MQ
		780	Fig. 14c, ME-MQ
Flame or induction hardened steels, Surface hardness: 520-620 HV10	0.65·HV10 + 830		Fig. 10, MQ
		0.25·HV10 + 580	Fig. 12, MQ
Alloyed through hardening steels, Surface hardness: 195-360 HV10	1.32·HV10 + 372		Fig. 5, MQ
		0.78 HV10 + 400	Fig. 7, MQ
Through hardened carbon steels, Surface hardness: 135-210 HV10	1.05·HV10 + 335		Fig. 5, Carbon steel, MQ
		0.50·HV10 + 320	Fig. 7, Carbon steel, MQ

SI units	$\sigma_{H\lim}$ N/mm²	σ_{FE} N/mm²	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Alloyed cast steels, Surface hardness: 198-358 HV10	1.30 HV10 + 295		Fig. 6, MQ-ML
		0.68 HV10 + 325	Fig. 8, MQ-ML
Cast carbon steels, Surface hardness: 135-210 HV10	0.87·HV10 + 290		Fig. 6, Carbon steel, MQ-ML
		0.50·HV10 + 225	Fig. 8, Carbon steel, MQ-ML

MKS units	$\sigma_{H\lim}$ kgf/mm²	σ_{FE} kgf/mm²	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Case hardened (carburized) CrNiMo steels:			
• of ordinary grade;	153	93.8	Fig. 9, MQ, Fig. 11, MQ (1)
• of specially approved high quality grade (to be based on review and verification of established testing procedure).			Fig. 9, ME
	168.3	107.1	Fig. 11, ME (2)
Other case hardened (carburized) steels	153		Fig. 9, MQ
		85.7	Fig. 11, MQ (3)
Gas nitrided steels: hardened, tempered and gas nitrided, Surface hardness: 700-850 HV10	127.5		Fig. 13a, MQ
		93.8	Fig. 14a, MQ
Through hardened steels: hardened, tempered and gas nitrided, Surface hardness: 500-650 HV10	102		Fig. 13b, MQ
		75.5	Fig. 14b, MQ
Through hardened steels: hardened, tempered or normalized and nitro-carburized, Surface hardness: 450-650 HV10	96.9		Fig. 13c, ME-MQ
		79.5	Fig. 14c, ME-MQ
Flame or induction hardened steels, Surface hardness: 520-620 HV10	0.0663·HV10 + 84.6		Fig. 10, MQ
		0.0255·HV10 + 59.1	Fig. 12, MQ
Alloyed through hardening steels, Surface hardness: 195-360 HV10	0.1346·HV10 + 37.9		Fig. 5, MQ
		0.0795 HV10 + 40.8	Fig. 7, MQ
Through hardened carbon steels, Surface hardness: 135-210 HV10	0.1071·HV10 + 34.2		Fig. 5, Carbon steel, MQ
		0.0510·HV10 + 32.6	Fig. 7, Carbon steel, MQ
Alloyed cast steels, Surface hardness: 198-358 HV10	0.1326 HV10 + 30.1		Fig. 6, MQ-ML
		0.0693 HV10 + 33.1	Fig. 8, MQ-ML
Cast carbon steels, Surface hardness: 135-210 HV10	0.0887·HV10 + 29.6		Fig. 6, Carbon steel, MQ-ML
		0.0510·HV10 + 22.9	Fig. 8, Carbon steel, MQ-ML

US units	$\sigma_{H\lim}$ psi	σ_{FE} psi	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Case hardened (carburized) CrNiMo steels:			
• of ordinary grade;	217557	133435	Fig. 9, MQ, Fig. 11, MQ (1)
• of specially approved high quality grade (to be based on review and verification of established testing procedure).			Fig. 9, ME
	239312	152290	Fig. 11, ME (2)
Other case hardened (carburized) steels	217557		Fig. 9, MQ
		121832	Fig. 11, MQ (3)
Gas nitrided steels: hardened, tempered and gas nitrided, Surface hardness: 700-850 HV10	181297		Fig. 13a, MQ
		133435	Fig. 14a, MQ

US units	$\sigma_{H\lim}$ psi	σ_{FE} psi	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Through hardened steels: hardened, tempered and gas nitrided, Surface hardness: 500-650 HV10	145038		Fig. 13b, MQ
		107328	Fig. 14b, MQ
Through hardened steels: hardened, tempered or normalized and nitro-carburized, Surface hardness: 450-650 HV10	137786		Fig. 13c, ME-MQ
		113129	Fig. 14c, ME-MQ
Flame or induction hardened steels, Surface hardness: 520-620 HV10	94.3·HV10 + 120381		Fig. 10, MQ
		36.3·HV10 + 84122	Fig. 12, MQ
Alloyed through hardening steels, Surface hardness: 195-360 HV10	191.5·HV10 + 53954		Fig. 5, MQ
		113.1 HV10 + 58015	Fig. 7, MQ
Through hardened carbon steels, Surface hardness: 135-210 HV10	152.3·HV10 + 48588		Fig. 5, Carbon steel, MQ
		72.5·HV10 + 46412	Fig. 7, Carbon steel, MQ
Alloyed cast steels, Surface hardness: 198-358 HV10	188.6 HV10 + 42786		Fig. 6, MQ-ML
		98.6 HV10 + 47137	Fig. 8, MQ-ML
Cast carbon steels, Surface hardness: 135-210 HV10	126.2·HV10 + 42061		Fig. 6, Carbon steel, MQ-ML
		72.5·HV10 + 32633	Fig. 8, Carbon steel, MQ-ML

Notes:

HV10: Vickers hardness at load *F*= 98.10 N, see ISO 6336-5

1 Core hardness ≥ 25 HRC, Jominy hardenability at *J* = 12 mm ≥ HRC 28 and Surface hardness: 640-780 HV10

2 Core hardness ≥ 30 HRC, Surface hardness: 660-780 HV10

3 Core hardness ≥ 25 HRC, Jominy hardenability at *J* = 12 mm < HRC 28 and Surface hardness: 640-780 HV10

TABLE 4
Determination of Life Factor for Contact Stress, Z_N

(Ref. 4-3-1-A1/21.21)

Material	Number of load cycles	Life factor Z_N
St, V, GGG (perl., bain.), GTS (perl.), Eh, IF; Only when a certain degree of pitting is permissible	N_L ≤ 6 x 10^5 , static	1.6
	N_L = 10^7	1.3
	N_L = 10^9	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
St, V, GGG (perl., bain.), GTS (perl.), Eh, IF	N_L ≤10^5 , static	1.6
	N_L = 5 x 10^7	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
GG, GGG (ferr.), NT (nitr.), NV (nitr.)	N_L ≤ 10^5 , static	1.3
	N_L = 2 x 10^6	1.0

Material	Number of load cycles	Life factor Z_N
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
NV (nitrocar.)	N_L ≤ 10^5, static	1.1
	N_L = 2 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication,material, manufacturing,and experience	0.85 1.0

St:	steel (U < 800 N/mm², 82 kgf/mm2, 1.16×10^5 psi)
V:	through-hardening steel, through-hardened (U ≥ 800 N/mm²)
GG:	gray cast iron
GGG (perl., bain., ferr.):	nodular cast iron (perlitic, bainitic, ferritic structure)
GTS (perl.):	black malleable cast iron (perlitic structure)
Eh:	case-hardening steel, case hardening
IF:	steel and GGG, flame or induction hardened
NT (nitr.):	nitriding steel, nitrided
NV (nitr.):	through-hardening and case-hardening steel, nitrided
NV (nitrocar.):	through-hardening and case-hardening steel, nitrocarburized

TABLE 5
Determination of Life Factor for Tooth Root Bending Stress, Y_N

(Ref. 4-3-1-A1/23.21)

Material	Number of load cycles	Life factor Y_N
V, GGG (perl., bain.), GTS (perl.)	N_L ≤ 10^4, static	2.5
	N_L = 3 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
Eh, IF (root)	N_L ≤ 10^3, static	2.5
	N_L = 3 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
St, NT, NV (nitr.), GG, GGG (ferr.)	N_L ≤ 10^3, static	1.6
	N_L = 3 x 10^6	1.0

Material	Number of load cycles	Life factor Y_N
	$N_L = 10^{10}$ Optimum lubrication, material, manufacturing, and experience	0.85 1.0
NV (nitrocar.)	$N_L \le 10^3$, static	1.0
	$N_L = 3 \times 10^6$	1.0
	$N_L = 10^{10}$ Optimum lubrication, material, manufacturing, and experience	0.85 1.0

Notes:

1. Abbreviations of materials are as explained in and of this Appendix.
2. $$N_L = n \cdot 60 \cdot HPD \cdot DPY \cdot YRS$$

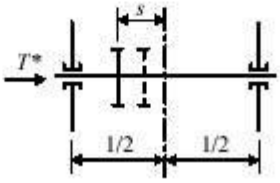
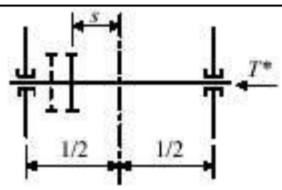
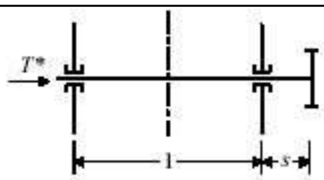
n = rotational speed, rpm.

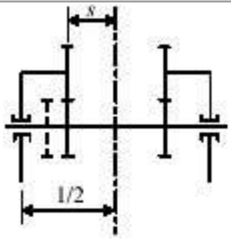
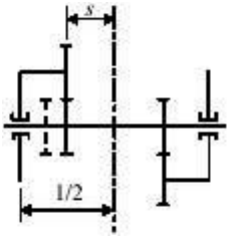
HPD = operation hours per day

DPY = days per year

YRS = years (normal life of vessel = 25 *years*

TABLE 6
Constant *K′* for the Calculation of the Pinion Offset Factor γ

Factor K'		Figure	Arrangement
With stiffening (1)	Without stiffening (1)		
0.48	0.8	a)	 with s/ℓ < 0.3
-0.48	-0.8	b)	 with s/ℓ < 0.3
1.33	1.33	c)	 with s/ℓ < 0.3

Factor K'		Figure	Arrangement
With stiffening [1]	Without stiffening [1]		
-0.36	-0.6	d)	 with s/ℓ < 0.3
-0.6	-1.0	e)	 with s/ℓ < 0.3

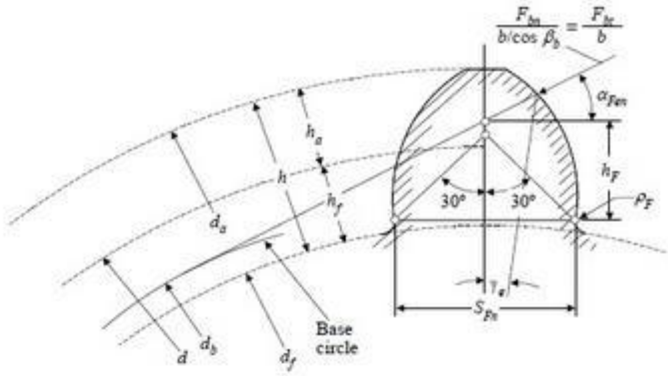
1. When d_1/d_{sh} ≥1.15, stiffening is assumed; when d_1/d_{sh} < 1.15, there is no stiffening. Furthermore, scarcely any or no stiffening at all is to be expected when the pinion slides on a shaft and feather key or a similar fitting, nor when normally shrink fitted.

T^* is the input or output torqued end,not dependent on direction of rotation.

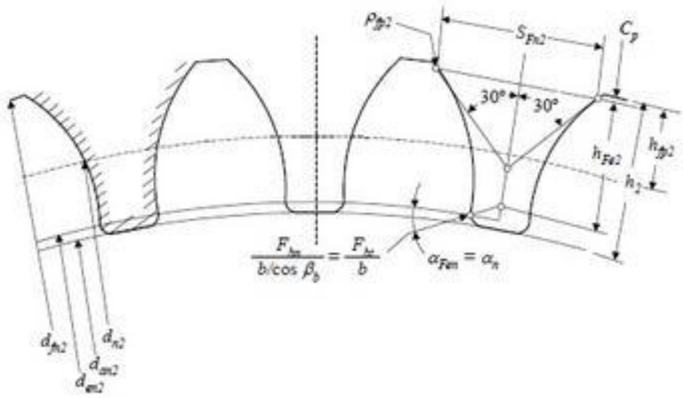
Dashed line indicates the less deformed helix of a double helical gear.

Determined t_{sh} from the diameter in the gaps of double helical gearing mounted centrally between bearings

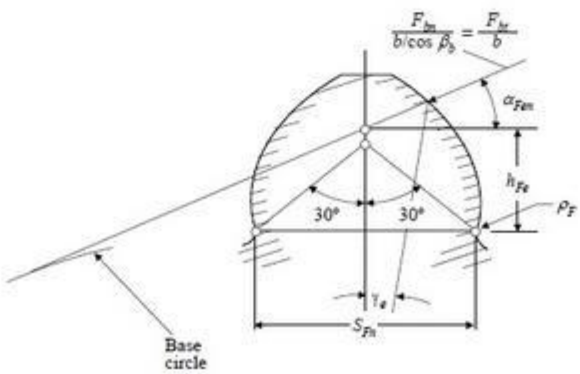
FIGURE 1
Tooth in Normal Section



External cylindrical gears

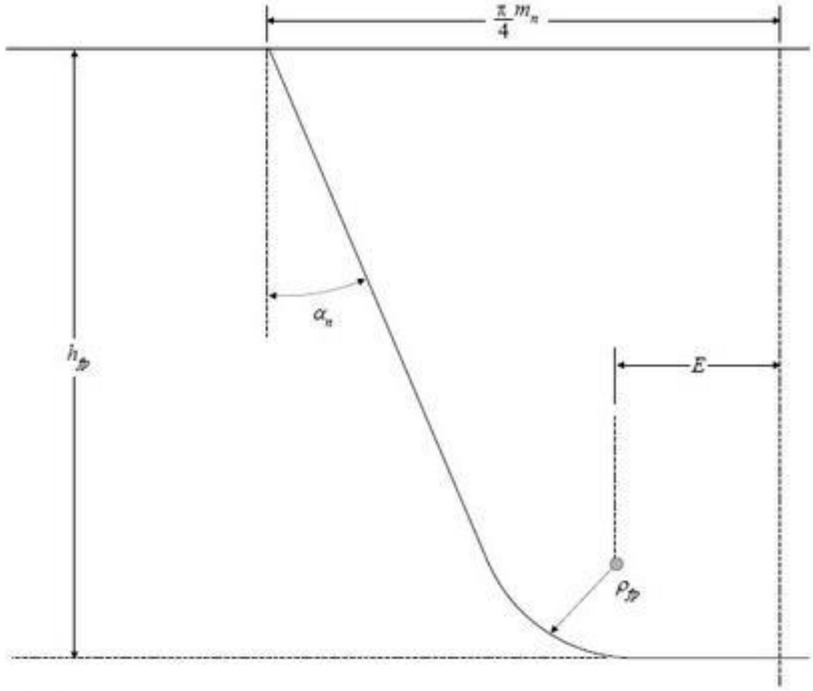


Internal cylindrical gears

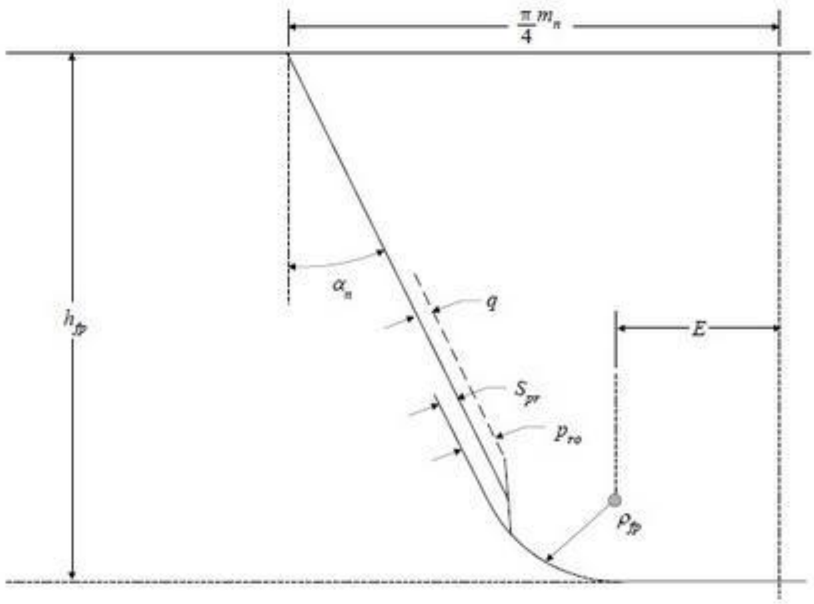


Bevel gears

FIGURE 2
Dimensions and Basic Rack Profile of the Tooth (Finished Profile)



Without undercut



With undercut

FIGURE 3
Wheel Blank Factor C_R , Mean Values for Mating Gears of Similar or Stiffer Wheel Blank Design.

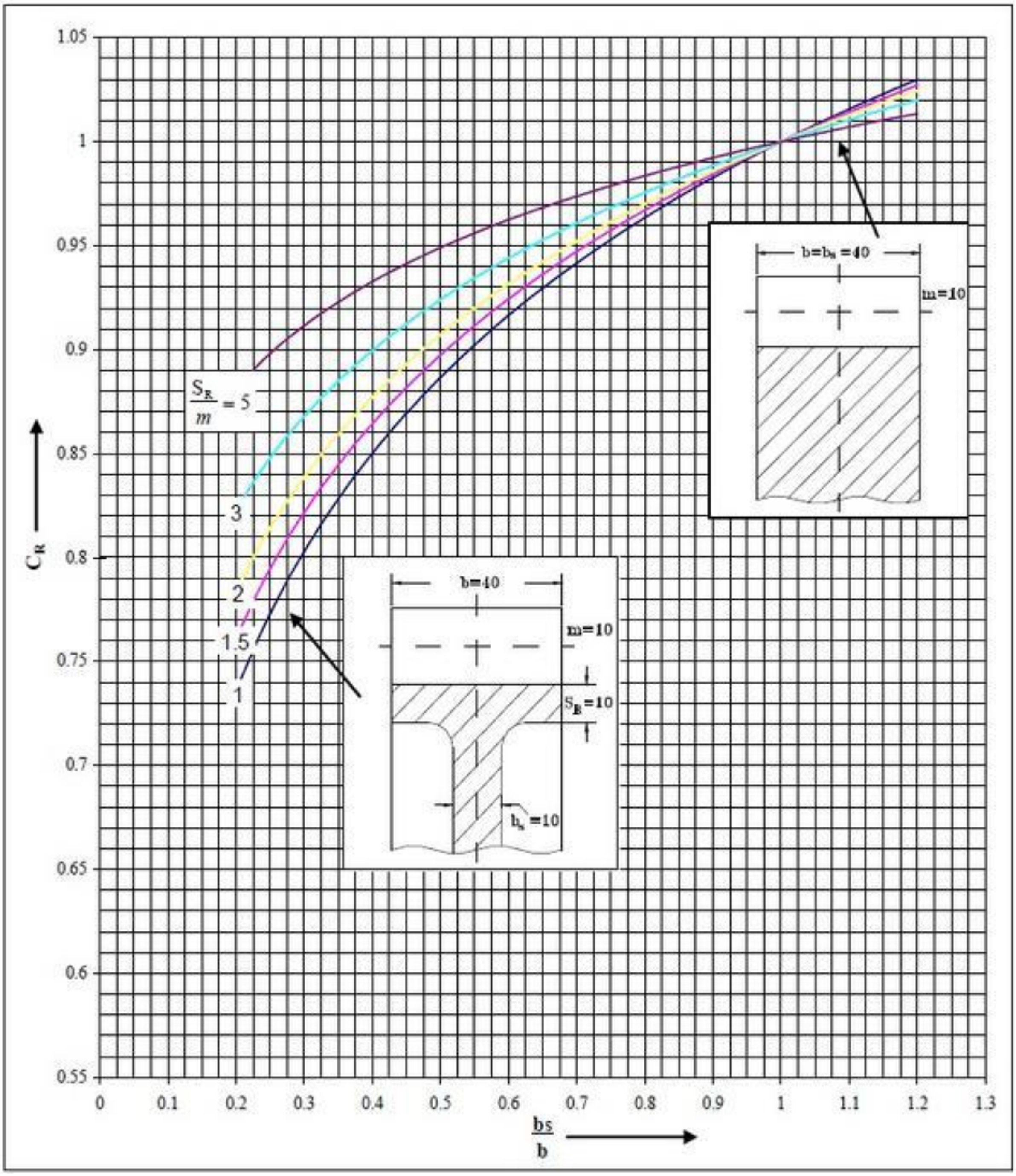


FIGURE 4
Bevel Gear Conversion to Equivalent Cylindrical Gear.

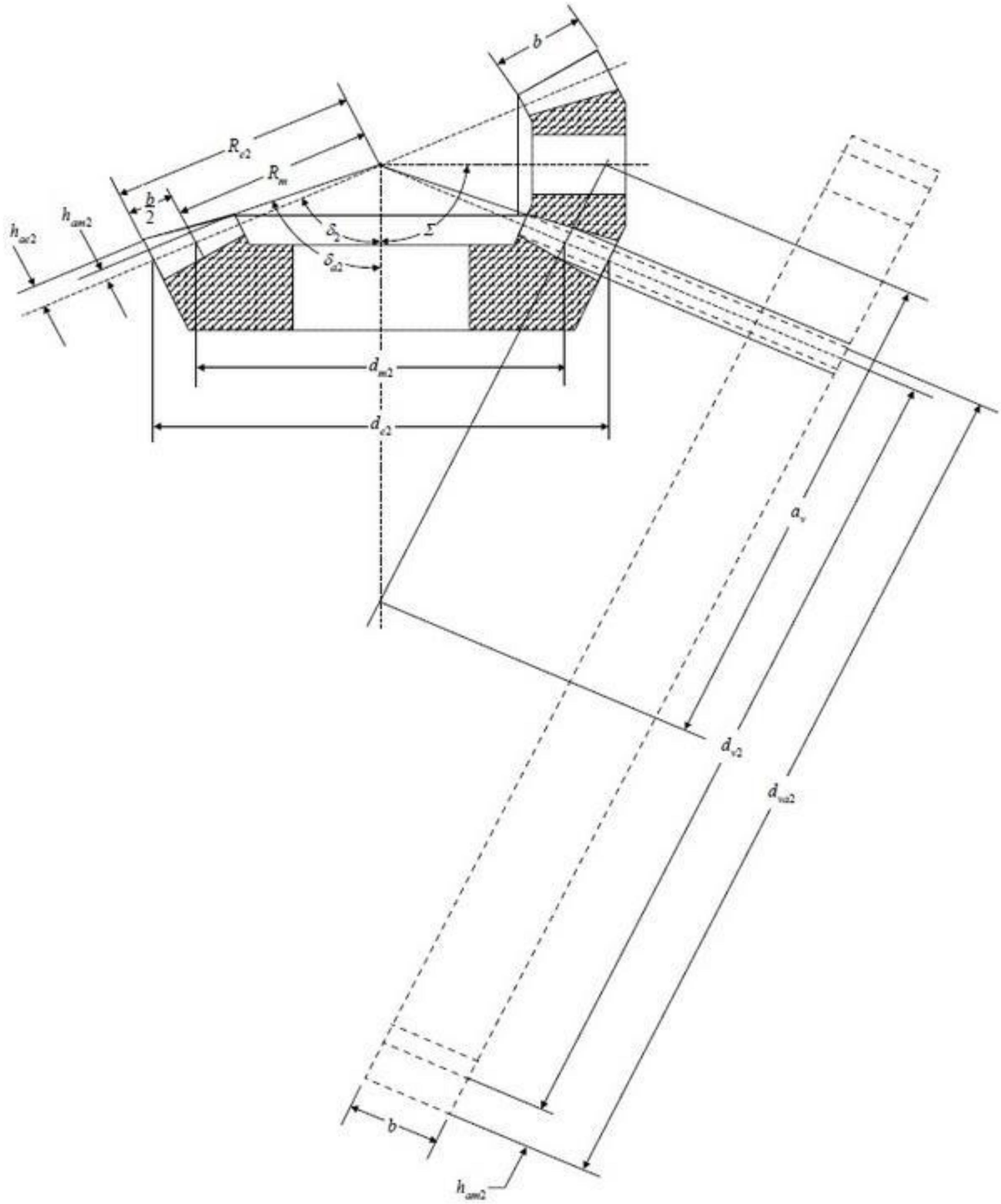
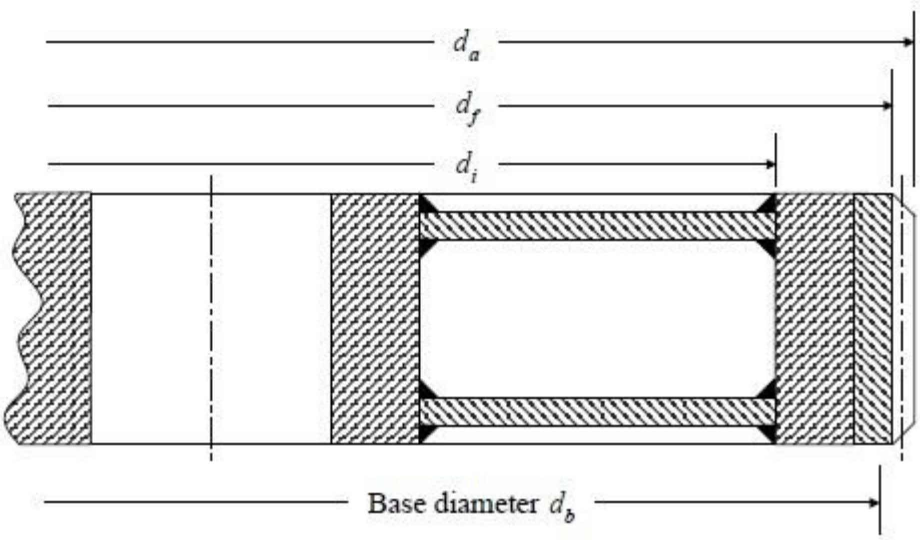


FIGURE 5
Definitions of the Various Diameters



Appendix 2 Guidance for Spare Parts

1 General

While spare parts are not required for purposes of classification, the spare parts listed below are provided as a guidance for vessels intended for unrestricted service. The maintenance of spare parts aboard each vessel is the responsibility of the owner.

3 Spare Parts for Gears

a. Sufficient packing rings with springs to repack one gland of each kind and size.
b. One (1) set of thrust pads or rings, also springs where fitted, for each size.
c. Bearing bushings sufficient to replace all the bushings on every pinion, and gear for main propulsion; spare bearing bushings sufficient to replace all the bushings on each non-identical pinion and gear having sleeve-type bearings or complete assemblies consisting of outer and inner races and cages complete with rollers or balls where these types of bearings are used.
d. One (1) set of bearing shoes for one face, for one single-collar type main thrust bearing where fitted. Where the ahead and astern pads differ, pads for both faces are to be provided.
e. One (1) set of strainer baskets or inserts for oil filters of special design of each type and size.
f. Necessary special tools.

Appendix 3 Gear Parameters

For purposes of submitting gear design for review, the following data and parameters may be used as a guide.

Gear manufacturer .... Year of build ....

Shipyard .... Hull number ....

Ship's name .... Shipowner ....

GENERAL DATA (1)

Gearing type ....

.... (non reversible, single reduction, double reduction, epicyclic, etc.)

Total gear ratio ....

Manufacturer and type of the main propulsion plant (or of the auxiliary machinery) ....

....

Power(2) .... kW, (PS, hp) Rotational speed (2) .... RPM

Maximum input torque for continuous service .... N-m, (kgf-m, lbf-ft)

Maximum input rotational speed for continuous service .... RPM

Type of coupling: (stiff coupling, hydraulic or equivalent coupling, high-elasticity coupling, other couplings, quill shafts, etc.) ....

Specified grade of lubricating oil ....

Expected oil temperature when operating at the classification power (mean values of temperature at inlet and outlet of reverse and/or reduction gearing) ....

Value of nominal kinematic viscosity, ν , at 40°C or 50°C of oil temperature .... (mm^2/s)

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Transmitted rated power, input in the gear drive – kW (mhp, hp) (3)	P				
Rotational speed, input in the gear drive (RPM) (2)	n				
Nominal transverse tangential load at reference cylinder – N, (kgf, lbf) (4)	F_t				
Number of teeth of pinion	z_1				
Number of teeth of wheel	z_2				
Gear ratio	u				
Center distance – mm (in.)	a				
Facewidth of pinion – mm (in.)	b_1				
Facewidth of wheel – mm (in.)	b_2				
Common facewidth – mm (in.)	b				
Overall facewidth, including the gap for double helical gears – mm (in.)	B				

1 The manufacturer can supply values, if available, supported by documents for stress numbers σ_{Hlim} and σ_{FE} involved in the formulas for gear strength with respect to the contact stress and with respect to the tooth root bending stress. See 4-3-1-A1/21 and 4-3-1-A1/23.

2 Maximum continuous performance of the machinery for which classification is requested.

3 It is intended the power mentioned under note (2) or fraction of it in case of divided power.

4 The nominal transverse tangential load is calculated on the basis of the above mentioned maximum continuous performance or on the basis of astern power when it gives a higher torque. In the case of navigation in ice, the nominal transverse tangential load is to be increased as required by 4-3-1-A1/9.

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Is the wheel an external or an internal teeth gear?	-				
Number of pinions	-				
Number of wheels	-				
Is the wheel an external or an internal teeth gear	-				
Is the pinion an intermediate gear?	-				
Is the wheel an intermediate gear?	-				
Is the carrier stationary or revolving (star or planetcarrier)?(5)	-				
Reference diameter of pinion – mm (in.)	d_1				
Reference diameter of wheel – mm (in.)	d_2				
Working pitch diameter of pinion – mm (in.)	d_{w1}				
Working pitch diameter of wheel – mm (in.)	d_{w2}				
Tip diameter of pinion – mm (in.)	d_{a1}				
Tip diameter of wheel – mm (in.)	d_{a2}				
Base diameter of pinion – mm (in.)	d_{b1}				
Base diameter of wheel – mm (in.)	d_{b2}				
Tooth depth of pinion – mm (in.)(6)	h_1				
Tooth depth of wheel – mm (in.)(6)	h_2				
Addendum of pinion – mm (in.)	h_{a1}				
Addendum of wheel – mm (in.)	h_{a2}				
Addendum of tool referred to normal module for pinion	h_{a01}				
Addendum of tool referred to normal module for wheel	h_{a02}				
Dedendum of pinion – mm (in.)	h_{f1}				
Dedendum of wheel – mm (in.)	h_{f2}				
Dedendum of basic rack [referred to m_n (= h_{a01})] for pinion	h_{fp1}				
Dedendum of basic rack [referred to m_n (= h_{a02})] for wheel	h_{fp2}				
Bending moment arm of tooth – mm (in.)(7)	h_F				
Bending moment arm of tooth – mm (in.)(8)	h_{F2}				
Bending moment arm of tooth – mm (in.)(9)	h_{Fa}				
Width of tooth at tooth-root normal chord – mm (in.)(7,9)	s_{Fn}				

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Width of tooth at tooth-root normal chord – mm (in.)(8)	S_{Fn2}				
Angle of application of load at the highest point of single tooth contact (degrees)	α_{Fen}				
Pressure angle at the highest point of single tooth contact (degrees)	α_{en}				

5 Only for epicyclic gears.

6 Measured from the tip circle (or circle passing through the point of beginning of the fillet at the tooth tip) to the beginning of the root fillet

7 Only for external gears.

8 Only for internal gears.

9 Only for bevel gears.

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Normal module – mm (in.)	m_n				
Outer normal module – mm (in.)	m_{na}				
Transverse module	m_t				
Addendum modification coefficient of pinion	x_1				
Addendum modification coefficient of wheel	x_2				
Addendum modification coefficient refers to the midsection(9)	x_{mn}				
Tooth thickness modification coefficient (midface)(9)	x_{sm}				
Addendum of tool – mm (in.)	h_{a0}				
Protuberance of tool – mm (in.)	P_{r0}				
Machining allowances – mm (in.)	q				
Tip radius of tool – mm (in.)	ρ_{a0}				
Profile form deviation – mm (in.)	f_{fa}				
Transverse base pitch deviation – mm (in.)	f_{pb}				
Root fillet radius at the critical section – mm (in.)	ρF				
Root radius of basic rack [referred to m_n (= ρ_{a0})] – mm (in.)	ρ_{fp}				
Radius of curvature at pitch surface – mm (in.)	ρ_c				
Normal pressure angle at reference cylinder (degrees)	α_n				
Transverse pressure angle at reference cylinder (degrees)	α_t				
Transverse pressure angle at working pitch cylinder (degrees)	α_{tw}				

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Helix angle at reference cylinder (degrees)	β				
Helix angle at base cylinder (degrees)	β_b				
Transverse contact ratio	ϵ_α				
Overlap ratio	ϵ_β				
Total contact ratio	ϵ_γ				
Angle of application of load at the highest point of single tooth contact (degrees)(9)	α_{Fan}				
Reference cone angle of pinion (degrees)(9)	δ_1				
Reference cone angle of wheel (degrees)(9)	δ_2				
Shaft angle (degrees)(9)	$\sum$				
Tip angle of pinion (degrees)(9)	$\delta_{\alpha 1}$				
Tip angle of wheel (degrees)(9)	$\delta_{\alpha 2}$				
Helix angle at reference cylinder (degrees)(9)	β_m				
Cone distance pinion. wheel – mm (in.)(9)	R				
Middle cone distance – mm (in.)(9)	R_m				

9 Only for bevel gears.

MATERIALS

First gear drive

Pinion: Material grade or specification ....

Complete chemical analysis ....

....

Minimum ultimate tensile strength(10) .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength(10).. N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... % Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Specified surface roughness RZ or Ra relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.[11] ....

<u>Wheel</u>: Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength[10] .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength[10]... N/mm², (kgf/mm², lbf/in²)

Elongation (A_{5}) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.[11] ....

10 Relevant to core of material

11 In case of shrink-fitted pinions, wheel rims or hubs

Second gear drive

Pinion: Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength (10) .... N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) .... N/mm², (kgf/mm², lbf/in.²)

Elongation (A_5) .... % Hardness (HB, HV10 or HRC)...

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.
(11) ....

Wheel: Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.
(11) ....

10 Relevant to core of material

11 In case of shrink-fitted pinions, wheel rims or hubs

Third gear drive

Pinion: Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) ....N/mm², (kgf/mm², lbf/in.²

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.
(11) ....

Wheel: Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Elongation (A_5)... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.
(11) ....

10 Relevant to core of material

11 In case of shrink-fitted pinions, wheel rims or hubs

Fourth gear drive

<u>Pinion:</u> Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength(10) .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength(10).. N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Specified surface hardness (HB, HV10 or HRC)...

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness RZ or Ra relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures

proposed to ensure the securing of rims.(11) ....

<u>Wheel:</u> Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength(10) .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength(10) .... N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures

proposed to ensure the securing of rims.(11) ....

VARIOUS ITEMS (indicate other elements, if any, in case of particular types of gears)

....

....

Date,..

Goals

Propulsion and maneuvering machinery for craft less than 24 meters (79 feet) in length covered in this section are to be designed, constructed, operated and maintained to:

Goal No.	Goal
PROP 1	provide sufficient thrust/power to move or maneuver the craft when required.
SAFE 1.1	minimize danger to persons on board, the vessel, and surrounding equipment/installations from hazards associated with machinery and systems.

Materials are to be suitable for the intended application in accordance with the following goals and support the Tier 1 Goals as listed above.

Goal No.	Goal
MAT 1	The selected materials' physical, mechanical and chemical properties are to meet the design requirements appropriate for the application, operating conditions and environment.

The goals in the cross-referenced Rules/Regulations are also to be met.

Functional Requirements

In order to achieve the above stated goals, the design, construction, installation and maintenance of the propulsion and maneuvering machinery for craft less than 24 meters (79 feet) in length are to be in accordance with the following functional requirements:

Functional Requirement No.	Functional Requirements
PROPULSION, MANEUVERING, STATION KEEPING (PROP)	
PROP-FR1	Propulsion systems including torque transmitting components are to withstand the maximum delivered power.
PROP-FR2	Propulsion units are to be arranged to maintain the steering of the craft in the worst operational conditions.
PROP-FR3	Mechanical steering gears and components are to be designed to withstand the anticipated severe operational conditions.
SAFETY OF PERSONNEL (SAFE)	
SAFE-FR1	Provide means to protect personnel from rotating parts of mechanical steering gear.
MATERIALS (MAT)	
MAT-FR1	Materials are to be suitable for the most severe operating conditions without any deformation or fatigue failure.

The functional requirements in the cross-referenced Rules/Regulations are also to be met.

Compliance

A craft is considered to comply with the goals and functional requirements within the scope of Classification when the applicable prescriptive requirements are complied with or when an alternative arrangement has been approved, refer to Part 1D, Chapter 2.

1 General

This Section is applicable to all craft less than 24 meters (79 feet) in Length. Section 4-3-3 are applicable except as modified herein.

3 Shafting and Gears

3.1 Outboard Engines

Outboard gasoline engines are to be built to an international or national standard acceptable to ABS or have at least two years of service in the marine industry. The installation is to be in accordance with manufacturer's recommendations.

3.3 Inboard/Outboard Installations

3.3.1 Inboard/Outboard Propulsion Systems

Inboard/outboard propulsion systems (gearing, transmission, etc.) are to be rated for the maximum rating and RPM of the associated prime mover. The installation is to be in accordance with manufacturer's recommendations.

3.3.2 Gears for Inboard and Inboard/Outboard Propulsion Systems

Gears for inboard and inboard/outboard propulsion systems are to be designed and built for installation in a marine environment and rated to the maximum rating and RPM of the associated prime mover. The gears are to be built to a standard acceptable to ABS or have at least two years of service in the marine industry. The installation is to be in accordance with manufacturer's recommendations.

3.5 Inboard Installations

Propulsion shafting installations are to comply with one of the following:

- [4-3-1/7],
- American Boat and Yacht Council (ABYC), Section P-6.5 using a design coefficient (C_d) of at least 15,
- Another National or International Standard acceptable to ABS, or
- Submission of a fatigue analysis showing that the proposed shafting size has a safety factor of at least two (2).

3.7 Propulsion Shaft Alignment and Vibrations

Section 4-3-1 is not applicable to craft under 24 meters (79 feet) in length.

3.9 Materials

Torque transmitting components are not required to be tested in the presence of a Surveyor. Material certificates are to be provided for these items.

5 Propellers

Propellers for craft under 24 meters (79 feet) in length are to be part of a manufacturer's standard product line and are not required to have a Surveyor's attendance or material testing and inspection.

7 Steering Gears

7.1 General **(2025)**

Hydraulic and electrical power operated steering gears and orbitrol systems are to comply with Section 4-3-3, except as modified below:

- Rudder actuators are acceptable based on Manufacturer's Certification
- Hydraulic power units are acceptable based on Manufacturer's Certification

or

Alternatively, the following ISO standards, as applicable, are acceptable in lieu of the requirements of this Section:

- ISO 8847 Small Craft – Steering Gear – Wire Rope and Pulley Systems
- ISO 8848 Small Craft – Remote Steering System
- ISO 9775 Small Craft – Remote Steering Systems for Single Outboard Motors of 15 kW to 40 kW Power
- ISO 10592 Small Craft – Hydraulic Steering Systems

7.3 Materials

Materials are to meet the requirements of 4-3-3/3.1, but are not required to be tested in accordance with 4-3-3/3.3.

7.5 System Arrangements **(2025)**

Steering systems are to meet the requirements of 4-3-3/7.7 and 4-3-3/11.3.1, except when the craft can meet one of the following requirements:

- For craft fitted with multiple propulsion units and the steering system disabled and/or locked in a neutral position, steering can be effected by varying the speed and direction of the propulsion units. The propulsion units are to be spaced as widely as practical and be capable of steering the craft in the worst anticipated operational conditions.
- When using mechanical steering systems, sufficient spare parts are to be carried so that crew can temporarily repair the steering system in a timely manner until permanent repairs can be effected at the nearest port of refuge.
- The steering can be effected manually under the worst anticipated operational conditions (i.e. physically turning the rudder, outboard motor, etc.).

7.7 Mechanical Steering Gears **(2025)**

Where mechanical steering systems are provided, the following are applicable.

7.7.1 Steering Chains and Wire Ropes

Steering chains and wire rope are to be tested, as required by of the ABS *Rules for Materials and Welding (Part 2)*.

7.7.2 Sheaves

Sheaves are to be of ample size and so placed as to provide a fairlead to the quadrant and avoid acute angles. Parts subjected to shock are not to be of cast iron. Guards are to be placed around the sheaves to protect against injury. For sheaves intended for use with ropes, the radius of the grooves is to be equal to that of the rope plus 0.8 mm (1/32 in.), and the sheave diameter is to be determined on the basis of wire rope flexibility. For 6 ´ 37 wire rope, the sheave diameter is to be not less than 18 times that of the rope. For wire ropes of lesser flexibility, the sheave diameter is to be increased accordingly. Sheave diameters for chain are to be not less than 30 times the chain diameter.

7.7.3 Buffers

Steering gears other than hydraulic type are to be designed with suitable buffer arrangement to relieve the gear from shocks to the rudder.

9 Inboard/Outboard and Outboard Installations

9.1 General

Steering installations for Inboard/Outboard and Outboard installations are to be in accordance manufacturer’s recommendations as well as the applicable requirements of this Section. Steering components which are provided as part of the Inboard/Outboard or Outboard installation are not required to be certified under 4-3-3/5.

9.3 Instrumentation

Instrumentation is to be in accordance with 4-3-3/11.9, as applicable, except that a rudder angle indicator is not required for that craft for which the rudder or outboard motor is in a direct line of sight from the navigation bridge and the steering angle can be determined visually.

9.5 Communications (2025)

A means of communication is not required for craft which have a direct line of sight between the main steering control station and the location of the steering gear. Otherwise, a means of communication is to be provided in accordance with 4-3-3/11.7.

9.7 Certification

Steering gears are not required to be certified under 4-3-3/3, 4-3-3/9.1, 4-3-3/9.3, and 4-3-3/15.1.1

9.9 Installation, Tests and Trials

Steering gear operation is to be demonstrated to the satisfaction of the Surveyor in accordance with 4-3-3/11.11, 4-3-3/15.1.2 and 4-3-3/15.3.

11 Waterjets

Waterjets are acceptable based on manufacturer's certification.

Light Warships, Patrol and High-Speed Naval Vessels (Jan 2026)

Part 4 Craft Systems and Machinery

Chapter 3 Propulsion and Maneuvering Machinery

Section 1 Gears

Goals

The gears covered in this Section is to be designed, constructed, operated, and maintained to:

Goal No.	Goal
PROP 1	provide sufficient thrust/power to move or maneuver the vessel when required.
PROP 3	*provide sufficient power for going astern...to secure proper control and bring the ship to rest in all normal circumstances. (SOLAS II-1/Reg 28)*
PROP 4	provide means to maneuver the vessel.
SAFE 1.1	minimize danger to persons on board, the vessel, and surrounding equipment/installations from hazards associated with machinery and systems.

The goals in the cross-referenced Rules /Regulations are also to be met.

Functional Requirements

In order to achieve the above-stated goals, the design, construction, and maintenance of the gears is to be in accordance with the following functional requirements:

Functional Requirement No.	Functional Requirements
PROPULSION, MANEUVERING, AND STATION KEEPING (PROP)	
PROP-FR1	Torque transmission parts are to withstand the design load not only by the steady torque, but also considering the effect from additional stress such as by vibration, shaft alignment and bending moments from gears.
PROP-FR2	Load-bearing parts are to be designed to resist deleterious deflection and prevent fatigue failure during the anticipated design life.
PROP-FR3	Gear teeth are to be designed to withstand the contact stress without progressive pitting or scuffing.

Functional Requirement No.	Functional Requirements
PROP-FR4	Gear teeth roots are to be designed to withstand the bending stress without causing tooth root fillet cracking or fracture.
PROP-FR5	Provide adequate lubrication to prevent friction as well as wear and tear such that the performance and lifespan of the bearings/gears can be maintained.
SAFETY OF PERSONNEL (SAFE)	
SAFE-FR1	Provide access for inspection of gear teeth contacts and bearings clearance for successful performance and operation.

The functional requirements in the cross-referenced Rules are also to be met.

Compliance

A vessel is considered to comply with the goals and functional requirements within the scope of Classification when the applicable prescriptive requirements are complied with or when an alternative arrangement has been approved. Refer to Part 1D, Chapter 2.

1 General

1.1 Application

Gears having a rated power of 100 kW (135 hp) and over, intended for propulsion and for auxiliary services essential for propulsion, maneuvering and safety (see 4-1-1/1.3) of the vessel, are to be designed, constructed, certified and installed in accordance with the requirements of this section.

Gears having a rated power of less than 100 kW (135 hp) are not required to comply with the requirements of this section but are to be designed, constructed and equipped in accordance with good commercial and marine practice. Acceptance of such gears will be based on the manufacturer’s affidavit, verification of gear nameplate data and subject to a satisfactory performance test after installation conducted in the presence of the Surveyor.

Gears having a rated power of 100 kW (135 hp) and over, intended for services considered not essential for propulsion, maneuvering and safety, are not required to be designed, constructed and certified by ABS in accordance with the requirements of this section, but are to be installed and tested to the satisfaction of the Surveyor.

Piping systems of gears, in particular, lubricating oil and hydraulic oil, are addressed in Section 4-6-5 and Section 4-6-6, respectively.

1.3 Definitions

For the purpose of this Section, the following definitions apply:

1.3.1 Gears

The term *Gears* as used in this Section covers external and internal involute spur and helical cylindrical gears having parallel axis as well as bevel gears used for either main propulsion or auxiliary services.

1.3.2 Rated Power

The *Rated Power* of a gear is the maximum transmitted power at which the gear is designed to operate continuously at its rated speed.

1.3.3 Rated Torque

The *Rated Torque* is defined by the rated power and speed and is the torque used in the gear rating calculations.

1.3.4 Gear Rating

The *Gear Rating* is the rating for which the gear is designed in order to carry its rated torque.

1.3.5 Intermittent Duty *(2012)*

Gear units intended for intermittent duty, that is at a higher rating than the rated power defined in 4-3-1/1.3.2 (the rated power at which the gear is designed to operate continuously at its rated speed), are to meet the requirements in 4-3-1/5.17 in addition to all applicable requirements contained in Section 4-3-1. When gear units rated for intermittent duty are installed on a vessel, the vessel's classification and operating profile are to the intermittent rating. Such ratings would normally be considered for only non-commercial services or for commercial service with limited operating time in the intermittent duty service as defined by the mission profile.

1.5 Plans and Particulars to be Submitted

1.5.1 Gear Construction

- General arrangement
- Sectional assembly
- Details of gear casings
- Bearing load diagram
- Quill shafts, gear shafts and hubs
- Shrink fit calculations and fitting instructions
- Pinions
- Wheels and rims
- Details of welded construction of gears

1.5.2 Gear Systems and Appurtenances

- Couplings
- Coupling bolts
- Lubricating oil system and oil spray arrangements

1.5.3 Data **(2020)**

- Transmitted rated power for each gear
- Revolutions per minute for each gear at rated transmitted power
- Bearing lengths and diameters
- Tooth data as per
- Mass of rotating parts
- Balancing data
- Spline data
- Shrink allowance for rims and hubs
- Type of coupling between prime mover and reduction gears
- Type and viscosity of lubricating oil recommended by manufacturer

1.5.4 Material Specifications **(2020)**

The following typical properties of gear materials are to be submitted:

- Range of chemical composition,
- Physical properties at room temperature,
- Endurance limits for pitting resistance, contact stress and tooth root bending stress,

- Heat treatment of gears, coupling elements, shafts, quill shafts, and gear cases.

1.5.5 Hardening Procedure (**2020**)

The hardening procedure for surface hardened gears is to be submitted for review. The submittal is to include materials, details for the procedure itself, quality assurance procedures and testing procedures. Surface hardness depth and core hardness (and their shape) are to be determined from sectioned test samples. These test samples are to be of sufficient size to provide for determination of core hardness and are to be of the same material and heat treated as the gears that they represent.

- Non Destructive Examination
  The non destructive examination procedures are to include surface hardness, roughness of critical fillet. Forgings are to be tested in accordance with Part 2, Chapter 3. Internal soundness verification for gears.

1.5.6 Calculations and Analyses (**2020**)

- Bearing life calculations
- Tooth coupling and spline connection calculations
- Shafts calculation

3 Materials

3.1 Material Specifications and Test Requirements

3.1.1 Material Specifications (**2025**)

Material for gears and gear units is to conform to specifications approved in connection with the design in each case. Specifications other than those given in Chapter 3 of the ABS *Rules for Materials and Welding (Part 2)* conforming to national standard or to Manufacturer standard, are to be submitted for review and are to be approved in connection with the designs. Minimum material properties are to comply with 4-2-3/3.3.1.

Copies of material specifications and purchase orders are to be submitted to the Surveyor for information. The Surveyor will inspect and test material manufactured to other specifications than those given in Chapter 3 of the ABS *Rules for Materials and Welding (Part 2)*, provided that such specifications are approved in connection with the designs and that they are clearly indicated on purchase orders.

3.1.2 Material Certification (**2025**)

For Gear Units for Steering of Azimuthing Thruster see 4-3-1/3.1.3.

For gear units whose power is in excess of 375 kW or more, materials for the following components materials are to be tested in the presence of and inspected by the Surveyor. The test results are to meet the specifications in Part 2, Chapter 3 or that approved in connection with the design.

- Forgings for shafting, couplings, coupling bolts.
- Forgings for through hardened, induction hardened, and nitrided gears.
- Forgings for carburized gears. See .
- Castings approved for use in place of any of the above forgings. (ref ABS Rules for Building and Classing Marine Vessel Section and).
- Plates used for fabricating gears, which transmit torque (refer to Section 2-3-2 of the ABS *Rules for Materials and Welding (Part 2)* or the plates can be manufactured to a recognized standard).

Material for gear units of 375 kW (500 hp) or less, including shafting, gears, couplings, and coupling bolts will be accepted on the basis of the manufacturer’s certified material test reports and a satisfactory surface inspection and hardness check witnessed by the Surveyor.

Materials for power takeoff gears and couplings that are:

- For transmission of power to drive auxiliaries that are for use in port only (e.g., cargo oil pump), and
- Declutchable from propulsion shafting and not essential for propulsion

are accepted on the basis of the manufacturer's certified material test reports and a satisfactory surface inspection and hardness check witnessed by the Surveyor, regardless of power rating.

Non fitted coupling bolts manufactured to a recognized bolt standard do not require material certification, they are to be traceable to material certificate by the manufacturer.

3.1.3 Gear Units for Steering of Azimuthing Thruster **(2025)**

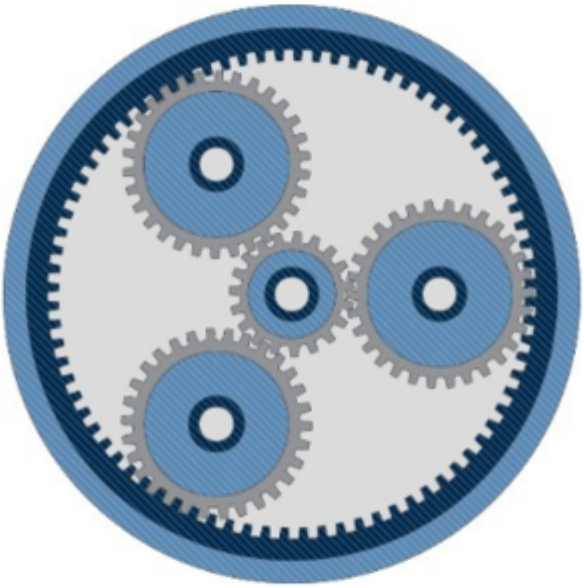
Gears for steering of azimuthing thruster (e.g., planetary gears, worm gears, etc.) are to have their material traceable to Manufacturer certificates except for output pinion and relevant shaft for which materials are to be tested in the presence of the Surveyor.

For small components of less than 50 kg weight, of mass produced gear units:

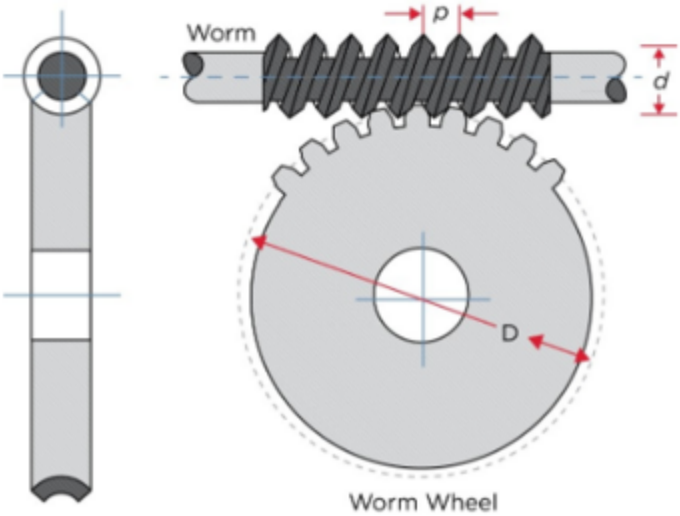
- For castings destructive examination of a sample for each batch for castings to prove casting process is to be carried out.
- For forging or rolled bars refer to ABS Rules for Material and Welding and .

FIGURE 1 (2025)

(a) Example of Planetary Gear Units



(b) Example of Worm Gear Units



3.3 Alternative Material Minimum Requirements (2020)

Materials for torque transmitting components other than grades listed in Part 2 Chapter 3 are to comply with the following minimum properties

3.3.1 Elongation (2020)

Material for torque transmitting components of gear units is to show a minimum elongation at core as follows:

- Case hardened steel 8%
- Surface hardened steel 10%
- Nitriding steels 10%
- Through hardened steels 12%
- For planet carriers and connecting flanges for planetary gears 10%

3.3.2 Cleanliness (2020)

Steel for gears is to conform to cleanliness equivalent to a grade MQ ISO 6336-5.

3.3.3 Heat Treatment (2020)

Heat treatment procedure and relevant controls are to comply with ABS Rules for Material and Welding 2-3-7.

5 Design

5.1 Gear Tooth (2025)

Gear teeth mesh capacity is to be calculated taking into consideration the pitting resistance and bending strength of the teeth. The teeth are to be designed for a load which can be transmitted without causing progressive pitting, and without causing tooth root fillet cracking or fracture. The calculations are to be carried out in accordance with Appendix 4-3-1-A1.

Where carburized case hardened teeth are shot peened at tooth root fillet to increase bending strength, an increase in the *allowable bending stress number* (ref Appendix 4-3-1-A1) of 10% is acceptable provided appropriate tests are carried out to demonstrate adequacy of the shot peening process or the gears have adequate service experience. Where cleaning after shot peening is carried out, an increase in the *allowable bending stress number* above 10% is acceptable provided this is demonstrated by appropriate fatigue testing or the gear design has adequate service experience. Gear drawings recalling shot peening are to specify areas to be peened and areas to be masked, peening intensity and location of intensity verification, media type and size for peening.

5.1.1 Finish (2025)

The gear teeth surface finish is not to be rougher than 1.05 μm (41 μin.) arithmetic or centerline average.
Commentary:

For gears having a rated power below 3728 kW (5000 hp), a surface finish rougher than 1.05 μm (41 μin.) arithmetic or centerline average may be accepted, provided the manufacturer can demonstrate a reliable operating history with similar designs including lubricant.

End of Commentary

5.3 Bearings (2025)

Bearings of gear units are to be so designed and arranged that their design lubrication rate is kept in service under working conditions.

5.3.1 Journal Bearings **(2025)**

For journal bearings, the maximum bearing pressures, minimum oil film thickness, maximum predicted internal bearing temperature and the maximum static unit load are to be in accordance with an applicable standard such as ISO 12130-1 (plain tilting plain thrust bearings), ISO 12131-1 (plain thrust pad bearings), ISO 12167-1 (plain journal bearings with drainage grooves), ISO 12168-1:2001 (plain journal bearings without drainage grooves), ANSI/AGMA 6032-A94, Table 8, etc.

Commentary:

Consideration will be given to bearing pressures exceeding the limits in the standards listed in 4-3-1/5.3.1 and 4-3-1/5.3.2 provided the manufacturer can demonstrate a reliable operating history with similar designs.

End of Commentary

5.3.2 Roller Bearings **(2025)**

The minimum L10 bearing life is not to be less than 20,000 hours for ahead drives and 5,000 hours for astern. Shorter life may be considered in conjunction with an approved bearing inspection and replacement program reflecting the actual calculated bearing life. See 4-3-5/5.9 for application to thrusters. Calculations are to be in accordance with an applicable standard such as ISO 76, ISO 281 (rolling bearings, for static and dynamic ratings, respectively).

5.3.3 Shaft Alignment Analysis **(2020)**

For gear-shafts which are directly connected to the propulsion shafting, the load on the gear shaft bearings is to be evaluated by taking into consideration the loads resulting from the shafting alignment condition. For generic approval in the absence of an alignment calculation, a multiplication factor of 1.1 will be considered for output shaft bearing load.

5.5 Gear Cases **(2025)**

Gear cases are to be of substantial construction in order to minimize elastic deflections and maintain accurate mounting of the gears.

They are to be designed to withstand without deleterious deflection: the tooth forces generated by the gear elements, thrust bearing arrangement, lineshaft alignment, prime mover(s), clutches, couplings and accessories, under all modes of operation.

Additionally, the inertial effects of the gears within the case, due to 1 **g** horizontal and 2 **g** vertical dynamic forces of the ship in a seaway, are to be considered.

Calculations, in accordance with the manufacturer's code of practice, substantiating these Rule requirements are to be submitted.

Commentary:

For gear case designs for which the manufacturer can provide a satisfactory service history, submission of calculations may be waived.

End of Commentary

5.5.1 Oil Sump **(2020)**

Where forced lubrication systems are installed, the position of suction for lubricating oil pump within the gear unit is to be such as to provide adequate immersion when the vessel is inclined, as per maximum inclination specified in 4-1-1/9 TABLE 2.

5.7 Access for Inspection **(2025)**

The construction of gear cases is to be such that a sufficient number of access points are provided for adequate inspection of gears, checking of gear teeth contacts and measurement of thrust bearing clearance. Alternative methods such as use of special viewing devices may be considered.

Commentary:

Alternative methods such as use of special viewing devices may be considered.

End of Commentary

5.9 Calculation of Shafts for Gears

5.9.1 General (**2025**)

The minimum diameter of shafts for gears is to be determined by the following equations:

$$d = k \cdot \sqrt[6]{(bT)^2 + (mM)^2}$$

$$b = 0.073 + \frac{n}{Y}$$

$$m = \frac{c_1}{c_2 + Y}$$

where (in SI (MKS and US units), respectively):

d	=	shaft diameter at section under consideration; mm (in.)
Y	=	yield strength (see 2-1-1/13.3); N/mm^2 (kgf/mm^2, psi)
T	=	torsional moment at rated speed; N-m (kgf-cm, lbf-in) (See also 4-3-1/5.9.7 to account for effect of torsional vibrations, where applicable.)
M	=	bending moment at section under consideration; N-m (kgf-cm, lbf-in)

k, n, c_1 and c_2 are constants given in the following table:

	SI units	MKS units	US unit s
k	5.25	2.42	0.10
n	191.7	19.5	27800
c_1	1186	121	172000
c_2	413.7	42.2	60000

5.9.2 Shaft Diameter in way of Keyways or Splines (**2025**)

The minimum diameter of the shaft in way of keyways or splines is to be increased not less than 10% of the minimum diameter calculated in accordance with 4-3-1/5.9.1.

5.9.3 Maximum Torsional Moment (**2020**)

In determining the required size of the gears, couplings and gear shafting, the maximum torsional moment is the sum of:

i. Maximum steady torque of the operating profile (including astern power where applicable), when it exceeds the transmitted rated torque.

ii. Alternating torques per the torsional vibration calculations.

For generic approval following amplification factor can be considered: (a) For direct diesel engine drives: 1.2 (b) For all other drives, including diesel engine drives with elastic couplings: 1.0

5.9.4 Fatigue Analysis **(2020)**

Where the design of shafts includes geometric discontinuities (e.g., shoulder fillet with radius less than 0.08d), a fatigue analysis is to be submitted by the designer in accordance with ANSI/AGMA 6101 or DIN 743. The safety factor for high cycle fatigue is not to be less than 1.5

5.9.5 Quill Shafts

In the specific case of quill shafts subjected to small stress raisers and no bending moments, the least diameter is to be determined by the following equation:

SI units	MKS units	US units
$d = 5.25 \cdot \sqrt[3]{bT}$	$d = 2.42 \cdot \sqrt[3]{bT}$	$d = 0.1 \cdot \sqrt[3]{bT}$
$b = 0.053 + \frac{187.8}{Y}$	$b = 0.053 + \frac{19.1}{Y}$	$b = 0.053 + \frac{27200}{Y}$

5.9.6 Shaft Couplings

For shaft couplings, coupling bolts, flexible coupling and clutches, see 4-3-2/5.17. For keys, see 4-3-2/5.7.

5.9.7 Vibration *(1 July 2006)*

The designer or builder is to evaluate:

i. The shafting system for different modes of vibrations (torsional, axial, lateral) and the their coupled effect, as appropriate,
ii. The diameter of shafts considering maximum total torque (steady and vibratory torque),
iii. The gears for gear chatter and harmful torsional vibrations stresses. See also .

5.9.8 Shrink Fitted Pinions and Wheels **(2020)**

For pinions and wheels shrink-fitted on shafts, preloading and stress calculations and fitting instructions are to be submitted for review. The torsional holding capacity is to be at least 2.8 times the transmitted mean torque plus vibratory torque due to torsional vibration. For calculation purpose, to take account of torsional vibratory torque, the following factors are to be applied to the transmitted torque, unless the actual measured vibratory torque is higher, in which case the actual vibratory torque is to be used.

- For direct diesel engine drives: 1.2;
- For all other drives, including diesel engine drives with elastic couplings: 1.0.

The preload stress based on the maximum available interference fit or maximum pull-up length is not to exceed 90% of the minimum specified yield strength.

The following friction coefficients are to be used

- Oil injection method of fit: 0.13;
- Dry method of fit: 0.18.

Shrink fit couplings are to be designed to avoid the risk of fretting corrosion. Where the length of contact within the coupling is more than 1.5 times the diameter of the shaft, or the shaft is subject to high bending moment, the shaft diameter is to be increased by 10% in way of the contact surface with the shoulder fillet, with a minimum radius of 0.2 times the shaft's smaller diameter.

5.9.9 Shrink Fitted Wheel Rim

For shrink-fitted wheel rims, preloading and stress calculations and fitting instructions are to be submitted for consideration. The preloading stress based on the maximum interference fit or maximum pull-up length is not to exceed 90% of the minimum specified yield strength.

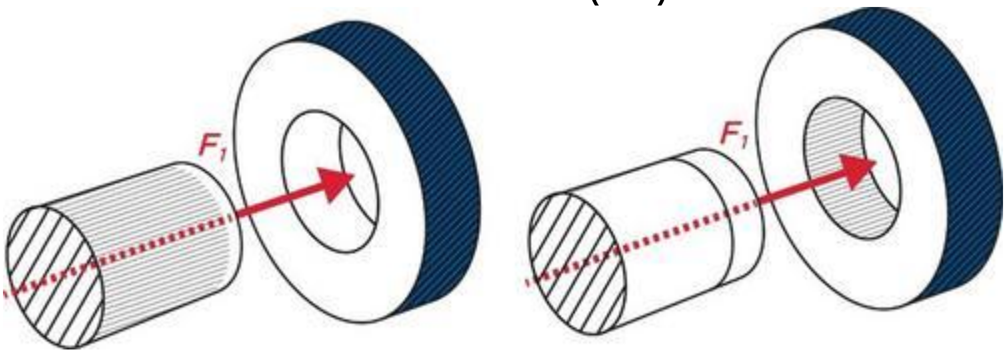
5.9.10 Knurled Shrink Fit **(2020)**

For small gears (pitch diameter less than 400mm) subject to low bending moment (e.g. ring gear of planetary gear units) where it is proposed to shrink a knurled profile into the hub to create a non-demountable coupling, the procedure is to be validated by testing to demonstrate adequate safety against slippage, avoidance of cracks on hub, adequate tolerances after shrinkage.

The difference in hardness between knurled male and hub is to be at least 100 HB

The knurled profile is to be used only to transmit torque. Adequate alignment of the fitted components is to be addressed by the tolerance of the plain cylindrical shoulder.

FIGURE 2
Knurled Shrink Fit (2025)



5.11 Rating of Cylindrical and Bevel Gears

The calculation procedures for the rating of external and internal involute spur and helical cylindrical gears having parallel axes and of bevel gears, with regard to surface durability (pitting) and tooth root bending strength, may be as given in Appendix 4-3-1-A1. These procedures are in substantial agreement with ISO 6336 and ISO-DIS 10300 for cylinder and bevel gears, respectively.

5.13 Alternative Gear Rating Standards *(2018)*

Consideration will be given to gears that are rated based on any of the following alternative standards. In which case, gear rating calculations and justification of the applied gear design coefficients in accordance with the applicable design standard are to be submitted to ABS for review.

Cylindrical Gears	Bevel Gears
ANSI/AGMA 6032	AGMA 2003
ISO 6336	ISO 10300
DIN 3990, Part 31	DIN 3991

5.15 Gears with Multiple **Prime Mover Inputs**

For single helical gears with arrangements utilizing multiple prime mover inputs, and single or multiple outputs, the following analyses for all operating modes are to be conducted:

- All bearing reactions
- Tooth modifications
- Load distributions on the gear teeth
- Contact and tooth root bending stresses

A summary of the results of these analyses for each operating mode is to be submitted for review.

5.17 Rating of Gears for Intermittent Duty *(2012)*

5.17.1 Gears Intended for Intermittent Duty **(2020)**

When gear units rated for intermittent duty are installed as part of the propulsion system of a vessel, the vessel's classification and operating profile are to reflect the intermittent rating. Such ratings would be considered for only non-commercial services or for commercial service with limited operating time in the intermittent duty service as defined by the mission profile.

Gears intended for propulsion and rated for intermittent duty are to have the conditions associated with the intermittent rating clearly defined and listed as part of the rating.

These conditions include, but are not limited to, the specific intermittent rating (power and rpm) or ratings if there will be multiple intermittent ratings, the limit on the number of hours associated with the particular phase of the rating, limitation of load factor or a combination thereof, load spectrum, the time between overhaul and servicing of the unit, and any other conditions that the manufacturer places on the unit.

5.17.2 Calculations **(2020)**

Calculations supporting the intermittent rating or ratings are to be submitted by the manufacturer to ABS for review.

Teeth calculations are to be based on ISO 6336-6 *Calculation of local capacity of spur and helical gears – Calculation of service life under variable load,* or an equivalent recognized national standard that uses a fatigue cycle approach.

Bearings for an intermittent duty gear unit are to be calculated with Miner approach as reported in ISO 281 or similar. Finally, the following is to be considered: maximum speed of bearings, maximum speed of clutches or clutch reduced capacity at maximum speed, pump speed limits, balance.

7 Piping Systems for Gears

The requirements of piping systems essential for operation of gears for propulsion, maneuvering, electric power generation and vessel's safety are in Section 4-6-5 for diesel engine and gas turbine installations. Additionally, requirements for hydraulic and pneumatic systems are provided in Section 4-6-5. Specifically, the following references are applicable:

Lubricating oil system:	4-6-5/7
Cooling system:	4-6-5/9
Hydraulic system:	4-6-6/3
Pneumatic system:	4-6-6/5
Piping system general requirements:	Section 4-6-1 and Section 4-6-2

9 Testing, Inspection and Certification of Gears (2020)

9.1 Material Tests

For testing of materials intended for gear construction, see 4-3-1/3.1 and 4-3-1/3.3.

9.3 Dynamic Balancing (2020)

Gears and wheel assemblies are to be free from injurious unbalance.

Where their pitch line velocity does not exceed 25 m/s (4920 ft/min) or where dynamic balance is impracticable due to size, weight, speed or construction of units, the parts may be statically balanced in a single plane.

The residual unbalance in each plane is not to exceed the value determined by the following equations:

SI units	MKS units	US units
$B = 24 \cdot W/N$	$B = 24000 \cdot W/N$	$B = 15.1 \cdot W/N$

where:

B	=	maximum allowable residual unbalance; N-mm (gf-mm, ozf-in)
W	=	weight of rotating part; N (kgf, lbf)
N	=	rpm at rated speed

For mass produced gear units whose rated power is below 3000 kW and rated speed below 3500 rpm, maximum unbalance of pinion and wheel assemblies can be determined by calculation taking into consideration machining tolerances of wheels, gear and shafts and relevant coupling: this is not applicable to casted, bolted or welded components or components with non-machined or manual-machined surfaces. A noise test and visual examination at maximum speed of the gear unit will be used as confirmation of adequacy of the gear assemblies: the maximum sound pressure level at 1 m from gearbox at a height between 1.2 m and 1.6 m above gear foundation is not to exceed 100 dB(A).

In all other cases, finished pinions and wheels are to be dynamically balanced as follows:

1. In two planes, when their pitch line velocity exceeds 25 m/s (4920 ft./min).
2. Where their pitch line velocity does not exceed 25 m/s (4920 ft./min) or where dynamic balance is impracticable due to size, weight, speed or construction of units, the parts may be statically balanced in a single plane.

9.5 Shop Inspection (2025)

Each gear unit that requires to be certified by 4-3-1/1.1 is to be inspected during manufacture by the Surveyor for conformance with the approved design. This is to include but not limited to checks on gear teeth hardness, and surface finish and dimension checks of main load bearing components. The accuracy of meshing is to be verified for all meshes and the initial tooth contact pattern is to be checked by the Surveyor.

Reports on pinions and wheels balancing as per 4-3-1/9.3 are to be made available to the Surveyor for verification.

9.7 Certification of Gears

9.7.1 General (2020)

Each gear required to be certified by 4-3-1/1.1before being installed is:

i. To have its design approved by ABS; for which purpose, plans and data as required by are to be submitted to ABS for approval;
ii. To be surveyed during its construction, which is to include, but not limited to, material tests as indicated in , meshing accuracy and tooth contact pattern checks as indicated in , gear teeth hardness check, teeth surface roughness, verification of gear backlash, and verification of dynamic balancing or noise test reports, as indicated in .

9.7.2 Approval Under the Type Approval Program

9.7.2(a) Product Design Assessment. **(2020)**

Upon application by the manufacturer, each rating of a type of gear may be design assessed as described in 1C-1-A2/5.1. For this purpose, each rating of a gear type is to be approved in accordance with 4-3-1/9.7.1.i). The prototype test, however, is to be conducted in accordance with an approved test schedule and is to be witnessed by the Surveyor. Gear so approved may be applied to ABS for listing on the ABS website as Products Design Assessed. Once listed, and subject to renewal and updating of certificate as required by 1C-1-A2/5.7, gear particulars will not be required to be submitted to ABS each time the engine is proposed for use onboard a vessel.

9.7.2(b) Mass Produced Gears.

Manufacturer of mass-produced gears, who operates a quality assurance system in the manufacturing facilities, may apply to ABS for quality assurance assessment described in 1C-1-A2/5.5 (PQA).

Upon satisfactory assessment under 1C-1-A2/5.5 (PQA), gears produced in those facilities will not require a Surveyor's attendance at the tests and inspections indicated in 4-3-1/9.7.1.ii). Such tests and inspections are to be carried out by the manufacturer whose quality control documents will be accepted. Certification of each gear will be based on verification of approval of the design and on continued effectiveness of the quality assurance system. See 1C-1-A2/5.7.1(a).

9.7.2(c) Non-mass Produced Gears.

Manufacturer of non-mass produced gears, who operates a quality assurance system in the manufacturing facilities, may apply to ABS for quality assurance assessment described in 1C-1-A2/5.3.1(a) (Manufacturers Procedure) and 1C-1-A2/5.3.1(b) (RQS). Certification to 1C-1-A2/5.5 (PQA) may also be considered in accordance with 4-1-1/9 TABLE 2.

9.7.2(d) Type Approval Program. **(2020)**

Gear types which have their ratings approved in accordance with 4-3-1/9.7.2(a) and the quality assurance system of their manufacturing and heat treatment facilities approved in accordance with 4-3-1/9.7.2(b) or 4-3-1/9.7.2(c) will be deemed Type Approved and will be eligible for listing on the ABS website as Type Approved.

9.9 Shipboard Trials for Propulsion Gears (2020)

After installation on board a vessel, the gear unit is to be operated in the presence of the Surveyor to demonstrate its ability to function satisfactorily under operating conditions and its freedom from harmful vibration at vessel speeds within the operating range. When the propeller or waterjet shafting is driven through reduction gears, the Surveyor is to ascertain that no gear-tooth chatter occurs throughout the operating range; otherwise, a barred speed range, as specified in 4-3-2/5.5, is to be provided.

For conventional propulsion gear units above 1120 kW (1500 hp), a record of gear-tooth contact is to be made at the trials. To facilitate the survey of extent and uniformity of gear-tooth contact, selected bands of pinion or gear teeth on each meshing are to be coated beforehand with copper or layout dye. See also 7-6-2/1.1.2 of the ABS *Rules for Survey After Construction (Part 7)* for the first annual survey after the vessel enters service.

Post-trial examination of spur and helical type gears is to indicate essentially uniform contact across 90% of the effective face width of the gear teeth, excluding end relief.

The gear-tooth examination for spur and helical type gear units of 1120 kW (1500 hp) and below, all epicyclical gear units and bevel type gears will be subject to special consideration. The gear manufacturers' recommendations will be considered.

11 Spare Parts

Spare parts are not required for purposes of classification.

Appendix 1 Rating of Cylindrical and Bevel Gears

1 Application

The following calculation procedures cover the rating of external and internal involute spur and helical cylindrical gears having parallel axis, and of bevel gears with regard to surface durability (pitting) and tooth root bending strength.

For normal working pressure angles in excess of 25° or helix angles in excess of 30°, the results obtained from these calculation procedures are to be confirmed by experience data which are to be submitted by the manufacturer.

The influence factors are defined regarding their physical interpretation. Some of the influence factors are determined by the gear geometry or have been established by conventions. These factors are to be calculated in accordance with the equations provided.

Other influence factors, which are approximations, and are indicated as such, can be calculated according to alternative methods for which engineering justification is to be provided for verification.

3 Symbols and Units (2025)

The main symbols used are listed below. Symbols specifically introduced in connection with the definition of influence factors are described in the appropriate sections.

Units of calculations are given in the sequence of SI units (MKS units, and US units.)

a	center distance	mm (in.)
b	common facewidth	mm (in.)
b_1, b_2	facewidth of pinion, wheel	mm (in.)
b_{eH}	effective facewidth	mm (in.)
b_s	web thickness	mm (in.)
b_B	facewidth of one helix on a double helical gear	mm (in.)
d_1, d_2	reference diameter of pinion, wheel	mm (in.)
d_{a1}, d_{a2}	tip diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{b1}, d_{b2}	base diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{f1}, d_{f2}	root diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{i1}, d_{i2}	rim inside diameter of pinion, wheel (refer to 4-3-1-A1/23.29 FIGURE 5)	mm (in.)
d_{sh}	external diameter of shaft	mm (in.)
d_{shi}	internal diameter of hollow shaft	mm (in.)
d_{w1}, d_{w2}	working pitch diameter of pinion, wheel	mm (in.)

f_{fa1}, f_{fa2}	profile form deviation of pinion, wheel	mm (in.)
f_{pb1}, f_{pb2}	transverse base pitch deviation of pinion, wheel	mm (in.)
h_1, h_2	tooth depth of pinion, wheel	mm (in.)
h_{a1}, h_{a2}	addendum of pinion, wheel	mm (in.)
h_{a01}, h_{a02}	addendum of tool of pinion, wheel	mm (in.)
h_{f1}, h_{f2}	dedendum of pinion, wheel	mm (in.)
h_{fp1}, h_{fp2}	dedendum of basic rack of pinion, wheel	mm (in.)
h_{F1}, h_{F2}	bending moment arm for tooth root bending stress for application of load at the outer point of single tooth pair contract for pinion, wheel	mm (in.)
ℓ	bearing span	mm (in.)
ℓ_b	length of contract	mm (in.)
m_n	normal module	mm (in.)
m_{na}	outer normal module	mm (in.)
m_t	transverse module	mm (in.)
n_1, n_2	rotational speed of pinion, wheel	rpm
p_d	outer diametral pitch	mm^{-1} (in^{-1})
p_{r01}, p_{r02}	protuberance of tool for pinion, wheel	mm (in.)
q_1, q_2	machining allowance of pinion, wheel	mm (in.)
S_{Fn1}, S_{Fn2}	tooth root chord in the critical section of pinion, wheel	mm (in.)
S	distance between mid-plane of pinion and the middle of the bearing span	mm (in.)
U	gear ratio	---
V	tangential speed at reference diameter	m/s (m/s, ft/min)
x_1, x_2	addendum modification coefficient of pinion, wheel	---
x_{sm}	tooth thickness modification coefficient (midface)	---
z_1, z_2	number of teeth of pinion, wheel	---
z_{n1}, z_{n2}	virtual number of teeth of pinion, wheel	---
B	total facewidth, of double helical gear including gap width	mm (in.)
F_{bt}	nominal tangential load on base cylinder in the transverse section	N (kgf, lbf)
F_t	nominal transverse tangential load at reference cylinder	N (kgf, lbf)
P	transmitted rated power	kW (mhp, hp)
Q	ISO grade of accuracy	---

R	cone distance	mm (in.)
R_m	middle cone distance	mm (in.)
R_{zf1}, R_{zf2}	flank roughness of pinion, wheel	$\mu m \ (\mu in.)$
R_{zr1}, R_{zr2}	fillet roughness of pinion. wheel	$\mu m \ (\mu in.)$
T_1, T_2	nominal torque of pinion, wheel	N-m (kgf-m, lbf-ft)
U	minimum ultimate tensile strength of core (applicable to through hardened, normalized and cast gears only)	N/mm^2 (kgf/mm^2, lbf/in^2)
$\alpha_{en1}, \alpha_{en2}$	form-factor pressure angle: pressure angle at the outer point of single pair tooth contact for pinion, wheel	degrees
$\alpha_{Fen1}, \alpha_{Fen2}$	load direction angle: relevant to direction of application of load at the outer point of single pair tooth contact of pinion, wheel	degrees
α_n	normal pressure angle at reference cylinder	degrees
α_t	transverse pressure angle at reference cylinder	degrees
α_{vt}	transverse pressure angle of virtual cylindrical gear	degrees
α_{wt}	transverse pressure angle at working pitch cylinder	degrees
β	helix angle at reference cylinder	degrees
β_b	helix angle at base cylinder	degrees
β_{vb}	helix angle at base circle	degrees
δ_1, δ_2	reference cone angle of pinion, wheel	degrees
δ_{a1}, δ_{a2}	tip angle of pinion, wheel	degrees
ϵ_a	transverse contact ratio	---
ϵ_β	overlap ratio	---
ϵ_γ	total contact ratio	---
ρ_{a01}, ρ_{a02}	tip radius of tool of pinion, wheel	mm (in.)
ρ_c	radius of curvature at pitch surface	mm (in.)
ρ_{fp}	root radius of basic rack	mm (in.)
ρ_{F1}, ρ_{F2}	root fillet radius at the critical section of pinion, wheel	mm (in.)
Subscripts	1 = pinion; 2 = wheel; 0 = tool	

5 Geometrical Definitions

For internal gearing $z_2, a, d_2, d_{a2}, d_{b2}, d_{w2}$ and u are negative.

The pinion is defined as the gear with the smaller number of teeth. Therefore the absolute value of the gear ratio, defined as follows, is always greater or equal to the unity:

$$u = z_2/z_1 = d_{w2}/d_{w1} = d_2/d_1$$

In the equation of surface durability, b is the common facewidth on the pitch diameter.

In the equation of tooth root bending stress, $b_1 \mathrm{or} b_2$ are the facewidths at the respective tooth roots. In any case, $b_1 \mathrm{and} b_2$ are not to be taken as greater than b by more than one normal module m_n on either side.

The common facewidth b may be used also in the equation of teeth root bending stress if significant crowning or end relief have been applied.

Additional geometrical definitions are given in the following expressions.

$$\tan\alpha_t = \tan\alpha_n / \cos\beta$$

$$\tan\beta_b = \tan\beta \cdot \cos\alpha_t$$

$$d_{1,2} = z_{1,2} \cdot m_n / \cos\beta$$

$$d_{b1,2} = d_{1,2} \cdot \cos\alpha_t = d_{w1,2} \cdot \cos\alpha_{tw}$$

$$a = 0.5(d_{w1} + d_{w2})$$

$$z_{n1,2} = z_{1,2} / (\cos^2\beta_b \cdot \cos\beta)$$

$$m_t = m_m / \cos\beta$$

$$\mathrm{inv}\alpha = \tan\alpha - \pi \cdot \alpha / 180, \alpha \mathrm{in degrees}$$

$$\mathrm{inv}\alpha_{wt} = \mathrm{inv}\alpha_t + 2 \cdot \tan\alpha_n \cdot \frac{(x_1 + x_2)}{(z_1 + z_2)}$$

$$\alpha_{wt} = \arccos\left(\frac{d_{b1} + d_{b2}}{2 \cdot a}\right), \alpha_{wt} \mathrm{in degrees}$$

$$x_1 + x_2 = \frac{(z_1 + z_2) \cdot (inv\alpha_{wt} - \mathrm{inv}\alpha_t)}{2 \cdot \tan\alpha_n}$$

$$x_1 = \frac{h_{a0}}{m_n} - \frac{d_1 - d_{f1}}{2 \cdot m_n}, x_2 = \frac{h_{a0}}{m_n} - \frac{d_2 - d_{f2}}{2 \cdot m_n}$$

$$\epsilon_a = \frac{0.5 \cdot \sqrt{d_{a1}^2 - d_{b1}^2} \pm 0.5 \cdot \sqrt{d_{a2}^2 - d_{b2}^2} - a \cdot \sin\alpha_{wt}}{\pi \cdot m_n \cdot \frac{\cos\alpha_t}{\cos\beta}}$$

(The positive sign is to be used for external gears; the negative sign for internal gears.)

$$\epsilon_\beta = \frac{b \cdot \sin\beta}{\pi \cdot m_n}$$

(For double helix, b is to be taken as the width of one helix.)

$$\epsilon_\gamma = \epsilon_a + \epsilon_\beta$$

$$\rho_c = \frac{a \cdot \sin\alpha_{wt} \cdot u}{\pi \cdot m_n}$$

$$v = d_{1,2} \cdot n_{1,2}/19099[SI\ and\ MKS\ units]$$

$$v = d_{1,2} \cdot n_{1,2}/3.82US\ units]$$

7 Bevel Gear Conversion and Specific Formulas *(2006)*

Conversion of bevel gears to virtual (equivalent) cylindrical gears is based on the bevel gear midsection.

Index v refers to the virtual (equivalent) cylindrical gear.

Index m refers to the midsection of bevel gear.

δ_1, δ_2	=	pitch angle pinion, wheel
δ_{a1}, δ_{a2}	=	face angle pinion, wheel
$\sum$	=	shaft angle
β_m	=	mean spiral angle
$d_{e1,2}$	=	outer pitch diameter pinion, wheel
$d_{m1,2}$	=	mean pitch diameter pinion, wheel
$d_{v1,2}$	=	reference diameter of virtual cylindrical gear pinion, wheel
$R_{e1,2}$	=	outer cone distance pinion, wheel
R_m	=	mean cone distance

Number of teeth of virtual cylindrical gear:

$$z_{v1} = \frac{z_{v1}}{\cos\delta_1}$$

$$z_{v2} = \frac{z_{v2}}{\cos\delta_2}$$

- For $\sum = 90^\circ$:

$$z_{v1} = z_1 \frac{\sqrt{u^2+1}}{u}$$

$$z_{v2} = z_2 \frac{\sqrt{u^2+1}}{u}$$

Gear ratio of virtual cylindrical gear

$$u_v = \frac{z_{v2}}{z_{v1}}$$

- For $\sum = 90^\circ$:

$$u_v = u^2$$

Geometrical definitions:

$$\delta_1 + \delta_1 = \Sigma$$

$$\mathbf{tan}\alpha_{vt} = \frac{\mathbf{tan}\alpha_n}{\cos\beta_m}$$

$$\mathbf{tan}\beta_{bm} = \mathbf{tan}\beta_m \cdot \cos\alpha_{vt}$$

$$\beta_{vb} = \mathbf{arcsin}\left(\sin\beta_m \cdot \cos\alpha_n\right)$$

$$R_e = \frac{d_{e1,2}}{2 \cdot \sin\delta_{1,2}}$$

$$R_m = R_e - \frac{b}{2}, b \leq \frac{R}{3}$$

Reference diameter of pinion, wheel refers to the midsection of the bevel gear:

$$d_{m1} = d_{e1} - b \cdot \sin\delta_1$$

$$d_{m2} = d_{e2} - b \cdot \sin\delta_2$$

Modules:

Outer transverse module:

$$m_{et} = \frac{d_{e2}}{z_2} = \frac{d_{e1}}{z_1} = \frac{25.4}{p_d}$$

Outer normal module:

$$m_{na} = m_t \cdot \cos\beta_m$$

Mean normal module:

$$m_{mn} = m_{mt} \cdot \cos\beta_m$$

$$m_{mn} = m_{et} \cdot \frac{R_m}{R_e} \cdot \cos\beta_m$$

$$m_{mn} = \frac{d_{m1}}{z_1} \cdot \cos\beta_m$$

$$m_{mn} = \frac{d_{m2}}{z_2} \cdot \cos\beta_m$$

Base pitch:

$$p_{btm} = \frac{\pi \cdot m_{mn} \cdot \cos\alpha_{vt}}{\cos\beta_m}$$

Reference diameter of pinion, wheel refers to the virtual (equivalent) cylindrical gear:

$$d_{v1} = \frac{d_{m1}}{\cos\delta_1}$$

$$d_{v2} = \frac{d_{m2}}{\cos\delta_2}$$

Base diameter of pinion, wheel

$$d_{vb1} = d_{v1} \cdot \cos\alpha_{vt}$$

$$d_{vb2} = d_{v2} \cdot \cos\alpha_{vt}$$

Center distance of virtual cylindrical gear:

$$a_v = 0.5 \cdot (d_{v1} + d_{v2})$$

Transverse pressure angle of virtual cylindrical gear:

$$\alpha_{vt} = \arccos\left(\frac{d_{vb1} + d_{vb2}}{2 \cdot a_v}\right), \alpha_{vt}\text{in degrees}$$

Mean Addendum:

For gears with constant addendum *Zyklo-Palloid(Klingelnberg)*:

$$h_{am1} = m_{mn} \cdot (1 + x_{hm1})$$

$$h_{am2} = m_{mn} \cdot (1 + x_{hm2})$$

For gears with variable addendum *(Gleason):*

$$h_{am1} = h_{ae1} - \frac{b}{2} \cdot \tan(\delta_{a1} - \delta_1)$$

$$h_{am2} = h_{ae2} - \frac{b}{2} \cdot \tan(\delta_{a2} - \delta_2)$$

where h_{ae} is the outer addendum.

Profile shift coefficients:

$$x_{hm1} = \frac{h_{am1} - h_{am2}}{2 \cdot m_{mn}}$$

$$x_{hm2} = \frac{h_{am2} - h_{am1}}{2 \cdot m_{mn}}$$

Mean Dedendum:

For gears with constant dedendum *Zyklo-Palloid (Klingelnberg):*

$$h_{fp} = (1.25\ldots1.30)\cdot m_{mn}$$

$$\rho_{fp} = (0.2\ldots0.3)\cdot m_{mn}$$

For gears with variable dedendum (*Gleason):*

$$h_{fm1} = h_{fe1} - \frac{b}{2}\cdot\tan(\delta_{a1}-\delta_1)$$

$$h_{fm2} = h_{fe2} - \frac{b}{2}\cdot\tan(\delta_{a2}-\delta_2)$$

$$h_{f1} = h_{fm1} + x_{hm1}\cdot m_{mn}$$

$$h_{f2} = h_{fm2} + x_{hm2}\cdot m_{mn}$$

where h_{fe} is the outer dedendum and h_{fm} is the mean dedendum.

Tip diameter of pinion, wheel:

$$d_{va1} = d_{v1} + 2\cdot h_{am1}$$

$$d_{va2} = d_{v2} + 2\cdot h_{am2}$$

Transverse contact ratio:

$$\epsilon_{va} = \frac{0.5\cdot\sqrt{d_{va1}^2 - d_{vb1}^2} + 0.5\cdot\sqrt{d_{va2}^2 - d_{vb2}^2} - a_v\cdot\sin\alpha_{vt}}{\pi\cdot m_{mn}\cdot\frac{\cos\alpha_{vt}}{\cos\beta_m}}$$

Overlap ratio:

$$\epsilon_{v\beta} = \frac{b\cdot\sin\beta_m}{\pi\cdot m_{mn}}$$

Modified contact ratio:

$$\epsilon_{v\gamma} = \sqrt{\epsilon_{va}^2 + \epsilon_{v\beta}^2}$$

Tangential speed at midsection:

$$v_{mt} = \frac{d_{m1,2}\cdot n_{1,2}}{19098}\ m/s\ [SI\ and\ MKS\ units]$$

$$v_{mt} = \frac{d_{m1,2}\cdot n_{1,2}}{3.82}\ ft/min\ [US\ units]$$

Radius of curvature (normal section):

$$\rho_{vc} = \frac{a_v\cdot\sin\alpha_{vt}}{\cos\beta_{bm}}\cdot\frac{u_v}{(1+u_v)^2}$$

Length of the middle line of contact:

$$\ell_{bm} = \frac{b\cdot\epsilon_{va}}{\cos\beta_{vb}}\cdot\frac{\sqrt{\epsilon_{v\gamma}^2 - [(2-\epsilon_{va})\cdot(1-\epsilon_{v\beta})]^2}}{\epsilon_{v\gamma}^2}\ \text{for}\epsilon_{v\beta} < 1$$

$$\ell_{bm} = \frac{b \cdot \epsilon_{va}}{\epsilon_{v\gamma} \cdot \cos\beta_{vb}} \text{ for} \epsilon_{v\beta} < 1$$

9 Nominal Tangential Load, F_t, F_{mt}

The nominal tangential load, F_t or F_{mt} , tangential to the reference cylinder and perpendicular to the relevant axial plane, is calculated directly from the rated power transmitted by the gear by means of the following equations:

	SI units	*MKS units*	*US units*
	$T_{1,2} = 9549 \cdot P/n_{1,2}$ N-m	$T_{1,2} = 716.2 \cdot P/n_{1,2}$ kgf-m	$T_{1,2} = 5252 \cdot P/n_{1,2}$ lbf-ft
Cylindrical gears	$F_t = \frac{2000T_{1,2}}{d_{1,2}} = \frac{19.1P \times 10^6}{n_{1,2}d_{1,2}}$ N	$F_t = \frac{2000T_{1,2}}{d_{1,2}} = \frac{1.4325P \times 10^6}{n_{1,2}d_{1,2}}$ kgf	$F_t = \frac{24T_{1,2}}{d_{1,2}} = \frac{126.05P \times 10^3}{n_{1,2}d_{1,2}}$ lbf
Bevel gears	$F_{mt} = \frac{2000T_{1,2}}{d_{m1,2}} = \frac{19.1P \times 10^6}{n_{1,2}d_{m1,2}}$ N	$F_{mt} = \frac{2000T_{1,2}}{d_{m1,2}} = \frac{1.4325P \times 10^6}{n_{1,2}d_{m1,2}}$ kgf	$F_{mt} = \frac{24T_{1,2}}{d_{m1,2}} = \frac{126.05P \times 10^3}{n_{1,2}d_{m1,2}}$ lbf

11 Application Factor, K_A

The application factor, K_A , accounts for dynamic overloads from sources external to the gearing.

The application factor, K_A , for gears designed for infinite life, is defined as the ratio between the maximum repetitive cyclic torque applied to the gear set and the nominal rated torque.

The factor mainly depends on:

- Characteristics of driving and driven machines;
- Ratio of masses;
- Type of couplings;
- Operating conditions as e.g., overspeeds, changes in propeller load conditions.
  When operating near a critical speed of the drive system, a careful analysis of these conditions is to be made.

  The application factor, K_A , is to be determined by measurements or by appropriate system analysis.

  Where a value determined in such a way cannot be provided, the following values are to be used

a)	*Main propulsion gears:*	
	Turbine and electric drive:	1.00
	Diesel engine with hydraulic or electromagnetic slip coupling:	1.00
	Diesel engine with high elasticity coupling:	1.30
	Diesel engine with other couplings:	1.50
b)	*Auxiliary gears:*	
	Electric motor, diesel engine with hydraulic or electromagnetic slip coupling:	1.00
	Diesel engine with high elasticity coupling:	1.20
	Diesel engine with other couplings:	1.40

13 Load Sharing Factor, $K\gamma$ *(2013)*

The load sharing factor $K\gamma$ accounts for the maldistribution of load in multiple path transmissions as e.g., dual tandem, epicyclical, double helix.

The load sharing factor $K\gamma$ is defined as the ratio between the maximum load through an actual path and the evenly shared load. The factor mainly depends on accuracy and flexibility of the branches.

The load sharing factor $K\gamma$ is to be determined by measurements or by appropriate system analysis. Where a value determined in such a way cannot be provided, the following values are to be used:

a)	*Epicyclical gears:*	
	Up to 3 planetary gears:	1.00
	4 planetary gears:	1.20
	5 planetary gears:	1.30
	6 planetary gears and over:	1.40
b)	*Other gear arrangements including bevel gears:*	1.00

15 Dynamic Factor, K_v

The dynamic factor, K_v , accounts for internally generated dynamic loads due to vibrations of pinion and wheel against each other.

The dynamic factor, K_v , is defined as the ratio between the maximum load which dynamically acts on the tooth flanks and the maximum externally applied load ($F_t \cdot K_A \cdot K_\gamma$).

The factor mainly depends on:

- Transmission errors (depending on pitch and profile errors)
- Masses of pinion and wheel
- Gear mesh stiffness variation as the gear teeth pass through the meshing cycle
- Transmitted load including application factor
- Pitch line velocity
- Dynamic unbalance of gears and shaft
- Shaft and bearing stiffnesses
- Damping characteristics of the gear system

  The dynamic factor, K_v , is to be advised by the manufacturer as supported by his measurements, analysis or experience data or is to be determined as per , except that where $vz_1/100$ is 3 m/s (590 ft/min.) or above, K_v may be obtained from .

15.1 Determination of K_v **– Simplified Method**

Where all of the following four conditions are satisfied, K_v may be determined in accordance with 4-3-1-A1/15.1.

a. Steel gears of heavy rims sections.

b. Values of F_t/b are in accordance with the following table:

SI units	MKS units	US units
> 150 N/mm	> 15 kgf/mm	> 856 lbf/in

c. $z_1 < 50$ d

d. Running speeds in the subcritical range are in accordance with the following table:

	$(v \cdot z_1)/100$	
	SI & MKS units	US units
Helical gears	< 14 m/s	< 2756 ft/min
Spur gears	< 10 m/s	< 1968 ft/min
All types of gears	< 3 m/s	< 590 ft/min

For gears other than specified above, the single resonance method, as per below, may be applied. The methods of calculation are as follows:

	SI and MKS units	US units
For helical gears of overlap ratio $\epsilon_\beta \geq$ unity	$K_v = K_{vh} = 1 + K_1 \cdot v \cdot z_1/100$	$K_v = K_{vh} = 1 + K_1 \cdot v \cdot z_1/19685$
For helical gears of overlap ratio $\epsilon_\beta <$ unity	K_v is obtained by means of linear interpolation: $K_v = K_{vs} = \epsilon_\beta(K_{vs} - K_{vh})$	
For spur gears	$K_v = K_{vs} = 1 + K_1 \cdot v \cdot z_1/100$	$K_v = K_{vs} = 1 + K_1 \cdot v \cdot z_1/19685$
For bevel gears	In the above conditions (b, c and d) and in the above formulas: ° The real z_1 is to be used instead of the virtual (equivalent) z_{v1} ; ° v is to be substituted by v_{mt} ; (tangential speed at midsection); and ° F_t is to be substituted by F_{mt} .	
For all gears	K_1 values are specified in table below.	

	K_1 for different values of Q (ISO Grades of accuracy)					
	3	4	5	6	7	8
Spur gears	0.0220	0.0300	0.0430	0.0620	0.0920	0.1250
Helical gears	0.0125	0.0165	0.0230	0.0330	0.0480	0.0700
Q is to be according to ISO 1328. In case of mating gears with different grades of accuracy, the grade corresponding to the lower accuracy is to be used.						

15.3 Determination of K_v – Single Resonance Method

For single stage gears, the dynamic factor K_v may be determined from 4-3-1-A1/15.3.3 through 4-3-1-A1/15.3.6 for the ratio *N* in 4-3-1-A1/15.3.1 and using the factors given in 4-3-1-A1/15.3.2.

15.3.1 Resonance Ratio, *N (2006)*

$$N = \frac{n_1}{n_{E1}}$$

$$n_{E1} = \frac{30 \times 10^3}{\pi \cdot z_1} \cdot \sqrt{\frac{C\gamma}{m_{red}}} \text{rpm}[SI\ units]$$

$$n_{E1} = \frac{93.947 \times 10^3}{\pi \cdot z_1} \cdot \sqrt{\frac{C\gamma}{m_{red}}} \text{rpm}[MKS\ units]$$

$$n_{E1} = \frac{589.474 \times 10^3}{\pi \cdot z_1} \cdot \sqrt{\frac{C\gamma}{m_{red}}} \text{rpm}[US\ units]$$

where

m_{red} = relative reduced mass of the gear pair, per unit facewidth referred to the line of action:

m_{red} = $$\frac{\pi}{8} \cdot \left(\frac{d_{m1}}{d_{b1}}\right)^2 \cdot \left[\frac{d_{m1}^2}{(1/(1-q_1^4) \cdot \rho_1) + (1/(1-q_2^4) \cdot \rho_2 \cdot u^2)}\right] kg/mm\ (lb/in)$$

m_{red} = $$\frac{J_1 \cdot J_2}{p \cdot J_1 \cdot \left(\frac{d_{b2}}{2}\right)^2 + J_2 \cdot \left(\frac{d_{b1}}{2}\right)^2} kg/mm\ (lb/in)$$

p = for planetary gears, number of planets

$$d_{m1,m2} = \frac{d_{a1,a2} + d_{f1,f2}}{2} mm\ (in.)$$

$$q_1 = \frac{d_{i1}}{d_{m1}}; q_2 = \frac{d_{i2}}{d_{m2}}$$

for reference, see 4-3-1-A1/23.29 FIGURE 5

J_1 = moment of inertia per unit facewidth for pinion:

= $$\frac{\pi \cdot \rho_1 \cdot d_{b1}^4}{32} \text{kg-mm}^2/mm\ (lb - in^2/in)$$

J_2 = moment of inertia per unit facewidth for wheel:

J_2 = $$\frac{\pi \cdot \rho_2 \cdot d_{b2}^4}{32} \text{kg-mm}^2/mm\ (lb - in^2/in)$$

$\rho_{1,2}$	=	density of pinion, wheel materials
ρ	=	density of steel material:
	=	$7.81 \times 10^{-6}\mathrm{kg/mm}^3[SI\ and\ MKS\ units]$
	=	$2.83 \times 10^{-1}\mathrm{lb/in}^3[US\ units]$

Bevel gears:

For bevel gears, the real z_1 (not the equivalent) is to be inserted in the above formulas. Determination of m_{red} is to be as follows:

$$m^*_{1,2} = \frac{\pi}{8} \cdot \rho_{1,2} \cdot \left[\frac{d^2_{m1,2}}{\cos^2 a_n}\right] kg/mm\ (lb/in.)$$

$$m_{red} = \frac{m^*_1 \cdot m^*_2}{m^*_1 + m^*_2}\ kg/mm\ (lb/in.)$$

Mesh stiffness per unit facewidth, C_γ :

$$C_\gamma = \frac{20}{0.85} \cdot B_b \mathrm{N/mm}\text{-}\mu\ \mathrm{m}[SI\ units]$$

$$C_\gamma = \frac{2.039}{0.85} \cdot B_b \mathrm{kgf/mm}\text{-}\mu\ \mathrm{m}[MKS\ units]$$

$$C_\gamma = \frac{2.901}{0.85} \cdot B_b \mathrm{lbf/in}\text{-}\mu\ \mathrm{in}[US\ units]$$

Tooth stiffness of one pair of teeth per unit facewidth (single stiffness), c' :

$$c' = \frac{14}{0.85} \cdot B_b \mathrm{N/mm}\text{-}\mu\ \mathrm{m}[SI\ units]$$

$$c' = \frac{1.428}{0.85} \cdot B_b \mathrm{kgf/mm}\text{-}\mu\ \mathrm{m}[MKS\ units]$$

$$c' = \frac{2.031}{0.85} \cdot B_b \mathrm{lbf/in}\text{-}\mu\ \mathrm{in}[US\ units]$$

For a combination of different materials for pinion and wheel, c' is to be multiplied by ξ ,

where

$$\xi = \frac{E}{E_{st}}$$

$$E = \frac{2E_1E_2}{E_1 + E_2}$$

where values of E_1 and E_2 are to be obtained from 4-3-1-A1/23.29 TABLE 1

E_{st} = Young's modulus of steel; see 4-3-1-A1/23.29 TABLE 1

Overall facewidth, B_b :

$$B_b = \frac{b_{eH}}{b}$$

where: $b_{eH} =$ effective facewidth, mm (in.)

Note: Higher values than $B_b = 0.85$ are not to be used.

Cylindrical gears:

Mesh stiffness per unit facewidth, C_γ :

$$C_\gamma = c' \cdot (0.75 \cdot \epsilon_a + 0.25) N/mm - \mu m \ (kgf/mm - \mu m,\ lbf/in - \mu in)$$

Tooth stiffness of one pair of teeth per unit facewidth (single stiffness), c' :

$$c' = 0.8 \cdot \frac{\cos\beta}{q'} C_{BS} \cdot C_R \mathrm{N/mm\text{-}}\mu \mathrm{\ m}[SI\ units]$$

$$c' = 8.158 \cdot 10^{-2} \cdot \frac{\cos\beta}{q'} \cdot C_{BS} \cdot C_R \mathrm{kgf/mm\text{-}}\mu \mathrm{\ m}[MKS\ units]$$

$$c' = 11.603 \cdot 10^{-2} \cdot \frac{\cos\beta}{q'} \cdot C_{BS} \cdot C_R \mathrm{lbf/in\text{-}}\mu \mathrm{\ in}[US\ units]$$

For a combination of different materials for pinion and wheel, c' is to be multiplied by ξ , where

$$\xi = \frac{E}{E_{st}}$$

$$E = \frac{2E_1E_2}{E_1 + E_2}$$

where values of E_1 and E_2 are to be obtained from 4-3-1-A1/23.29 TABLE 1

E_{st} = Young's modulus of steel; see 4-3-1-A1/23.29 TABLE 1

$$q' = 0.04723 + \frac{0.15551}{z_{n1}} + \frac{0.25791}{z_{n2}} - 0.00635 \cdot x_1 - \frac{0.11654 \cdot x_1}{z_{n1}} - 0.00193 \cdot x_2 - \frac{0.24188 \cdot x_2}{z_{n2}} + 0.00529 \cdot x_1^2 + 0.00182 \cdot x_2^2$$

$$z_{n1} = \frac{z_1}{\cos^2\beta_b \cdot \cos\beta}$$

$$z_{n2} = \frac{z_2}{\cos^2\beta_b \cdot \cos\beta}$$

Note: For internal gears, use $z_{n2} (= \infty)$ equal infinite and $x_2 = 0$.

$$C_{BS} = \left[1 + 0.5 \cdot \left(1.2 - \frac{h_{fP}}{m_n}\right)\right] \cdot [1 - 0.0 \cdot (20 - a_n)]$$

When the pinion basic rack dedendum is different from that of the wheel, the arithmetic mean of C_{BS1} for a gear pair conjugate to the pinion basic rack and C_{BS2} for a gear pair conjugate to the basic rack of the wheel:

$$C_{BS} = 0.5 \cdot (C_{BS1} + C_{BS2})$$

Gear blank factor, C_R :

$$C_R = 1 + \frac{\ln(b_s/b)}{5 \cdot e^{(s_R/5 \cdot m_n)}}$$

when:

1. $b_s/b < 0.2, \text{use } 0.2 \text{ for} b_s/b$
2. $b_s/b < 1.2, \text{use } 1.2 \text{ for} b_s/b$
3. $S_R/m_n < 1, \text{use } 0.2 \text{ for} S_R/m_n$

For b_s and S_R see 4-3-1-A1/23.29 FIGURE 3.

15.3.2 Factors $B_p, B_j,$ *and* B_k *(2006)*

a. Values for f_{pt} and y_a according to ISO grades of accuracy Q :

$Q^{(1)}$		3	4	5	6	7	8	9	10	11(4)	12
f_{pt}	µm	3	6	12	25	45	70	100	150	201	282
	µin	118	236	472	984	1772	2756	3937	5906	7913	11102
$y_a^{(2)}$	µm	0	0.5	1.5	4	7	15	25	40	55	75
	µin	0	19.7	59.1	157	276	591	984	1575	2165	2953

Notes:

1. ISO grades of accuracy according to ISO 1328. In case of mating gears with different grades of accuracy, the grade corresponding to the lower accuracy is to be used.
2. For specific determination of y_a , see b) through d) below.
3. Hardened nitrided.
4. Tempered normalized.

b. Determination of y_a for structural steels, through hardened steels and nodular cast iron (perlite, bainite):

SI units	MKS units	US units
$y_a = \frac{160}{\sigma_{Hlim}} \cdot f_{pb}$	$y_a = \frac{16.315}{\sigma_{Hlim}} \cdot f_{pb}$	$y_a = \frac{2.331 \times 10^4}{\sigma_{Hlim}} \cdot f_{pb}$
$\mathrm{For} v \leq 5\mathrm{m/s}:$ no restriction	$\mathrm{For} v \leq 5\mathrm{m/s}:$ no restriction	$\mathrm{For} v \leq 984\mathrm{ft/min}:$ no restriction
$\text{For } 5 \text{ m/s} < v \leq 10\text{m/s}:$ $y_{amax} = \frac{12800}{\sigma_{Hlim}}$ and $f_{pbmax} = 80\mu$ m	$\text{For } 5 \text{ m/s} < v < 10\text{m/s}:$ $y_{amax} = \frac{1.305 \times 10^3}{\sigma_{Hlim}}$ and $f_{pbmax} = 80\mu$ m	$\text{For } 984 \text{ ft/min} < v < 1968\text{ft/min}:$ $f_{pbmax} = 3150\mu$ in
$\mathrm{For} v > 10\mathrm{m/s}:$	$\mathrm{For} v > 10\mathrm{m/s}:$	$\mathrm{For} v > 1968\mathrm{ft/min}:$
$y_{amax} = \frac{6400}{\sigma_{Hlim}}$ and $f_{pbmax} = 40\mu$ m	$y_{amax} = \frac{652.618}{\sigma_{Hlim}}$ and $f_{pbmax} = 40\mu$ m	$y_{amax} = \frac{3.67 \times 10^7}{\sigma_{Hlim}}$ and $f_{pbmax} = 1575\mu$ in

c. Determination of y_a for gray cast iron and nodular cast iron (ferritic):

SI and MKS units	US units
$y_a = 0.275 \cdot f_{pb}\mu$ m	$y_a = 0.275 \cdot f_{pb}\mu$ in
$\mathrm{For} v \leq 5\mathrm{m/s}:$ no restriction	$\mathrm{For} v \leq 984\mathrm{ft/min}:$ no restriction
$\text{For } 5 \text{ m/s} < v \leq 10\text{m/s}:$ $y_{amax} = 22 \text{and} f_{pbmax} = 80\mu$ m	$\text{For } 984 \text{ ft/min} < v < 1968\text{ft/min}:$ $y_{amax} = 866 \text{and} f_{pbmax} = 3150\mu$ in
$\mathrm{For} v > 10\mathrm{m/s}:$ $y_{amax} = 11 \text{and} f_{pbmax} = 40\mu$ m	$\mathrm{For} v > 1968\mathrm{ft/min}:$ $y_{amax} = 433 \text{and} f_{pbmax} = 1575\mu$ in

d. Determination of y_a for case hardened, nitrided or nitrocarburized steels:

SI and MKS units	US units
$y_a = 0.075 \cdot f_{pb}\mu$ m	$y_a = 0.075 \cdot f_{pb}\mu$ in
For all velocities but with the restriction: $y_{amax} = 3 \text{and}$ $f_{pbmax} = 40\mu$ m	For all velocities but with the restriction: $y_{amax} = 118 \text{and}$ $f_{pbmax} = 1575\mu$ in

When the material of pinion differs from that of the wheel, y_{a1} for pinion and y_{a2} for wheel are to be determined separately. The mean value:

$$y_a = 0.5(y_{a1} + y_{a2})$$

is to be used for the calculation.

For bevel gears, f_{pt} is substituted for fpb when determining y_{a2} in *b), c)* and *d)*.

e. Determination of factors B_p, B_f, B_k

$$B_p = \frac{c' \cdot f_{pbeff} \cdot b}{F_t \cdot K_A \cdot K_\gamma}$$

$$B_f = \frac{c' \cdot f_{fbeff} \cdot b}{F_t \cdot K_A \cdot K_\gamma}$$

where

$$f_{pbeff} = f_{pb} - y_a \mu m \ (\mu in)$$

$$f_{feff} = f_{fa} - y_f \mu m \ (\mu in)$$

$$f_{pb} = f_{pt} \cdot \cos a_t \mu m \ (\mu in)$$

y_f can be determined in the same way as y_a when the profile deviation f_{fa} is used instead of the base pitch deviation f_{pb} .

$$B_k = \left|1 - \frac{C_a}{C_{eff}}\right|$$

where

$$C_a = \frac{1}{18} \cdot \left(\frac{\sigma_{Hlim}}{97} - 18.45\right)^2 + 1.5 \mathrm{SI units} [\mu \mathrm{m}]$$

$$C_a = \frac{1}{18} \cdot \left(\frac{\sigma_{Hlim}}{9.891} - 18.45\right)^2 + 1.5 \mathrm{MKS units} [\mu \mathrm{m}]$$

$$C_a = \frac{1}{0.457} \cdot \left(\frac{\sigma_{Hlim}}{1.407 \times 10^4} - 18.45\right)^2 + 59 \mathrm{US units} [\mu \mathrm{in}]$$

When the materials differ, C_{a1} is to be determined for the pinion material and C_{a2} for the wheel material using the following equations. The average value $C_a = \dfrac{C_{a1} + C_{a2}}{2}$ is used for the calculation.

$$C_{a1} = \frac{1}{18} \cdot \left(\frac{\sigma_{Hlim1}}{97} - 18.45\right)^2 + 1.5\mathrm{SI units}[\mu\mathrm{m}]$$

$$C_{a1} = \frac{1}{18} \cdot \left(\frac{\sigma_{Hlim1}}{9.891} - 18.45\right)^2 + 1.5\mathrm{MKS units}[\mu\mathrm{m}]$$

$$C_{a1} = \frac{1}{0.457} \cdot \left(\frac{\sigma_{Hlim1}}{1.407 \times 10^4} - 18.45\right)^2 + 59\mathrm{US units}[\mu\mathrm{in}]$$

$$C_{a2} = \frac{1}{18} \cdot \left(\frac{\sigma_{Hlim2}}{97} - 18.45\right)^2 + 1.5\mathrm{SI units}[\mu\mathrm{m}]$$

$$C_{a2} = \frac{1}{18} \cdot \left(\frac{\sigma_{Hlim2}}{9.891} - 18.45\right)^2 + 1.5\mathrm{MKS units}[\mu\mathrm{m}]$$

$$C_{a2} = \frac{1}{0.457} \cdot \left(\frac{\sigma_{Hlim2}}{1.407 \times 10^4} - 18.45\right)^2 + 59\mathrm{US units}[\mu\mathrm{in}]$$

For cylindrical gears:

$$C_{eff} = \frac{F_t \cdot K_A \cdot K_\gamma}{b \cdot c'} \mu m(\mu in)$$

For bevel gears:

$$C_{eff} = \frac{F_{mbt} \cdot K_A}{b_{eH} \cdot c'} \mu m(\mu in)$$

15.3.3 Dynamic Factor, K_v , in the Subcritical Range **(2025)**

Cylindrical gears: (N ≤ 0.85)

$$C_{v1} = 0.32$$

$$C_{v2} = 0.34 \mathrm{for} \epsilon_\gamma > 2$$

$$C_{v2} = \frac{0.57}{\epsilon_\gamma - 0.3} \mathrm{for} \epsilon_\gamma > 2$$

$$C_{v3} = 0.23 \mathrm{for} \epsilon_\gamma > 2$$

$$C_{v3} = \frac{0.096}{\epsilon_\gamma - 1.56} \mathrm{for} \epsilon_\gamma > 2$$

$$K = (C_{v1} \cdot B_p) + (C_{v2} \cdot B_f) + (C_{v3} \cdot B_k)$$

Bevel gears: (N ≤ 0.75)

For bevel gears, $\epsilon_{v\gamma}$ are to be substituted for ϵ_γ .

$$K = \frac{b \cdot f_{peff} \cdot c'}{F_{mt} \cdot K_A} \cdot c_{v1,2} + c_{v3}$$

$$f_{peff} = f_{pt-y_p} \text{with} y_p \approx y_a$$

$$c_{v1,2} = c_{v1} + c_{v2}$$

$$K_v = (N \cdot K) + 1$$

15.3.4 Dynamic Factor, K_v , in the Main Resonance Range *(2006)*

$$C_{v4} = 0.90 \text{for} \epsilon_\gamma \leq 2$$

$$C_{v4} = \frac{0.57 - 0.05 \cdot \epsilon_\gamma}{\epsilon_\gamma - 1.44} \text{for} \epsilon_\gamma > 2$$

Cylindrical gears: (0.85 < N ≤ 1.15)

$$K_{v(N=1.15)} = (C_{v1} \cdot B_p) + (C_{v2} \cdot B_f) + (C_{v4} \cdot B_k) + 1$$

Bevel gears: (0.75 < N ≤ 1.25)

For bevel gears, $\epsilon_{v\gamma}$ are to be substituted for ϵ_γ .

$$K_{v(N=1.25)} = \frac{b \cdot f_{peff} \cdot c'}{F_{mt} \cdot K_A} \cdot c_{v1,2} + c_{v4} + 1$$

For $C_{v1}, C_{v2}, C_{v1,2},$ and $f_{peff},$ see 4-3-1-A1/15.3.3 above.

15.3.5 Dynamic Factor, K_v , in the Supercritical Range *(N ≥ 1.5) (2006)*

$$C_{v5} = 0.47$$

$$C_{v6} = 0.47 \text{for} \epsilon_\gamma \leq 2$$

$$C_{v6} = \frac{0.12}{\epsilon_\gamma - 1.74} \text{for} \epsilon_\gamma > 2$$

$$C_{v7} = 0.75 \text{for} 1.0 < \epsilon_\gamma \leq 1.5$$

$$C_{v7} = 0.125 \cdot \sin[\pi \cdot (\epsilon_\gamma - 2)] + 0.875 \text{for} 1.5 < \epsilon_\gamma \leq 2.5$$

$$C_{v7} = 1.0 \text{for} \epsilon_\gamma > 2.5$$

Cylindrical gears:

$$K_{v(N=1.5)} = (C_{v5} \cdot B_p) + (C_{v6} \cdot B_f) + C_{v7}$$

Bevel gears:

For bevel gears, $\epsilon_{v\gamma}$ are to be substituted for ϵ_{γ} .

$$K_{v(N=1.5)} = \frac{b \cdot f_{peff} \cdot c'}{F_{mt} \cdot K_A} \cdot c_{v5,6} + c_{v7}$$

$$c_{v5,6} = c_{v5} + c_{v6}$$

For f_{peff} , see 4-3-1-A1/15.3.3 above.

15.3.6 Dynamic Factor, K_v , in the Intermediate Range *(2006)*

Cylindrical gears:

In this range, the dynamic factor is determined by linear interpolation between K_v at *N* = 1.15 as specified in 4-3-1-A1/15.3.4 and K_v at *N* = 1.5 as specified in 4-3-1-A1/15.3.5.

$$K_v = K_{v(N=1.5)} = \frac{K_{v(N=1.15)} - K_{v(N=1.5)}}{0.35} \cdot (1.5 - N)$$

Bevel gears:

In this range, the dynamic factor is determined by linear interpolation between K_v at *N* = 1.25 as specified in 4-3-1-A1/15.3.4 and K_v at *N* = 1.5 as specified in 4-3-1-A1/15.3.5.

$$K_v = K_{v(N=1.5)} = \frac{K_{v(N=1.25)} - K_{v(N=1.5)}}{0.25} \cdot (1.5 - N)$$

17 Face Load Distribution Factors, $K_{H\beta}$ and $K_{F\beta}$

The face load distribution factors, $K_{H\beta}$ for contact stress and $K_{F\beta}$ for tooth root bending stress, account for the effects of non-uniform distribution of load across the facewidth.

$K_{H\beta}$ is defined as follows:

$$K_{H\beta} = \frac{\text{maximum load per unit facewidth}}{\text{mean load per unit facewidth}}$$

$K_{F\beta}$ is defined as follows:

$$K_{F\beta} = \frac{\text{maximum bending stress at tooth root per unit facewidth}}{\text{mean bending stress at tooth root per unit facewidth}}$$

Note: The mean bending stress at tooth root relates to the considered facewidth b_1 or b_2

$K_{F\beta}$ can be expressed as a function of the factor $K_{H\beta}$.

The factors $K_{H\beta}$ and $K_{F\beta}$ mainly depend on:

- Gear tooth manufacturing accuracy
- Errors in mounting due to bore errors
- Bearing clearances
- Wheel and pinion shaft alignment errors
- Elastic deflections of gear elements, shafts, bearings, housing and foundations which support the gear elements
- Thermal expansion and distortion due to operating temperature
- Compensating design elements (tooth crowning, end relief, etc.)

These factors can be obtained from and , using the factors in .

17.1 Factors Used for the Determination of $K_{H\beta}$

17.1.1 Helix Deviation F_β

The helix deviation, F_β , is to be determined by the designer or by the following equations and tables:

SI and MKS units, µm	US units, µin.
$F_\beta - 0.1\sqrt{d_{geom}} + 0.63\sqrt{b_{geom}} + 4.2$ for ISO Grade of accuracy $Q = 5$	$F_\beta - 19.8\sqrt{d_{geom}} + 125.0\sqrt{b_{geom}} + 165.4$ for ISO Grade of accuracy $Q = 5$
For other accuracy grades, multiply F_β by the following formula: $2^{0.5(Q-5)} \text{where} 0 \leq Q \leq 12$	

SI and MKS units		US units	
Reference diameter d mm	Corresponding geometric mean diameter d_{geom} mm	Reference diameter d in	Corresponding geometric mean diameter d_{geom} in.
5 $\leq d \leq$ 20	10.00	0.2 $\leq d \leq$ 0.79	0.3937
20 $< d \leq$ 50	31.62	0.79 $\leq d \leq$ 2.0	1.245
50 $< d \leq$ 125	79.06	2.0 $\leq d \leq$ 4.92	3.112
125 $< d \leq$ 280	187.1	4.92 $\leq d \leq$ 11.0	7.365
280 $< d \leq$ 560	396.0	11.0 $\leq d \leq$ 22.0	15.59
560 $< d \leq$ 1000	748.3	22.0 $\leq d \leq$ 39.37	29.46
1000 $< d \leq$ 1600	1265	39.37 $\leq d \leq$ 62.99	49.80
1600 $< d \leq$ 2500	2000	62.99 $\leq d \leq$ 98.43	78.74

SI and MKS units		US units	
Reference diameter d mm	**Corresponding geometric mean diameter** d_{geom} **mm**	**Reference diameter d in**	**Corresponding geometric mean diameter** d_{geom} **in.**
2500 $< d \leq$ 4000	3162	98.43 $\leq d \leq$ 157.5	124.5
4000 $< d \leq$ 6000	4899	157.5 $\leq d \leq$ 236.2	192.9
6000 $< d \leq$ 8000	6928	236.2 $\leq d \leq$ 315.0	272.8
8000 $< d \leq$ 10000	8944	315.0 $\leq d \leq$ 393.70	352.1

SI and MKS units		US units	
Facewidth b mm	**Corresponding geometric mean facewidth** b_{geom} **mm**	**Facewidth b in**	**Corresponding geometric mean facewidth** b_{geom} **in.**
4 $< b \leq$ 10	6.325	0.16 $< b \leq$ 0.39	0.2490
10 $< b \leq$ 20	14.14	0.39 $< b \leq$ 0.79	0.5568
20 $< b \leq$ 40	28.28	0.79 $< b \leq$ 1.6	1.114
40 $< b \leq$ 80	56.57	1.6 $< b \leq$ 3.15	2.227
80 $< b \leq$ 160	113.1	3.15 $< b \leq$ 6.30	4.454
160 $< b \leq$ 250	200.0	6.30 $< b \leq$ 9.84	7.874
250 $< b \leq$ 400	316.2	9.84 $< b \leq$ 15.7	12.45
400 $< b \leq$ 650	509.9	15.7 $< b \leq$ 25.6	20.07
650 $< b \leq$ 1000	806.2	25.6 $< b \leq$ 39.37	31.74

Rounding Rules			
For resulting F_β	$F_\beta < 5\mu\ \mathrm{m}$	$5\mu\ \mathrm{m} \leq F_\beta \leq 10\mu\ \mathrm{m}$	$F_\beta > 10\mu m\ (\mu in)$
	Round to the nearest 0.1 µm value or integer number	Round to the nearest 0.5 µm value or integer number	Round to the nearest integer number

17.1.2 Mesh Alignment f_{ma} [µm (µin)] (**2025**)

For gear pairs with well-designed end relief: $f_{ma} = 0.7 \cdot F_\beta$.

For gear pairs with provision for adjustment (lapping or running-in under light load, adjustment bearings or appropriate helix modification) and gear pairs suitablycrowned: $f_{ma} = 0.5 \cdot F_\beta$.

For helix deviation due to manufacturing inaccuracies: $f_{ma} = 0.5 \cdot F_\beta$.

In all other cases the following is to used:

$$f_{ma} = 1.0 \cdot F_\beta$$

17.1.3 Initial Equivalent Misalignment $F_{\beta\chi}$ [µm (µin)]

$F_{\beta\chi}$ is the absolute value of the sum of manufacturing deviations and pinion and shaft deflections, measured in the plane of action.

$$F_{\beta\chi} = 1.33 \cdot f_{sh} + f_{ma}$$

$$f_{sh} = f_{sh0} \cdot \frac{F_t \cdot K_A \cdot K_\gamma \cdot K_v}{b}$$

$$f_{sh0min} = 0.005\mu \text{ m-mm/N}[MKS\ units]$$

$$f_{sh0min} = 0.049\mu \text{ m-mm/kgf}[MKS\ units]$$

$$f_{sh0min} = 0.03445\mu \text{ in-in/lbf}[US\ units]$$

	SI units [µm-mm/N]	**MKS units** [µm-mm/kgf]	**US units** [µin-in/lbf]
For spur and helical gears without crowning or end relief	$f_{sh0} = 0.023 \cdot \gamma$	$f_{sh0} = 0.22555 \cdot \gamma$	$f_{sh0} = 0.15858 \cdot \gamma$
For spur and helical gears without crowning but with end relief	$f_{sh0} = 0.016 \cdot \gamma$	$f_{sh0} = 0.15691 \cdot \gamma$	$f_{sh0} = 0.11032 \cdot \gamma$
For spur and helical gears with crowning	$f_{sh0} = 0.012 \cdot \gamma$	$f_{sh0} = 0.11768 \cdot \gamma$	$f_{sh0} = 0.08274 \cdot \gamma$
For spur and helical gears with crowning and end relief	$f_{sh0} = 0.010 \cdot \gamma$	$f_{sh0} = 0.09807 \cdot \gamma$	$f_{sh0} = 0.06895 \cdot \gamma$

$$\gamma = \left[\left|1 + K\text{'} \cdot \frac{\ell \cdot S}{d_1^2} \cdot \left(\frac{d_1}{d_{sh}}\right)^4 - 0.3\right| + 0.3\right] \cdot \left(\frac{b}{d_1}\right)^2$$ for spur and single helical gears

$$\gamma = 2 \cdot \left[\left|1.5 + K\text{'} \cdot \frac{\ell \cdot S}{d_1^2} \cdot \left(\frac{d_1}{d_{sh}}\right)^4 - 0.3\right| + 0.3\right] \cdot \left(\frac{b_B}{d_1}\right)^2$$ for double helical gears

where $b_B = b/2$ is the width of one helix.

The constant K' makes allowances for the position of the pinion in relation to the torqued end. It can be taken from 4-3-1-A1/23.29 TABLE 6.

17.1.4 Determination of y_β and χ_β for Structural Steels, Through Hardened Steels and Nodular Cast Iron (Perlite, Bainite)

SI units	MKS units.	US units
$y_\beta = \frac{320}{\sigma_{Hlim}} \cdot F_{\beta\chi} \mu\ \mathrm{m}$	$y_\beta = \frac{32.63}{\sigma_{Hlim}} \cdot F_{\beta\chi} \mu\ \mathrm{m}$	$y_\beta = \frac{4.662 \cdot 10^4}{\sigma_{Hlim}} \cdot F_{\beta\chi} \mu\ \mathrm{in}$
$\chi_\beta = 1 - \frac{320}{\sigma_{Hlim}}$ $\mathrm{with} y_\beta \leq F_{\beta\chi}; \chi_\beta \geq 0$	$\chi_\beta = 1 - \frac{32.63}{\sigma_{Hlim}}$ $\mathrm{with} y_\beta \leq F_{\beta\chi}; \chi_\beta \geq 0$	$\chi_\beta = 1 - \frac{4.662 \cdot 10^4}{\sigma_{Hlim}}$ $\mathrm{with} y_\beta \leq F_{\beta\chi}; \chi_\beta \geq 0$
For $v \leq$ 5 m/s no restriction	For $v \leq$ 5 m/s no restriction	For $v \leq$ 984 ft/min no restriction
For 5 m/s $< v \leq$ 10 m/s: $y_{\beta max} = \frac{25600}{\sigma_{Hlim}}$ corresponding to $F_{\beta\chi}$ = 80 µm	For 5 m/s $< v \leq$ 10 m/s: $y_{\beta max} = \frac{2.61 \cdot 10^3}{\sigma_{Hlim}}$ corresponding to $F_{\beta\chi}$ = 80 µm	For 984 ft/min $< v \leq$ 1968 ft/min: $y_{\beta max} = \frac{14.682 \cdot 10^7}{\sigma_{Hlim}}$ corresponding to $F_{\beta\chi}$ = 3150 $\mu \mathrm{in}$
For $v >$ 10 m/s: $y_{\beta max} = \frac{12800}{\sigma_{Hlim}}$ corresponding to $F_{\beta\chi}$ = 40 µm	For $v >$ 10 m/s: $y_{\beta max} = \frac{1.305 \cdot 10^3}{\sigma_{Hlim}}$ corresponding to $F_{\beta\chi}$ = 40 µm	For $v >$ 1968 ft/min: $y_{\beta max} = \frac{7.341 \cdot 10^7}{\sigma_{Hlim}}$ corresponding to $F_{\beta\chi}$ = 1575 $\mu \mathrm{in}$

For σ_{Hlim}, see 4-3-1-A1/23.29 TABLE 3.

17.1.5 Determination of y_β *and* χ_β for Gray Cast Iron and Nodular Cast Iron (Ferritic):

SI & MKS units, µm	US units, µin
$Y_\beta = 0.55 \cdot F_{\beta\chi}$	$Y_\beta = 0.55 \cdot F_{\beta\chi}$
$\chi_\beta = 0.45$	$\chi_\beta = 0.45$
For $v \leq$ 5 m/s, no restriction	For $v \leq$ 984 ft/min, no restriction

SI & MKS units, µm	US units, µin
For 5 m/s $< v \leq$ 10 m/s: $y_{\beta max} = 45$ corresponding to $F_{\beta\chi}$ = 80 µm	For 984 ft/min $< v \leq$ 1968 ft/min: $y_{\beta max} = 1771$ corresponding to $F_{\beta\chi}$ = 3150 µin
For $v >$ 10 m/s: $y_{\beta max} = 22$ corresponding to $F_{\beta\chi}$ = 40 µm	For $v >$ 1968 ft/min: $y_{\beta max} = 866$ corresponding to $F_{\beta\chi}$ = 1575 µin

17.1.6 Determination of y_β and χ_β for Case Hardened, Nitrided or Nitrocarburized Steels:

SI & MKS units	US units
$Y_\beta = 0.15 \cdot F_{\beta\chi}\mu \text{ m}$	$Y_\beta = 0.15 \cdot F_{\beta\chi}\mu \text{ in}$
$\chi_\beta = 0.85$	$\chi_\beta = 0.85$
For all velocities but with the restriction: $y_{\beta max} = 6$ corresponding to $F_{\beta\chi}$ = 40 µm	For all velocities but with the restriction: $y_{\beta max} = 236$ corresponding to $F_{\beta\chi}$ = 1575 µin

When the material of the pinion differs from that of the wheel, $y_{\beta 1}$ and $\chi_{\beta 1}$ for pinion, and $y_{\beta 2}$ and $\chi_{\beta 2}$ for wheel are to be determined separately. The mean of either value:

$$y_\beta = 0.5 \cdot (y_{\beta 1} + y_{\beta 2})$$

$$\chi_\beta = 0.5 \cdot (\chi_{\beta 1} + \chi_{\beta 2})$$

is to be used for the calculation.

17.3 Face Load Distribution Factor for Contact Stress $K_{H\beta}$

17.3.1 $K_{H\beta}$ for Helical and Spur Gears

$$K_{H\beta} = 1 + \frac{b \cdot F_{\beta y} \cdot C_\gamma}{2 \cdot F_t \cdot K_A \cdot K_\gamma \cdot K_v} < 2, \text{for } \frac{b \cdot F_{\beta y} \cdot C_\gamma}{2 \cdot F_t \cdot K_A \cdot K_\gamma \cdot K_v} < 1$$

$$K_{H\beta} = \sqrt{\frac{2 \cdot b \cdot F_{\beta y} \cdot C_\gamma}{F_t \cdot K_A \cdot K_\gamma \cdot K_v}} \geq 2, \text{for } \frac{b \cdot F_{\beta y} \cdot C_\gamma}{2 \cdot F_t \cdot K_A \cdot K_\gamma \cdot K_v} \geq 1$$

where:

$$F_{\beta y} = F_{\beta x} - y_\beta \text{or} F_\beta = F_{\beta x} \cdot \chi_\beta$$

Calculated values of $K_{H\beta} \geq 2$ are to be reduced by improvement accuracy and helix deviation.

17.3.2 $K_{H\beta}$ for Bevel Gears

$$K_{H\beta} = 1.5 \cdot \frac{0.85}{B_b} \cdot K_{H\beta be}$$

The bearing factor, $K_{H\beta be}$, representing the influence of the bearing arrangement on the faceload distribution, is given in the following table:

Mounting conditions of pinion and wheel		
Both members straddle mounted	One member straddle mounted	Neither member straddle mounted
1.10	1.25	1.50
Based on optimum tooth contact pattern under maximum operating load as evidenced by results of a deflection test on the gears in their mountings.		

17.5 Face Load Distribution Factor for Tooth Root Bending Stress $K_{F\beta}$

17.5.1 In Case the Hardest Contact is at the End of the Face-width $K_{F\beta}$ is Given by the Following Equations

$$K_{F\beta} = (K_{H\beta})^N$$

$$N = \frac{(b/h)^2}{1 + (b/h) + (b/h)^2} = \frac{1}{1 + (h/b) + (h/b)^2}$$

$(b/h) =$ (facewidth/tooth depth), the lesser of b_1/h_1 or b_2/h_2. For double helical gears, the facewidth of only one helix is to be used, i.e., $b_B = b/2$ is to be substituted for b in the equation for *N*.

17.5.2 In Case of Gears Where the Ends of the Facewidth are Lightly Loaded or Unloaded (End Relief or Crowning)

$$K_{F\beta} = K_{H\beta}$$

17.5.3 Bevel Gears

$$K_{F\beta} = \frac{K_{H\beta}}{K_{FO}}$$

$$K_{FO} = 0.211 \cdot \left(\frac{r_{eo}}{R_m}\right)^q + 0.789 \text{for spiral bevel gears.}$$

$$q = \frac{0.279}{\log(\sin\beta_m)}$$

where

K_{FO}	=	for straight or zero bevel gears.
r_{eo}	=	cutter radius, mm (in.)
R_m	=	mean cone distance, mm (in.)

Limitations of K_{FO} :

If K_{FO} < unity, use K_{FO} = unity

If K_{FO} > 1.15, use K_{FO} = 1.15

19 Transverse Load Distribution Factors, K_{Ha} and K_{Fa}

The transverse load distribution factors, K_{Ha} for contact stress and K_{Fa} for tooth root bending stress, account for the effects of pitch and profile errors on the transversal load distribution between two or more pairs of teeth in mesh.

The factors K_{Ha} and K_{Fa} mainly depend on:

- Total mesh stiffness
- Total tangential load $F_t, K_A, K_\gamma, K_v, K_{H\beta}$
- Base pitch error
- Tip relief
- Running-in allowances

The load distribution factors, K_{Ha} and K_{Fa} are to be advised by the manufacturer as supported by his measurements, analysis or experience data or are to be determined as follows.

19.1 Determination of K_{Ha} for Contact Stress K_{Fa} for Tooth Root Bending Stress *(2006)*

$$K_{Ha} = K_{Fa} = 0.9 + 0.4 \cdot \sqrt{\frac{2 \cdot (\epsilon_\gamma - 1)}{\epsilon_\gamma}} \cdot \frac{C_\gamma \cdot (f_{pbe} + -y_a) \cdot b}{F_{tH}} \text{for} \epsilon_\gamma > 2$$

$$K_{Ha} = K_{Fa} = \frac{\epsilon_\gamma}{2} \cdot \left[0.9 + 0.4 \frac{C_\gamma \cdot (f_{pbe} + -y_a) \cdot b}{F_{tH}}\right] \text{for} \epsilon_\gamma > 2$$

Cylindrical gears:

$$F_{tH} = F_t \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{H\beta} N \ (kgf,\ lbf)$$

$$f_{pbe} = f_{pb} \cdot \cos\alpha_t$$

Bevel gears:

$$F_{mtH} = F_{mt} \cdot K_A \cdot K_v \cdot K_{H\beta} N \ (kgf,\ lbf)$$

For bevel gears, $f_{pt}, \epsilon_{v\gamma}, F_{mtH}, F_{mt}$ and α_{vt} (equivalent) are to be substituted for $f_{pbe}, \epsilon_\gamma, F_{tH}, F_t$ and α_t in the above formulas.

19.3 Limitations of K_{Ha} and K_{Fa}

19.3.1 K_{Ha} *(2006)*

When K_{Ha} < 1, use 1.0 for K_{Ha}

Cylindrical gears:

$$\text{When} K_{Ha} > \frac{\epsilon_\gamma}{\epsilon_a \cdot Z_\epsilon^2}, \text{use} \frac{\epsilon_\gamma}{\epsilon_a \cdot Z_\epsilon^2} \text{for} K_{Ha};$$

$$Z_\epsilon = \sqrt{\frac{4-\epsilon_a}{3} \cdot (1-\epsilon_\beta) + \frac{\epsilon_\beta}{\epsilon_a}},$$ contact ratio factor (pitting) for helical gears for $\epsilon_\beta < 1$

$$Z_\epsilon = \sqrt{\frac{1}{\epsilon_a}},$$ contact ratio factor (pitting) for helical gears for $\epsilon_\beta \geq 1$

$$Z_\epsilon = \sqrt{\frac{4-\epsilon_a}{3}},$$ contact ratio factor (pitting) for spur gears

Bevel gears:

$$\text{When} K_{Ha} > \frac{\epsilon_{v\gamma}}{\epsilon_{va} \cdot Z_{LS}^2}, \text{use} \frac{\epsilon_{v\gamma}}{\epsilon_{va} \cdot Z_{LS}^2} \text{for} K_{Ha}.$$

For the calculation of Z_{LS} , see 4-3-1-A1/23.13.

19.3.2 K_{Fa} *(2006)*

$$\text{When} K_{Fa} < 1, \text{use} 1.0 \text{for} K_{Fa}.$$

$$\text{When} K_{Fa} > \frac{\epsilon_\gamma}{\epsilon_a \cdot Y_\epsilon}, \text{use} \frac{\epsilon_\gamma}{\epsilon_a \cdot Y_\epsilon} \text{for} K_{Fa};$$

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_a},$$ contact ratio factor for $\epsilon_\beta = 0$

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_a} - \left(\frac{0.75}{\epsilon_a} - 0.375\right) \cdot \epsilon_\beta,$$ contact ratio factor for $0 < \epsilon_\beta < 1$

$$Y_\epsilon = 0.625,$$ contact ratio factor for $\epsilon_\beta \geq 1$

or:

$$Y_\epsilon = 0.25 + \frac{0.75 \cdot \cos^2 \beta_b}{\epsilon_a}$$ for cylindrical gears only

For bevel gears, $\epsilon_{v\gamma}, \epsilon_{v\beta},$ and $\epsilon_{va},$ (equivalent) are to be substituted for $\epsilon_\gamma, \epsilon_\beta$, and ϵ_a, in the above formulas.

21 Surface Durability

The criterion for surface durability is based on the Hertzian pressure on the operating pitch point or at the inner point of single pair contact. The contact stress σ_H is not to exceed the permissible contact stress σ_{HP}.

21.1 Contact Stress *(2006)* **(2025)**

$$\sigma_{H1} = \sigma_{HO1} \cdot \sqrt{K_A \cdot K_\gamma \cdot K_v \cdot K_{Ha} \cdot K_{H\beta}} \leq \sigma_{HP1}$$

$$\sigma_{H2} = \sigma_{HO2} \cdot \sqrt{K_A \cdot K_\gamma \cdot K_v \cdot K_{Ha} \cdot K_{H\beta}} \leq \sigma_{HP2}$$

Cylindrical gears:

$$\sigma_{HO1,2} = \text{basic value of contact stress for pinion and wheel}$$

$$\sigma_{HO1} = Z_B \cdot Z_H \cdot Z_E \cdot Z_\epsilon \cdot Z_\beta \sqrt{\frac{F_t}{d_1 \cdot b} \cdot \frac{u+1}{u}} \text{for pinion}$$

$$\sigma_{HO2} = Z_D \cdot Z_H \cdot Z_E \cdot Z_\epsilon \cdot Z_\beta \sqrt{\frac{F_t}{d_1 \cdot b} \cdot \frac{u+1}{u}} \text{for wheel}$$

where

Z_B	=	single pair mesh factor for pinion, see 4-3-1-A1/21.5 below
Z_D	=	single pair mesh factor for wheel, see 4-3-1-A1/21.5 below
Z_H	=	zone factor, see 4-3-1-A1/21.27 below
Z_E	=	elasticity factor, see 4-3-1-A1/21.9 below
Z_ϵ	=	contact ratio factor (pitting), see 4-3-1-A1/21.11 below
Z_β	=	helix angle factor, see 4-3-1-A1/21.17 below
F_t	=	nominal transverse tangential load, see 4-3-1-A1/9 of this Appendix.

Gear ratio u for external gears is positive, for internal gears, u is negative.

Regarding factors $K_A, K_\gamma, K_v, K_{Ha}$ and $K_{H\beta}$, see 4-3-1-A1/11, 4-3-1-A1/13, 4-3-1-A1/15, 4-3-1-A1/19 and 4-3-1-A1/17 of this Appendix.

Bevel gears:

$$\sigma_{HO1} = \text{basic value of contact stress for pinion and wheel}$$

$$\sigma_{HO1} = Z_{M-B} \cdot Z_H \cdot Z_E \cdot Z_{LS} \cdot Z_\beta \cdot Z_K \cdot \sqrt{\frac{F_{mt}}{d_{v1} \cdot \ell_{bm}} \cdot \frac{u_v + 1}{u_v}}$$

For the shaft angle $\sum = \delta_1 + \delta_2 = 90^\circ$ the following applies:

$$\sigma_{HO1} = Z_{M-B} \cdot Z_H \cdot Z_E \cdot Z_{LS} \cdot Z_\beta \cdot Z_K \cdot \sqrt{\frac{F_{mt}}{d_{m1} \cdot \ell_{bm}} \cdot \frac{\sqrt{u^2+1}}{u}}$$

where

Z_{M-B}	=	mid-zone factor, see 4-3-1-A1/21.5 below
Z_H	=	zone factor, see 4-3-1-A1/21.7 below
Z_E	=	elasticity factor, see 4-3-1-A1/21.9 below
Z_{LS}	=	load sharing factor, see 4-3-1-A1/21.13 below
Z_β	=	helix angle factor, see 4-3-1-A1/21.17 below
Z_K	=	bevel gear factor (flank), see 4-3-1-A1/21.15 below
F_{mt}	=	nominal transverse tangential load, see 4-3-1-A1/9 of this appendix.
d_{m1}	=	mean pitch diameter of pinion of bevel gear
d_{v1}	=	reference diameter of pinion of virtual (equivalent) cylindrical gear

ℓ_{bm} = length of middle line of contact

u_v = gear ratio of virtual (equivalent) cylindrical gear

u = gear ratio of bevel gear

21.3 Permissible Contact Stress (2025)

The permissible contact stress, σ_{HP} , is to be evaluated separately for pinion and wheel.

$$\sigma_{HP} = \frac{\sigma_{Hlim}}{S_H} \cdot Z_N \cdot Z_L \cdot Z_V \cdot Z_R \cdot Z_W \cdot Z_X \mathrm{N/mm^2} (kgf/mm^2,\ psi)$$

where

σ_{Hlim} = endurance limit for contact stress, see 4-3-1-A1/9 below

Z_N = life factor for contact stress, see 4-3-1-A1/21.21 below

Z_L = lubrication factor, see 4-3-1-A1/23 below

Z_V = speed factor, see 4-3-1-A1/21.23 below

Z_R = roughness factor, see 4-3-1-A1/21.23 below

Z_W = hardness ratio factor, see 4-3-1-A1/23.25 below

Z_X = size factor for contact stress, see 4-3-1-A1/23.27 below

S_H = safety factor for contact stress, see 4-3-1-A1/23.29 below

For shrink-fitted wheel rims, σ_{HP} is to be at least K_S times the mean contact stress σ_H , where

K_S = safety factor available for induced contact stresses, and is to be calculated as follows:

$$K_S = \begin{cases} 1 + \dfrac{\delta_{max} \cdot 2.2 \times 10^5}{Y} \cdot \dfrac{d_{ri}}{d_{w2}} \cdot \dfrac{0.25 m_n}{\rho_{F2}} \text{ [SI units]} \\ 1 + \dfrac{\delta_{max} \cdot 2.243 \times 10^4}{Y} \cdot \dfrac{d_{ri}}{d_{w2}^2} \cdot \dfrac{0.25 m_n}{\rho_{F2}} \text{ [MKS units]} \\ 1 + \dfrac{\delta_{max} \cdot 3.194 \times 10^7}{Y} \cdot \dfrac{d_{ri}}{d_{w2}^2} \cdot \dfrac{0.25 m_n}{\rho_{F2}} \text{ [US units]} \end{cases}$$

where

δ_{max} = maximum available interference fit or maximum pull-up length; mm (in.)

d_{ri} = inner diameter of wheel rim; mm (in.)

d_{w2}	=	working pitch diameter of wheel; mm (in.)
m_n	=	normal module; mm (in.)
Y	=	yield strength of wheel rim material is to be as follows:

- minimum specified yield strength for through hardened (quenched and tempered) steels
- 500 N/mm2 (51 kgf/mm^2, 72520 psi) for case hardened, nitrided steels

21.5 Single Pair Mesh Factors, Z_B, Z_D and Mid-zone Factor Z_{M-B} *(2012)*

The single pair mesh factors, Z_B for pinion and Z_D for wheel, account for the influence on contact stresses of the tooth flank curvature at the inner point of single pair contact in relation to Z_H .

The factors transform the contact stresses determined at the pitch point to contact stresses considering the flank curvature at the inner point of single pair contact.

21.5.1 For Cylindrical and Bevel Gears when $\epsilon_\beta = 0$ *(2006)*

21.5.1(a) Cylindrical Gears

$$Z_B = M_1 \text{or } 1, \text{whichever is the larger value}$$

$$Z_D = M_2 \text{or } 1, \text{whichever is the larger value}$$

$$M_1 = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{(d_{a1}/d_{b1})^2 - 1} - (2\pi/z_1)\right] \cdot \left[\sqrt{(d_{a2}/d_{b2})^2 - 1} - (\epsilon_\alpha - 1) \cdot (2\pi/z_2)\right]}}$$

$$M_2 = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{(d_{a2}/d_{b2})^2 - 1} - (2\pi/z_2)\right] \cdot \left[\sqrt{(d_{a1}/d_{b1})^2 - 1} - (\epsilon_\alpha - 1) \cdot (2\pi/z_1)\right]}}$$

21.5.1(b) Bevel Gears

$$Z_{M-B} = M$$

$$M = \frac{\tan\alpha_{wt}}{\sqrt{\left[\sqrt{(d_{va1}/d_{vb1})^2 - 1} - (2\pi/z_{v1})\right] \cdot \left[\sqrt{(d_{va2}/d_{vb2})^2 - 1} - (\epsilon_{v\alpha} - 1) \cdot (2\pi/z_{v2})\right]}}$$

21.5.2 For Cylindrical and Bevel Gears when $\epsilon_\beta \geq 1$

21.5.2(a) Cylindrical Gears

$Z_B = Z_D =$ 1 for cylindrical gears

21.5.2(b) Bevel Gears

$$Z_{M-B} = M$$

$$M = \frac{\tan\alpha_{vt}}{\sqrt{\left[\sqrt{(d_{va1}/d_{vb1})^2 - 1} - \epsilon_{v\alpha} \cdot (\pi/z_{v1})\right] \cdot \left[\sqrt{(d_{va2}/d_{vb2})^2 - 1} - \epsilon_{v\alpha} \cdot (\pi/z_{v2})\right]}}$$

21.5.3 For Cylindrical and Bevel Gears when $0 < \epsilon_\beta < 1$

21.5.3(a) Cylindrical Gears (**2025**)

The values of Z_B, Z_A are determined by linear interpolation between Z_B, Z_A for spur gears and Z_B, Z_A for helical gears having $\epsilon_\beta \geq 1$

Thus:

$$Z_B = M_1 - \epsilon_\beta \cdot (M_1 - 1) \text{and} Z_B \geq 1$$

$$Z_D = M_2 - \epsilon_\beta \cdot (M_2 - 1) \text{and} Z_D \geq 1$$

21.5.3(b) Bevel Gears

$$Z_{M-B} = M$$

$$M = \frac{\tan\alpha_{vt}}{\sqrt{\left[\sqrt{(d_{va1}/d_{vb1})^2 - 1} - (2 + (\epsilon_{v\alpha} - 2) \cdot \epsilon_{v\beta})(\pi/z_{v1})\right] \cdot \left[\sqrt{(d_{va2}/d_{vb2})^2 - 1} - (2(\epsilon_{v\alpha} - 1) + (2 - \epsilon_{v\alpha}) \cdot \epsilon_{v\beta}(\pi/z_{v2})\right]}}$$

21.7 Zone Factor, Z_H *(2006)*

Cylindrical gears:

The zone factor, Z_H , accounts for the influence on the Hertzian pressure of tooth flank curvature at pitch point and relates the tangential force at the reference cylinder to the normal force at the pitch cylinder.

$$Z_H = \sqrt{\frac{2 \cdot \cos\beta_b \cdot \cos a_{wt}}{\cos^2 a_t \cdot \sin a_{wt}}}$$

Bevel gears:

$$Z_H = 2 \cdot \sqrt{\frac{\cos\beta_{vb}}{\sin(2 \cdot a_{vt})}}$$

21.9 Elasticity Factor, Z_E

The elasticity factor, Z_E , accounts for the influence of the material properties E (modulus of elasticity) and v (Poisson’s ratio) on the Hertzian pressure.

$$Z_E = \sqrt{\frac{1}{\pi \cdot \left[\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}\right]}}$$

With Poisson's ratio of 0.3 and same E and v for pinion and wheel, Z_E may be obtained from the following:

$$Z_E = \sqrt{0.175 \cdot E}$$

with E = Young's modulus of elasticity.

The elasticity factor, Z_E, for steel gears [E_{st} = 206000 N/mm^2 (2.101 x 10^4 kgf/mm^2, 2.988 x 10^7 psi) is:

Elasticity factor Z_E		
SI units	MKS units	US units
189.8 N$^{1/2}$/mm	60.61 kgf$^{1/2}$/mm	2.286 x 10^3 lbf$^{1/2}$/in

For other material combinations, refer to 4-3-1-A1/23.29 TABLE 1.

21.11 Contact Ratio Factor (Pitting), Z_ϵ

The contact ratio factor, Z_ϵ **,** accounts for the influence of the transverse contact ratio and the overlap ratio on the specific surface load of gears.

Spur gears:

$$Z_\epsilon = \sqrt{\frac{4-\epsilon_\alpha}{3}}$$

Helical gears:

$$\text{For}\,\epsilon_\beta < 1 : Z_\epsilon = \sqrt{\frac{4-\epsilon_\alpha}{3} \cdot (1-\epsilon_\beta) + \frac{\epsilon_\beta}{\epsilon_\alpha}}$$

$$\text{For}\,\epsilon_\beta \geq 1 : Z_\epsilon = \sqrt{\frac{1}{\epsilon_a}}$$

21.13 Bevel Gear Load Sharing Factor, Z_{LS} *(2006)*

The load sharing factor, Z_{LS}, accounts for the load sharing between two or more pair of teeth in contact.

$$\text{For}\,\epsilon_{v\gamma} \leq 2 : Z_{LS} = 1$$

$$\text{For}\,\epsilon_{v\gamma} > 2\text{and}\epsilon_{v\beta} > 1 : Z_{LS} = \left[1 + 2 \cdot \{1 - (2/\epsilon_{v\gamma})^{1.5}\} \cdot \sqrt{1 - (4/\epsilon_{v\gamma}^2}\right]^{-0.5}$$

For other cases: The calculation of Z_{LS} can be based upon the method used in ISO 10300-2, Annex A, Load Sharing Factor, Z_{LS} .

21.15 Bevel Gear Factor (Flank), Z_K

The bevel gear factor (flank), Z_K , accounts the difference between bevel and cylindrical loading and adjusts the contact stresses so that the same permissible stresses apply.

$$Z_K = 0.8$$

21.17 Helix Angle Factor, Z_β *(1 July 2009)*

The helix angle factor, Z_β , accounts for the influence of helix angle on surface durability, allowing for such variables as the distribution of load along the lines of contact. Z_β is dependent only on the helix angle.

$$\text{Cylindrical gears}: Z_\beta = \frac{1}{\sqrt{\cos\beta}}$$

$$\text{Bevel gears}: Z_\beta = \frac{1}{\sqrt{\cos\beta_m}}$$

21.19 Allowable Stress Number (Contact), σ_{Hlim}

For a given material, σ_{Hlim} is the limit of repeated contact stress that can be sustained without progressive pitting. For most materials, their load cycles may be taken at 50 x 10^6, unless otherwise specified.

For this purpose, pitting is defined as follows:

- Not surface hardened gears: pitted area > 2% of total active flank area.
- Surface hardened gears: pitted area > 0.5% of total active flank area, or > 4% of one particular tooth flank area.

The endurance limit depends mainly on:

- Material composition, cleanliness and defects
- Mechanical properties
- Residual stresses
- Hardening process, depth of hardened zone, hardness gradient
- Material structure (forged, rolled bar, cast)

The σ_{Hlim} values correspond to a failure probability of 1% or less. The values of σ_{Hlim} are to be determined from 4-3-1-A1/23.29 TABLE 1 or to be advised by the manufacturer together with technical justification for the proposed values.

21.21 Life Factor, Z_N

The life factor, Z_N , accounts for the higher permissible contact stress, including static stress in case a limited life (number of load cycles) is specified.

The factor depends mainly on:

- Material and hardening
- Number of cycles
- Influence factors Z_R, Z_V, Z_L, Z_W, Z_X

The life factor, Z_N , can be determined from 4-3-1-A1/23.29 TABLE 4.

21.23 Influence Factors on Lubrication Film, Z_L, Z_V and Z_R

The lubricant factor, Z_L , accounts for the influence of the type of lubricant and its viscosity.

The speed factor, Z_V , accounts for the influence of the pitch line velocity.

The roughness factor, Z_R , accounts for the influence of the surface roughness on the surface endurance capacity.

The factors mainly depend on:

- Viscosity of lubricant in the contact zone
- The sum of the instantaneous velocities of the tooth surfaces
- Load
- Relative radius of curvature at the pitch point
- Surface roughness of teeth flanks
- Hardness of pinion and gear

Where gear pairs are of different hardness, the factors may be based on the less hardened material.

21.23.1 Lubricant Factor, Z_L

$$Z_L = C_{ZL} = + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (134/v_{40})]^2} \; [SI \; and \; MKS \; units]$$

$$Z_L = C_{ZL} = + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (0.208/v_{40})]^2} \; [US \; units]$$

or

$$Z_L = C_{ZL} = + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (80/v_{50})]^2} \; [SI \; and \; MKS \; units]$$

$$Z_L = C_{ZL} = + \frac{4 \cdot (1.0 - C_{ZL})}{[1.2 + (0.127/v_{50})]^2} \; [US \; units]$$

where σ_{Hlim} is the allowable stress number (contact) of the softer material.

a. with σ_{Hlim} in the range of:

850 N/mm^2 (87 kgf/mm^2, 1.23 × 10^5 psi) < σ_{Hlim} < 1200 N/mm^2 (122 kgf/mm^2, 1.74 × 10^5 psi)

$$C_{ZL} = \left(0.08 \cdot \frac{\sigma_{Hlim} - 850}{350}\right) + 0.83[SI\ units]$$

$$C_{ZL} = \left(0.08 \cdot \frac{\sigma_{Hlim} - 87}{35.7}\right) + 0.83[MKS\ units]$$

$$C_{ZL} = \left(0.08 \cdot \frac{\sigma_{Hlim} - 1.23 \cdot 10^5}{5.076 \times 10^4}\right) + 0.83[SI\ units]$$

b. with σ_{Hlim} in the range of σ_{Hlim} < 850 N/mm^2 (87 kgf/mm^2, 1.23 × 10^5 psi), C_{ZL} = 0.83;

c. with σ_{Hlim} in the range of σ_{Hlim} > 1200 N/mm^2 (122 kgf/mm^2, 1.74 × 10^5 psi), C_{ZL} = 0.91.

and

$v_{40} =$ nominal kinematic viscosity of the oil at 40°C (104°F), mm^2/s; see table below.

$v_{50} =$ nominal kinematic viscosity of the oil at 50°C (122°F), mm^2/s; see table below.

ISO lubricant viscosity grade		VG 32 (1)	VG 46 (1)	VG 68 (1)	VG 100	VG 150	VG 220	VG 320
SI/MKS units	average viscosity V_{40} mm^2/s	32	46	68	100	150	220	320
	average viscosity V_{50} mm^2/s	21	30	43	61	89	125	180
US units	average viscosity V_{40} in^2/s	0.0496	0.0713	0.1054	0.1550	0.2325	0.3410	0.4960
	average viscosity V_{50} in^2/s	0.0326	0.0465	0.0667	0.0945	0.1380	0.1938	0.2790
1) Only for high speed (> 1400 rpm) transmission.								

21.23.2 Speed Factor, Z_V *(2006)*

$$Z_V = C_{Zv} + \frac{2(1.0 - C_{Zv})}{\sqrt{0.8 + (32/v)}}\ [SI\ and\ MKS\ units]$$

$$Z_V = C_{Zv} + \frac{2(1.0 - C_{Zv})}{\sqrt{0.8 + (6.4 \cdot 10^3/v)}}\ [US\ units]$$

where σ_{Hlim} is the allowable stress number (contact) of the softer material.

For bevel gears, v is to be substituted by v_{mt} in the above formula.

a. with σ_{Hlim} in the range of:

850 N/mm^2 (87 kgf/mm^2, 1.23 × 10^5 psi) < σ_{Hlim} < 1200 N/mm^2 (122 kgf/mm^2, 1.74 × 10^5 psi)

$$C_{Zv} = \left(0.08 \cdot \frac{\sigma_{Hlim} - 850}{350}\right) + 0.85[SI\ units]$$

$$C_{Zv} = \left(0.08 \cdot \frac{\sigma_{Hlim} - 87}{35.7}\right) + 0.85[MKS\ units]$$

$$C_{Zv} = \left(0.08 \cdot \frac{\sigma_{Hlim} - 1.23 \cdot 10^5}{5.076 \times 10^4}\right) + 0.85[US\ units]$$

b. with σ_{Hlim} < 850 N/mm^2 (87 kgf/mm^2, 1.23 × 10^5 psi), C_{Zv} = 0.85.

c. with σ_{Hlim} > 1200 N/mm^2 (122 kgf/mm^2, 1.74 × 10^5 psi), C_{Zv} = 0.93.

21.23.3 Roughness Factor, Z_R **(2025)**

$$Z_R = \left(\frac{3}{R_{Z10}}\right)^{C_{ZR}}$$

The peak-to-valley roughness, R_Z, is to be advised by the manufacturer or to be determined as a mean value of R_Z, measured on several tooth flanks of the pinion and the gear, as given by the following expression:

$$R_Z = \frac{R_{Zf1} + R_{Zf2}}{2}$$

Where roughness values are not available, roughness of the pinion $R_{Zf1} = 6.3\ \mu m\ (248\ \mu in)$ and of the wheel $R_{Zf2} = 6.3\ \mu m\ (248\ \mu in)$ may be used.

R_{Z10} is to be given by:

$$R_{Z10} = R_Z \sqrt[3]{\frac{10}{\rho_{red}}}[SI\ and\ MKS\ units]$$

$$R_{Z10} = R_Z \sqrt[3]{\frac{6.4516 \cdot 10^{-6}}{\rho_{red}}}[US\ units]$$

and the relative radius of curvature is to be given by:

$$\rho_{red} = \frac{\rho_1 \cdot \rho_2}{\rho_1 \cdot \rho_2} \text{ for cylindrical gears}$$

$$\rho_{red} = \frac{\rho_{v1} \cdot \rho_{v2}}{\rho_{v1} \cdot \rho_{v2}} \text{ for bevel gears}$$

$$\rho_1 = 0.5 \cdot d_{b1} \cdot \tan\alpha_{tw}$$

$$\rho_{v1} = 0.5 \cdot d_{vb1} \cdot \tan\alpha_{tw}$$

$$\rho_2 = 0.5 \cdot d_{b2} \cdot \tan\alpha_{tw}$$

$$\rho_{v2} = 0.5 \cdot d_{vb2} \cdot \tan\alpha_{tw}$$

If the stated roughness is an R_a value, also known as arithmetic average (*AA*) and centerline average (*CLA*), the following approximate relationship may be applied:

$$R_a = CLA = AA = R_{Zf}/6$$

Where R_{Zf} is either R_{Zf1} for pinion or R_{Zf2} for gear and σ_{Hlim} is the allowable stress number (contact) of the softer material.

In the range of 850 N/mm^2 ≤ σ_{Hlim} ≤ 1200 N/mm^2 (87 kgf/N/mm^2 ≤ σ_{Hlim} ≤ 122 kgf/mm^2; 1.23 × 10^5 psi ≤ σ_{Hlim} ≤ 1.74 × 10^5 psi) , C_{ZR} can be calculated as follows:

$$C_{ZR} = 0.32 - 2.00 \times 10^{-4} \cdot \sigma_{Hlim}[SI\ units]$$

$$C_{ZR} = 0.32 - 1.96 \times 10^{-3} \cdot \sigma_{Hlim}[MKS\ units]$$

$$C_{ZR} = 0.32 - 1.38 \times 10^{-6} \cdot \sigma_{Hlim}[US\ units]$$

If σ_{Hlim} < 850 N/mm^2 (87 kgf/N/mm^2, 1.23 × 10^5 psi), take C_{ZR} = 0.150

If σ_{Hlim} > 1200 N/mm^2 (122 kgf/N/mm^2, 1.74 × 10^5 psi), take C_{ZR} = 0.080

21.25 Hardness Ratio Factor; Z_W

The hardness ratio factor, Z_W , accounts for the increase of surface durability of a soft steel gear when meshing with a surface hardened gear with a smooth surface.

The hardness ratio factor, Z_W , applies to the soft gear only and depends mainly on:

- Hardness of the soft gear
- Alloying elements of the soft gear
- Tooth flank roughness of the harder gear

$$Z_W = 1.2 - \frac{HB - 130}{1700}$$

where

$$HB = \text{Brinell hardness of the softer material}$$
$$HV10 = \text{Vickers hardness with} F = 98.1\ \text{N}$$

$$Forunalloyedsteels$$

$$HB \approx HV10 \approx U/3.6[SI\ units]$$

$$HB \approx HV10 \approx U/3.67[MKS\ units]$$

$$HB \approx HV10 \approx U/522[US\ units]$$

$$Foralloyedsteels$$

$$HB \approx HV10 \approx U/3.4[SI\ units]$$

$$HB \approx HV10 \approx U/3.47[MKS\ units]$$

$$HB \approx HV10 \approx U/493[US\ units]$$

For $HB < 130, Z_W = 1.2$ is to be used.

For $HB < 470, Z_W = 1.0$ is to be used.

21.27 Size Factor, Z_X

The size factor, Z_X , accounts for the influence of tooth dimensions on permissible contact stress and reflects the non-uniformity of material properties.

The factor mainly depends on:

- Material and heat treatment
- Tooth and gear dimensions
- Ratio of case depth to tooth size
- Ratio of case depth to equivalent radius of curvature

The size factors, Z_X , are to be obtained from .

For through-hardened gears and for surface-hardened gears with minimum required effective case depth including root of 1.14 mm (0.045 in.) relative to tooth size and radius curvature Z_X = 1. When the case depth is relatively shallow, then a smaller value of Z_X is to be chosen.

The size factors, Z_X , are to be obtained from 4-3-1-A1/23.29 TABLE 4.

21.29 Safety Factor for Contact Stress, S_H

Based on the application, the following safety factors for contact stress, S_H , are to be applied:

Main propulsion gears (including PTO): 1.40

Duplicated (or more) independent main propulsion gears (including azimuthing thrusters): 1.25

Main propulsion gears for yachts, single screw: 1.25

Main propulsion gears for yachts, multiple screw: 1.20

Auxiliary gears: 1.15

Note:

For the above purposes, yachts are considered pleasure craft not engaged in trade or carrying passengers, and not intended for charter-service.

23 Tooth Root Bending Strength

The criterion for tooth root bending strength is the permissible limit of local tensile strength in the root fillet. The tooth root stress, σ_F , and the permissible tooth root stress, σ_{FP} , are to be calculated separately for the pinion and the wheel, whereby σ_F is not to exceed the permissible tooth root stress σ_{FP} .

The following formulas apply to gears having a rim thickness greater than 3.5 m and further for all involute basic rack profiles, with or without protuberance, however, with the following restrictions:

- The 30° tangents contact the tooth-root curve generated by the basic rack of the tool
- The basic rack of the tool has a root radius $\rho_{fp} > 0$
- The gear teeth are generated using a rack type tool.

23.1 Tooth Root Bending Stress for Pinion and Wheel

Cylindrical gears:

$$\sigma_{F1,2} = \frac{F_t}{b \cdot m_n} \cdot Y_F \cdot Y_S \cdot Y_\beta \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{Fa} \cdot K_{F\beta} \leq \sigma_{FP1,2} \mathrm{N/mm}^2 (kgf/mm^2,\ psi)$$

Bevel gears:

$$\sigma_{F1,2} = \frac{F_{mt}}{b \cdot m_{mn}} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\epsilon \cdot Y_K \cdot Y_{LS} \cdot K_A \cdot K_\gamma \cdot K_v \cdot K_{Fa} \cdot K_{F\beta} \leq \sigma_{FP1,2} \mathrm{N/mm}^2 (kgf/mm^2,\ psi)$$

where

Y_F, Y_{Fa}	=	tooth form factor, see 4-3-1-A1/23.5 below
Y_S, Y_{Sa}	=	stress correction factor, see 4-3-1-A1/23.7 below
Y_β	=	helix angle factor, see 4-3-1-A1/23.9 below
Y_ϵ	=	contact ratio factor, see 4-3-1-A1/23.11 below
Y_K	=	bevel gear factor, see 4-3-1-A1/23.13 below
Y_{LS}	=	load sharing factor, see 4-3-1-A1/23.15 below

$F_t, F_{mt}, K_A, K_\gamma, K_v, K_{Fa}, K_{F\beta}, b, m_n, m_{mn}$, see 4-3-1-A1/9, 4-3-1-A1/11, 4-3-1-A1/13, 4-3-1-A1/15, 4-3-1-A1/19, 4-3-1-A1/17, 4-3-1-A1/5, and 4-3-1-A1/7 of this Appendix, respectively.

23.3 Permissible Tooth Root Bending Stress

$$\sigma_{F1,2} = \frac{\sigma_{FE}}{S_F} \cdot Y_d \cdot Y_N \cdot Y_{\delta relT} \cdot Y_{RelT} \cdot Y_X \mathrm{N/mm}^2 (kgf/mm^2,\ psi)$$

where

σ_{FE}	=	bending endurance limit, see 4-3-1-A1/23.17 below
Y_d	=	design factor, see 4-3-1-A1/23.19 below
Y_N	=	life factor, see 4-3-1-A1/23.21 below
$Y_{\delta relT}$	=	relative notch sensitivity factor, see 4-3-1-A1/23.23 below
Y_{RrelT}	=	relative surface factor, see 4-3-1-A1/23.25 below
Y_X	=	size factor, see 4-3-1-A1/23.27 below
S_F	=	safety factor for tooth root bending stress, see 4-3-1-A1/23.29 below

23.5 Tooth Form Factor, Y_F, Y_{Fa}

The tooth form factors, Y_F and Y_{Fa} , represent the influence on nominal bending stress of the tooth form with load applied at the outer point of single pair tooth contact.

The tooth form factors, Y_F and Y_{Fa} , are to be determined separately for the pinion and the wheel. In the case of helical gears, the form factors for gearing are to be determined in the normal section, i.e., for the virtual spur gear with virtual number of teeth, z .

C ylindrical gears:

$$Y_F = \frac{6 \cdot (h_F/m_n) \cdot \cos\alpha_{Fen}}{(S_{Fn}/m_n)^2 \cdot \cos\alpha_n}$$

Bevel gears:

$$Y_{Fa} = \frac{6 \cdot (h_{Fa}/m_{mn}) \cdot \cos\alpha_{Fan}}{(S_{Fn}/m_{mn})^2 \cdot \cos\alpha_n}$$

where

h_F, h_{Fa}	=	bending moment arm for tooth root bending stress for application of load at the outer point of single tooth pair contact; mm (in.)
s_{Fn}	=	width of tooth at highest stressed section; mm (in.)
$\alpha_{Fen}, \alpha_{Fan}$	=	normal load pressure angle at tip of tooth; degrees

For determination of h_F, h_{Fa}, S_{Fn} and $\alpha_{Fen}, \alpha_{Fan}$, see 4-3-1-A1/23.5.1.1, 4-3-1-A1/23.5.2, 4-3-1-A1/23.5.33 below and 4-3-1-A1/23.29 FIGURE 1.

23.5.1 External Gears

Width of tooth, S_{Fn} , at tooth-root normal chord:

$$\frac{S_{Fn}}{m_n} = z_n \cdot \sin\left(\frac{\pi}{3} - \vartheta\right) + \sqrt{3} \cdot \left(\frac{G}{\cos\vartheta} - \frac{\rho_{a0}}{m_n}\right)$$

$$\vartheta = 2 \cdot \frac{G}{z_n} \cdot \tan\vartheta - H \text{degrees; to be solved iteratively}$$

$$G = \frac{\rho_{a0}}{m_n} - \frac{h_{a0}}{m_n} + x$$

$$H = \frac{2}{z_n} \cdot \left(\frac{\pi}{2} - \frac{E}{m_n}\right) - \frac{\pi}{3} [SI\ and\ MKS\ units]$$

$$H = \frac{2}{z_n} \cdot \left(\frac{\pi}{2} - \frac{E}{25.4 \cdot m_n}\right) - \frac{\pi}{3} [US\ units]$$

$$z_n = \frac{z}{\cos^2\beta_b \cdot \cos\beta}$$

$$\beta_b = \arccos\sqrt{1 - (\sin\beta \cdot \cos\alpha_n)^2} \text{degrees}$$

$$E = \frac{\pi}{4} \cdot m_n - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n} [SI\ and\ MKS\ units]$$

$$E = 25.4 \cdot \left(\frac{\pi}{4} \cdot m_n - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n}\right) [US\ units]$$

$$S_{pr} = p_{r0} - q$$

$$S_{pr} = 0 when\ gears\ are\ not\ undercut\ (non - protuberance\ hob)$$

where:

$E, h_{a0}, a_n, S_{pr}, p_{r0}, q$ and ρ_{a0} are shown in 4-3-1-A1/23.29 FIGURE 2

h_{a0}	=	addendum of tool; mm (in.)
S_{pr}	=	residual undercut left by protuberance; mm (in.)
p_{r0}	=	protuberance of tool; mm (in.)
q	=	material allowances for finish machining; mm (in.)
ρ_{a0}	=	tip radius of tool; mm (in.)
z_n	=	virtual number of teeth
x	=	addendum modification coefficient
α_{Fen}	=	angle for application of load at the highest point of single tooth contact
α_{en}	=	pressure angle at the highest point of single tooth contact

Bending moment arm h_F :

$$\frac{h_f}{m_n} = \frac{1}{2} \cdot \left[(\cos\gamma_e - \sin\gamma_e \cdot \tan\alpha_{Fen}) \cdot \frac{d_{en}}{m_n} - z_n \cdot \cos\left(\frac{\pi}{3} - \vartheta\right) - \frac{G}{\cos\vartheta} + \frac{\rho_{a0}}{m_n} \right]$$

$$\frac{\rho_F}{m_n} = \frac{\rho_{a0}}{m_n} + \frac{2 \cdot G^2}{\cos\vartheta \cdot (z_n \cos^2\vartheta - 2G)}$$

where

ρ_F = root fillet radius in the critical section at 30° tangent; mm (in.); see 4-3-1-A1/23.29 FIGURE 1.

Normal load pressure angle at tip of tooth, $\alpha_{Fen}, \alpha_{Fan}$:

$$\alpha_{Fen} = \alpha_{en} - \gamma_e \text{degrees}$$

$$\alpha_{Fen} = \arccos\left(\frac{d_{bn}}{d_{en}}\right) \text{ degrees}$$

$$\gamma_e = \left(\frac{0.5 \cdot \pi + 2 \cdot x \cdot \tan\alpha_n}{z_n} + \text{inv}\alpha_n - \text{inv}\alpha_{en}\right) \cdot \frac{180}{\pi} \text{ degrees}$$

$$d_{an1} = d_{n1} + d_{a1} - d_1 mm\ (in.)$$

$$d_{an2} = d_{n2} + d_{a2} - d_2 mm\ (in.)$$

$$d_{n1,2} = Z_{n1,2} \cdot m_n mm\ (in.)$$

$$d_{\mathrm{bn1,2}} = d_{\mathrm{n1,2}} \cdot \cos\alpha_n mm\ (in.)$$

$$d_{en1} = \frac{2 \cdot z_1}{|z_1|} \cdot \sqrt{\left[\sqrt{\left(\frac{d_{an1}}{2}\right)^2 - \left(\frac{d_{bn1}}{2}\right)^2} - \frac{\pi \cdot d_1 \cdot \cos\beta \cdot \cos\alpha_n}{|z_1|} \cdot (\epsilon_{\alpha n} - 1)\right]^2 + \left(\frac{d_{bn1}}{2}\right)^2} mm\ (in.)$$

$$d_{en2} = \frac{2 \cdot z_2}{|z_2|} \cdot \sqrt{\left[\sqrt{\left(\frac{d_{an2}}{2}\right)^2 - \left(\frac{d_{bn2}}{2}\right)^2} - \frac{\pi \cdot d_2 \cdot \cos\beta \cdot \cos\alpha_n}{|z_2|} \cdot (\epsilon_{\alpha n} - 1)\right]^2 + \left(\frac{d_{bn1}}{2}\right)^2} mm\ (in.)$$

$$\epsilon_{an} = \frac{\epsilon_\alpha}{\cos^2\beta_b} \text{ degrees}$$

Note:

z_1, z_2 are positive for external gears and negative for internal gears.

23.5.2 Internal Gears *(2002)*

The tooth form factor of a special rack can be substituted as an approximate value of the form of an internal gear The profile of such a rack is to be a version of the basic rack profile, so modified that it would generate the normal profile, including tip and root circles, of an exact counterpart of the internal gear. The tip load angle is $\alpha_{Fen} = \alpha_n$.

Width of tooth, S_{Fn2} , at tooth-root normal chord:

$$\frac{S_{Fn2}}{m_n} = 2 \cdot \left[\frac{\pi}{4} + \tan\alpha_n \cdot \left(\frac{h_{fp2} - \rho_{fp2}}{m_n}\right) + \frac{\rho_{fp2} - S_{pr2}}{m_n \cdot \cos\alpha_n} - \frac{\rho_{fp2}}{m_n} \cdot \cos\frac{\pi}{6}\right]$$

Bending moment arm h_{F2} :

$$\frac{h_{F2}}{m_n} = \frac{d_{en} - d_{fn2}}{2 \cdot m_n} - \left[\frac{\pi}{4} + \left(\frac{h_{fp2}}{m_n} - \frac{d_{en2} - d_{fn2}}{2 \cdot m_n}\right) \cdot \tan\alpha_n\right] \cdot \tan\alpha_n - \frac{\rho_{fp2}}{m_n} \cdot \left(1 - \sin\frac{\pi}{6}\right)$$

$$d_{fn2} = d_{n2} + d_{f2} - d_2 mm\ (in.)$$

$$h_{fp2} = \frac{d_{n2} - d_{fn2}}{2} mm\ (in.)$$

$$\rho_{fp2} = \rho_{F2} = \frac{\rho_{a02}}{2} mm\ (in.)$$

Note: In the case of a full root fillet $\rho_{F2} = \rho_{a02}$ is to be used, or as an approximation:

$$\rho_{a02} = 0.30 \cdot m_n mm\ (in.)$$

$$\rho_{fp2} = \rho_{F2} = 0.15 \cdot m_n mm\ (in.)$$

$$h_{f2} = (1.25...1.30) \cdot m_n mm\ (in.)$$

$$\rho_{fp2} = \frac{C_p}{1 - \sin\alpha_n} = \frac{h_{f2} - h_{Nf2}}{1 - \sin\alpha_n} = \frac{d_{Nf2} - d_{f2}}{2 \cdot (1 - \sin\alpha_n)} mm\ (in.)$$

where

d_{f2}	=	root diameter of wheel; mm (in.); see 4-3-1-A1/23.29 FIGURE 1.
d_{Nf2}, h_{Nf2}	=	is the diameter, dedendum of basic rack at which the usable flank and root fillet of the annulus gear meet; mm (in.)
c_p	=	is the bottom clearance between basic rack and mating profile; mm (in.)

Note:

The diameters d_{n2} and d_{en2} are to be calculated with the same formulas as for external gears.

23.5.3 Bevel Gears *(2006)*

Width of tooth, s_{Fn} , at tooth-root normal chord:

$$\frac{S_{Fn}}{m_{mn}} = z_{vn} \cdot \sin\left(\frac{\pi}{3} - \vartheta\right) + \sqrt{3} \cdot \left(\frac{G}{\cos\vartheta} - \frac{\rho_{a0}}{m_{mn}}\right)$$

$$\vartheta = 2 \cdot \frac{G}{z_{vn}} \cdot \tan\vartheta - H \text{degrees; to be solved iteratively}$$

$$G = \frac{\rho_{a0}}{m_{mn}} - \frac{h_{a0}}{m_{mn}} + x_{hm}$$

$$H = \frac{2}{z_{vn}} \cdot \left(\frac{\pi}{2} - \frac{E}{m_{mn}}\right) - \frac{\pi}{3} \, [SI \ and \ MKS \ units]$$

$$H = \frac{2}{z_{vn}} \cdot \left(\frac{\pi}{2} - \frac{E}{25.4 \cdot m_{mn}}\right) - \frac{\pi}{3} \, [US \ units]$$

$$E = \frac{\pi}{4} \cdot m_{mn} - x_{sm} \cdot m_{mn} - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n} \, [SI \ and \ MKS \ units]$$

$$E = 25.4 \cdot \left(\frac{\pi}{4} \cdot m_{mn} - x_{sm} \cdot m_{mn} - h_{a0} \cdot \tan\alpha_n + \frac{S_{pr}}{\cos\alpha_n} - (1 - \sin\alpha_n) \cdot \frac{\rho_{a0}}{\cos\alpha_n}\right) [US \ units]$$

$$S_{pr} = p_{r0} - q$$

$$S_{pr} = 0 when \ gears \ are \ not \ undercut \ (non - protuberance \ hob)$$

where:

$E, h_{a0}, \alpha_n, S_{pr}, p_{r0}, q$ and ρ_{a0} are shown in 4-3-1-A1/23.29 FIGURE 2.

h_{a0}	=	addendum of tool; mm (in.)
S_{pr}	=	residual undercut left by protuberance; mm (in.)
p_{r0}	=	protuberance of tool; mm (in.)
q	=	material allowances for finish machining; mm (in.)
ρ_{a0}	=	tip radius of tool; mm (in.)
z_{vn}	=	virtual number of teeth
x_{hm}	=	profile shift coefficient
x_{sm}	=	tooth thickness modification coefficient (midface)
α_{Fan}	=	angle for application of load at the highest point of single tooth contact

α_{an} = pressure angle at the highest point of single tooth contact

Bending moment arm h_{Fa} :

$$\frac{h_{Fa}}{m_{mn}} = \frac{1}{2} \cdot \left[(\cos\gamma_a - \sin\gamma_a \cdot \tan\alpha_{Fan}) \cdot \frac{d_{van}}{m_{mn}} - z_{vn} \cdot \cos\left(\frac{\pi}{3} - \vartheta\right) - \frac{G}{\cos\vartheta} + \frac{\rho_{a0}}{m_{mn}} \right]$$

$$\frac{\rho_F}{m_{mn}} = \frac{\rho_{a0}}{m_{mn}} + \frac{2 \cdot G^2}{\cos\vartheta \cdot (z_{vn}\cos^2\vartheta - 2G)}$$

where

ρ_F = root fillet radius in the critical section at 30° tangent; mm (mm, in.); see 4-3-1-A1/23.29 FIGURE 1.

Normal load pressure angle at tip of tooth $\alpha_{Fan}, \alpha_{an}$:

$$\alpha_{Fan} = \alpha_{an} - \gamma_a \text{degrees}$$

$$\alpha_{an} = \arccos\left(\frac{d_{vbn}}{d_{van}}\right) \text{ degrees}$$

$$\gamma_a = \left(\frac{0.5 \cdot \pi + 2 \cdot (x_{hm} \cdot \tan\alpha_n + x_{sm}}{z_{vn}} + inv\alpha_n - inv\alpha_{an}\right) \cdot \frac{180}{\pi} \text{ degrees}$$

$$\beta_{bm} = \arccos\sqrt{1 - (\sin\beta_m \cdot \cos\alpha_n)^2} \text{ degrees}$$

(See also 4-3-1A1/7 of this Appendix)

$$d_{van1} = d_{vn1} + d_{va1} - d_{v1} mm \ (in.)$$

$$d_{van2} = d_{vn2} + d_{va2} - d_{v2} mm \ (in.)$$

$$d_{vn1} = Z_{vn1} \cdot m_{mn} mm \ (in.)$$

$$d_{vn2} = Z_{vn2} \cdot m_{mn} mm \ (in.)$$

$$d_{vbn1} = d_{\mathrm{vn1}} \cdot \cos\alpha_n mm \ (in.)$$

$$d_{vbn2} = d_{\mathrm{vn2}} \cdot \cos\alpha_n mm \ (in.)$$

23.7 Stress Correction Factor, Y_S, Y_{Sa} ,

The stress correction factors, Y_S and Y_{Sa} , are used to convert the nominal bending stress to the local tooth root stress.

Y_S applies to the load application at the outer point of single tooth pair contact. Y_S is to be determined for pinion and wheel, separately.

For notch parameter q_s within a range of ($1 \leq q_s < 8$):

$$q_s = \frac{S_{Fn}}{2 \cdot \rho_F}$$

Cylindrical gears:

$$Y_S = (1.2 + 0.13 \cdot L) \cdot q_s^{\left(\frac{1}{1.21+(2.3/L)}\right)}$$

Bevel gears:

$$Y_{Sa} = (1.2 + 0.13 \cdot L_a) \cdot q_s^{\left(\frac{1}{1.21+(2.3/La)}\right)}$$

where:

ρ_F	=	root fillet radius in the critical section at 30° tangent mm (in.)
L	=	s_{Fn}/h_F for cylindrical gears
L_a	=	s_{Fn}/h_{Fa} for bevel gears bevel gears

$h_F, h_{Fa}, s_{Fn}, \rho_F$ see 4-3-1-A1/23.29 FIGURE 3.

23.9 Helix Angle Factor, Y_β

The helix angle factor, Y_β , converts the stress calculated for a point loaded cantilever beam representing the substitute gear tooth to the stress induced by a load along an oblique load line into a cantilever plate which represents a helical gear tooth.

$$Y_\beta = 1 - \epsilon_\beta \cdot \frac{\beta}{120}$$

where:

β	=	reference helix angle in degrees for cylindrical gears

$\epsilon_\beta > 1.0$ a value of 1.0 is to be substituted for ϵ_β

$\beta > 30^\circ$ an angle of 30° is to be substituted for β

23.11 Contact Ratio Factor, Y_ϵ

The contact factor, Y_ϵ **,** covers the conversion from load application at the tooth tip to the load application for bevel gears.

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_a}\,; \text{for} \epsilon_\beta = 0$$

$$Y_\epsilon = 0.25 + \frac{0.75}{\epsilon_a} - \left(\frac{0.75}{\epsilon_a} - 0.375\right) \cdot \epsilon_\beta; \text{for} 0 < \epsilon_\beta < 1$$

$$Y_\epsilon = 0.625; \text{for} \epsilon_\beta \geq 1$$

23.13 Bevel Gear Factor, Y_K

The bevel gear factor, Y_K , accounts for the differences between bevel and cylindrical gears.

$$Y_K = \frac{1}{2} + \frac{b}{4 \cdot \ell'_{bm}} + \frac{\ell'_{bm}}{4 \cdot b}$$

$$\ell'_{bm} = \ell_{bm} \cdot \cos\beta_{vb}$$

23.15 Load Sharing Factor, Y_{LS}

The load sharing factor, Y_{LS} accounts for the differences between two or more pair of teeth for $\epsilon_\gamma > 2.$

$$Y_{LS} = Z_{LS}^2 \geq 0.7$$

for Y_{LS} , see 4-3-1-A1/21.13 of this Appendix.

23.17 Allowable Stress Number (Bending), σ_{FE} *(2012)*

For a given material, σ_{FE} is the limit of repeated tooth root stress that can be sustained. For most materials, their stress cycles may be taken at 3 × 10^6 as the beginning of the endurance limit, unless otherwise specified.

The endurance limit, σ_{FE} , is defined as the unidirectional pulsating stress with a minimum stress of zero (disregarding residual stresses due to heat treatment). Other conditions such as e.g., alternating stress or prestressing are covered by the design factor Y_d.

The endurance limit mainly depends on:

- Material composition, cleanliness and defects
- Mechanical properties
- Residual stress
- Hardening process, depth of hardened zone, hardness gradient
- Material structure (forged, rolled bar, cast)

The σ_{FE} values are to correspond to a failure probability of 1% or less. The values of σ_{FE} are to be determined from 4-3-1-A1/23.29 TABLE 3 or to be advised by the manufacturer, together with technical justification for the proposed values. For gears treated with controlled shot peening process, the value, σ_{FE} , can be increased by 10%.

23.19 Design Factor, Y_d **(2025)**

The design factor, Y_d , takes into account the influence of load reversing and shrink fit prestressing on the tooth root strength, relative to the tooth root strength with unidirectional load as defined for σ_{FE} .

Y_d	=	0.9	for gear with part load in reversed direction, such as main wheel in reversing gearboxes;
Y_d	=	0.7	for idler gears.
Y_d	=	1.0	for all other cases.

23.21 Life Factor, Y_N

The life factor, Y_N , accounts for the higher permissible tooth root bending stress in case a limited life (number of load cycles) is specified.

The factor mainly depends on:

- Material and hardening
- Number of cycles
- Influence factors ($Y_{\delta relT}, Y_{RrelT}, Y_X$)

The life factor, Y_N can be determined from 4-3-1-A1/23.29 TABLE 5.

23.23 Relative Notch Sensitivity Factor, $Y_{\delta relT}$ *(2002)*

The relative notch sensitivity factor, $Y_{\delta relT}$, indicates the extent to which the theoretically concentrated stress lies above the fatigue endurance limit.

The factor mainly depends on the material and relative stress gradient.

For notch parameter values within the range of $1.5 \leq q_s < 4, Y_{\delta relT} = 1.0$

For $q_s < 1.5, Y_{\delta relT} = 0.95$

For notch parameter $q_s \geq 4, Y_{\delta relT}$ can be determined by the methods outlined in ISO 6336-3, Section 11 Sensitivity factors, $Y_\delta, Y_{\delta T}, Y_{\delta k}$ and relative notch sensitivity factors, $Y_{\delta relT}, Y_{\delta relk}$.

23.25 Relative Surface Factor, Y_{RrelT}

The relative surface factor, Y_{RrelT} , as given in the following table, takes into account the dependence of the tooth root bending strength on the surface condition in the tooth root fillet, but mainly the dependence on the peak to valley surface roughness.

	$R_{zr} < 1\mu$ m	SI & MKS units	US units
	$R_{zr} < 39\mu$ in	1μ m $\leq R_{zr} \leq 40\mu$ m	39μ in $\leq R_{zr} \leq 1575\mu$ in
Case hardened steels, through - hardened steels *U* ≥ 800 N/mm^2 (82 kgf/mm^2, 1.16 × 10^5 psi)	1.120	$1.675 - 0.53 \cdot (R_{zr}+1)^{0.1}$	$1.675 - 0.53 \cdot (0.024 \cdot R_{zr}+1)^{0.1}$
Normalized steels *U* < 800 N/mm^2 (82 kgf/mm^2, 1.16 × 10^5 psi)	1.070	$5.3 - 4.2 \cdot (R_{zr}+1)^{0.01}$	$5.3 - 4.2(0.0254R_{zr}+1)^{0.01}$
Nitrided steels	1.025	$4.3 - 3.26 \cdot (R_{zr}+1)^{0.005}$	$4.3 - 3.26 \cdot (0.0254R_{zr}+1)^{0.005}$
R_{zr} = mean peak to-valley roughness of tooth root fillets; μm (μm, μin)			

This method is only applicable where scratches or similar defects deeper than 2 R_{zr} are not present.

If the stated roughness is an R_a value, also known as arithmetic average (*AA*) and centerline average (*CLA*), the following approximate relationship may be applied:

$$R_a = CLA = AA = R_{zr}/6$$

23.27 Size Factor (Root), Y_X **(2025)**

The size factor (root), Y_X , takes into account the decrease of the strength with increasing size.

The factor mainly depends on:

- Material and heat treatment
- Tooth and gear dimensions
- Ratio of case depth to tooth size

SI and MKS units		
$Y_X = 100$	for $m_n \leq 5$	For all cases
$Y_X = 1.03 - 0.006 \cdot m_n$	for $5 < m_n < 30$	Normalized and through tempered-hardened steels
$Y_X = 0.85$	for $m_n \geq 30$	
$Y_X = 1.05 - 0.010 \cdot m_n$	for $5 < m_n < 25$	Surface hardened steels
$Y_X = 0.80$	for $m_n \geq 25$	

US units		
$Y_X = 100$	for $m_n \leq 0.1968$	For all cases
$Y_X = 1.03 - 0.1524 \cdot m_n$	for $0.1968 < m_n < 1.181$	Normalized and through tempered- hardened steels
$Y_X = 0.85$	for $m_n \geq 1.181$	
$Y_X = 1.03 - 0.254 \cdot m_n$	for $0.1968 < m_n < 0.9842$	Surface hardened steels
$Y_X = 0.80$	for $m_n \geq 0.9842$	

Note: *For Bevel gears*, the m_n (normal module) is to be substituted by m_{mn} (normal module at mid-facewidth).

23.29 Safety Factor for Tooth Root Bending Stress, S_F

Based on the application, the following safety factors for tooth root bending stress, S_F , are to be applied:

Main propulsion gears (including PTO):	1.80
Duplicated (or more) independent main propulsion gears (including azimuthing thrusters):	1.60
Main propulsion gears for yachts, single screw:	1.50
Main propulsion gears for yachts, multiple screw:	1.45
Auxiliary gears:	1.40

Note:

For the above purposes, yachts are considered pleasure craft not engaged in trade or carrying passengers, and not intended for charter-service.

TABLE 1
Values of the Elasticity Factor Z_E and Young's Modulus of Elasticity E

(Ref. 4-3-1-A1/21.29)

The value of E for combination of different materials for pinion and wheel is to be calculated by:

$$E = \frac{2 \cdot E_1 \cdot E_2}{E_1 + E_2}$$

SI units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 N/mm²	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 N/mm²	Poisson's ratio ν	Elasticity Factor Z_E $N^{1/2}$/mm
Steel	206000	0.3	Steel	206000	0.3	189.8

SI units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 N/mm²	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 N/mm²	Poisson's ratio ν	Elasticity Factor Z_E $N^{1/2}$/mm
			Cast steel	202000		188.9
			Nodular cast iron	173000		181.4
			Cast tin bronze	103000		155.0
			Tin bronze	113000		159.8
			Lamellar graphite cast iron (gray cast iron)	126000 to 118000		165.4 to 162.0
Cast steel	202000		Cast steel	202000		188.0
			Nodular cast iron	173000		180.5
			Lamellar graphite cast iron (gray cast iron)	118000		161.4
Nodular cast iron	173000	0.3	Nodular cast iron	173000	0.3	173.9
			Lamellar graphite cast iron (gray cast iron)	118000		156.6
Lamellar graphite cast iron (gray cast iron)	126000 to 118000		Lamellar graphite cast iron (gray cast iron)	118000		146.0 to 143.7
Steel	206000	0.3	Nylon	7850 (mean value)	0.5	56.4

MKS units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 kgf/mm²	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 kgf/mm²	Poisson's ratio ν	Elasticity Factor Z_E $kgf^{1/2}$/mm
Steel	2.101×10^4	0.3	Steel	2.101×10^4	0.3	60.609
			Cast steel	2.060×10^4		60.321
			Nodular cast iron	1.764×10^4		57.926
			Cast tin bronze	1.050×10^4		49.496
			Tin bronze	1.152×10^4		51.029
			Lamellar graphite cast iron (gray cast iron)	1.285×10^4 to 1.203×10^4		52.817 to 51.731
Cast steel	2.060×10^4		Cast steel	2.060×10^4		60.034
			Nodular cast iron	1.764×10^4		57.639
			Lamellar graphite cast iron (gray cast iron)	1.203×10^4		51.540
Nodular cast iron	1.764×10^4	0.3	Nodular cast iron	1.764×10^4	0.3	55.531

MKS units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 kgf/mm²	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 kgf/mm²	Poisson's ratio ν	Elasticity Factor Z_E $kgf^{1/2}/mm$
			Lamellar graphite cast iron (gray cast iron)	1.203×10^4		50.007
Lamellar graphite cast iron (gray cast iron)	1.285×10^4 to 1.203×10^4		Lamellar graphite cast iron (gray cast iron)	1.203×10^4		46.622 to 45.888
Steel	2.101×10^4	0.3	Nylon	800.477 (mean value)	0.5	18.010

US units						
Pinion			Wheel			
Material	Young's Modulus of elasticity E_1 psi	Poisson's ratio ν	Material	Young's Modulus of elasticity E_2 psi	Poisson's ratio ν	Elasticity factor Z_E $lbf^{1/2}/in.$
Steel	2.988×10^7	0.3	Steel	2.988×10^7	0.3	2.286×10^3
			Cast steel	2.930×10^7		2.275×10^3
			Nodular cast iron	2.509×10^7		2.185×10^3
			Cast tin bronze	1.494×10^7		1.867×10^3
			Tin bronze	1.639×10^7		1.924×10^3
			Lamellar graphite cast iron (gray cast iron)	1.827×10^7 to 1.711×10^7		1.992×10^3 to 1.951×10^3
Cast steel	2.930×10^7		Cast steel	2.930×10^7		2.264×10^3
			Nodular cast iron	2.509×10^7		2.174×10^3
			Lamellar graphite cast iron (gray cast iron)	1.711×10^7		1.944×10^3
Nodular cast iron	2.509×10^7	0.3	Nodular cast iron	2.509×10^7	0.3	2.094×10^3
			Lamellar graphite cast iron (gray cast iron)	1.711×10^7		1.886×10^3
Lamellar graphite cast iron (gray cast iron)	1.827×10^7 to 1.711×10^7		Lamellar graphite cast iron (gray cast iron)	1.711×10^7		1.758×10^3 to 1.731×10^3
Steel	2.988×10^7	0.3	Nylon	1.139×10^6 (mean value)	0.5	679.234

TABLE 2
Size Factor Z_X for Contact Stress
(Ref. 4-3-1A1/21.27)

SI and MKS units	
Z_X , size factor for contact stress	Material
1.0	For through-hardened pinion treatment All modules (m_n)
1.0 1.05 – 0.005 m_n 0.9	For carburized and induction-hardened pinion heat treatment m_n ≤10 m_n < 30 m_n ≥30
1.0 1.08 – 0.011 m_n 0.75	For nitrided pinion treatment m_n < 7.5 m_n < 30 m_n ≥30
For ***Bevel gears*** the m_n (normal module) is to be substituted by m_{mn} (normal module at mid-facewidth).	

US units	
Z_X , size factor for contact stress	Material
1.0	For through-hardened pinion treatment All modules (m_n)
1.0 1.05 – 0.127 m_n 0.9	For carburized and induction-hardened pinion heat treatment m_n ≤0.394 m_n < 1.181 m_n ≥1.181
1.0 1.08 – 0.279 m_n 0.75	For nitrided pinion treatment m_n < 0.295 m_n < 1.181 m_n ≥1.181
For ***Bevel gears*** the m_n (normal module) is to be substituted by m_{mn} (normal module at mid-facewidth).	

TABLE 3
Allowable Stress Number (contact) $\sigma_{H\lim}$ and
Allowable Stress Number (bending) σ_{FE}

(Ref. 4-3-1-A1/21.19, 4-3-1-A1/23.17)

SI units	$\sigma_{H\lim}$ N/mm²	σ_{FE} N/mm²	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Case hardened (carburized) CrNiMo steels:			Fig. 9, MQ
• of ordinary grade;	1500	920	Fig. 11, MQ (1)
• of specially approved high quality grade (to be based on review and verification of established testing procedure).			Fig. 9, ME
	1650	1050	Fig. 11, ME (2)
Other case hardened (carburized) steels	1500		Fig. 9, MQ
		840	Fig. 11, MQ (3)
Gas nitrided steels: hardened, tempered and gas nitrided, Surface hardness: 700-850 HV10	1250		Fig. 13a, MQ
		920	Fig. 14a, MQ
Through hardened steels: hardened, tempered and gas nitrided, Surface hardness: 500-650 HV10	1000		Fig. 13b, MQ
		740	Fig. 14b, MQ
Through hardened steels: hardened, tempered or normalized and nitro-carburized, Surface hardness: 450-650 HV10	950		Fig. 13c, ME-MQ
		780	Fig. 14c, ME-MQ
Flame or induction hardened steels, Surface hardness: 520-620 HV10	0.65·HV10 + 830		Fig. 10, MQ
		0.25·HV10 + 580	Fig. 12, MQ
Alloyed through hardening steels, Surface hardness: 195-360 HV10	1.32·HV10 + 372		Fig. 5, MQ
		0.78 HV10 + 400	Fig. 7, MQ
Through hardened carbon steels, Surface hardness: 135-210 HV10	1.05·HV10 + 335		Fig. 5, Carbon steel, MQ
		0.50·HV10 + 320	Fig. 7, Carbon steel, MQ
Alloyed cast steels, Surface hardness: 198-358 HV10	1.30 HV10 + 295		Fig. 6, MQ-ML
		0.68 HV10 + 325	Fig. 8, MQ-ML
Cast carbon steels, Surface hardness: 135-210 HV10	0.87·HV10 + 290		Fig. 6, Carbon steel, MQ-ML
		0.50·HV10 + 225	Fig. 8, Carbon steel, MQ-ML

MKS units	$\sigma_{H\lim}$ kgf/mm²	σ_{FE} kgf/mm²	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Case hardened (carburized) CrNiMo steels:			Fig. 9, MQ
• of ordinary grade;	153	93.8	Fig. 11, MQ (1)
• of specially approved high quality grade (to be based on review and verification of established testing procedure).			Fig. 9, ME
	168.3	107.1	Fig. 11, ME (2)
Other case hardened (carburized) steels	153		Fig. 9, MQ
		85.7	Fig. 11, MQ (3)
Gas nitrided steels: hardened, tempered and gas nitrided, Surface hardness: 700-850 HV10	127.5		Fig. 13a, MQ
		93.8	Fig. 14a, MQ
Through hardened steels: hardened, tempered and gas nitrided, Surface hardness: 500-650 HV10	102		Fig. 13b, MQ
		75.5	Fig. 14b, MQ
Through hardened steels: hardened, tempered or normalized and nitro-carburized, Surface hardness: 450-650 HV10	96.9		Fig. 13c, ME-MQ
		79.5	Fig. 14c, ME-MQ
Flame or induction hardened steels, Surface hardness: 520-620 HV10	0.0663·HV10 + 84.6		Fig. 10, MQ
		0.0255·HV10 + 59.1	Fig. 12, MQ
Alloyed through hardening steels, Surface hardness: 195-360 HV10	0.1346·HV10 + 37.9		Fig. 5, MQ
		0.0795 HV10 + 40.8	Fig. 7, MQ

MKS units	$\sigma_{H\lim}$ kgf/mm²	σ_{FE} kgf/mm²	Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality
Through hardened carbon steels, Surface hardness: 135-210 HV10	0.1071·HV10 + 34.2		Fig. 5, Carbon steel, MQ
		0.0510·HV10 + 32.6	Fig. 7, Carbon steel, MQ
Alloyed cast steels, Surface hardness: 198-358 HV10	0.1326 HV10 + 30.1		Fig. 6, MQ-ML
		0.0693 HV10 + 33.1	Fig. 8, MQ-ML
Cast carbon steels, Surface hardness: 135-210 HV10	0.0887·HV10 + 29.6		Fig. 6, Carbon steel, MQ-ML
		0.0510·HV10 + 22.9	Fig. 8, Carbon steel, MQ-ML
US units	**$\sigma_{H\lim}$ psi**	**σ_{FE} psi**	**Reference Standard ISO 6336-5:1996(E) ISO Figure and Material Quality**
Case hardened (carburized) CrNiMo steels:			Fig. 9, MQ
• of ordinary grade;	217557	133435	Fig. 11, MQ (1)
			Fig. 9, ME
• of specially approved high quality grade (to be based on review and verification of established testing procedure).	239312	152290	Fig. 11, ME (2)
Other case hardened (carburized) steels	217557		Fig. 9, MQ
		121832	Fig. 11, MQ (3)
Gas nitrided steels: hardened, tempered and gas nitrided, Surface hardness: 700-850 HV10	181297		Fig. 13a, MQ
		133435	Fig. 14a, MQ
Through hardened steels: hardened, tempered and gas nitrided, Surface hardness: 500-650 HV10	145038		Fig. 13b, MQ
		107328	Fig. 14b, MQ
Through hardened steels: hardened, tempered or normalized and nitro-carburized, Surface hardness: 450-650 HV10	137786		Fig. 13c, ME-MQ
		113129	Fig. 14c, ME-MQ
Flame or induction hardened steels, Surface hardness: 520-620 HV10	94.3·HV10 + 120381		Fig. 10, MQ
		36.3·HV10 + 84122	Fig. 12, MQ
Alloyed through hardening steels, Surface hardness: 195-360 HV10	191.5·HV10 + 53954		Fig. 5, MQ
		113.1 HV10 + 58015	Fig. 7, MQ
Through hardened carbon steels, Surface hardness: 135-210 HV10	152.3·HV10 + 48588		Fig. 5, Carbon steel, MQ
		72.5·HV10 + 46412	Fig. 7, Carbon steel, MQ
Alloyed cast steels, Surface hardness: 198-358 HV10	188.6 HV10 + 42786		Fig. 6, MQ-ML
		98.6 HV10 + 47137	Fig. 8, MQ-ML
Cast carbon steels, Surface hardness: 135-210 HV10	126.2·HV10 + 42061		Fig. 6, Carbon steel, MQ-ML
		72.5·HV10 + 32633	Fig. 8, Carbon steel, MQ-ML

Notes:

HV10: Vickers hardness at load F = 98.10 N, see ISO 6336-5

1. Core hardness ≥ 25 HRC, Jominy hardenability at J = 12 mm ≥ HRC 28 and Surface hardness: 640-780 HV10
2. Core hardness ≥ 30 HRC, Surface hardness: 660-780 HV10
3. Core hardness ≥ 25 HRC, Jominy hardenability at J = 12 mm < HRC 28 and Surface hardness: 640-780 HV10

TABLE 4
Determination of Life Factor for Contact Stress, Z_N

(Ref. 4-3-1-A1/21.21)

Material	Number of load cycles	Life factor Z_N
St, V, GGG (perl., bain), GTS (perl.), Eh, IF; Only when a certain degree of pitting is permissible	N_L ≤ 6 x 10^5 , static	1.6
	N_L = 10^7	1.3
	N_L = 10^9	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
St, V, GGG (perl., bain), GTS (perl.), Eh, IF	N_L ≤10^5 , static	1.6
	N_L = 5 x 10^7	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
GG, GGG (ferr.), NT (nitr.), NV (nitr.)	N_L ≤10^5 , static	1.3
	N_L = 2 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
NV (nitrocar.)	N_L ≤10^5 , static	1.1
	N_L = 2 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication,material, manufacturing,and experience	0.85 1.0
St:	steel (U < 800 N/mm^2, 82 kgf/mm^2, 1.16 × 10^5 psi)	
V:	through-hardening steel, through-hardened (U ≥ 800 N/mm^2)	
GG:	gray cast iron	
GGG (perl., bain., ferr.):	nodular cast iron (perlitic, bainitic, ferritic structure)	
GTS (perl.):	black malleable cast iron (perlitic structure)	
Eh:	case-hardening steel, case hardening	
IF:	steel and GGG, flame or induction hardened	
NT (nitr.):	nitriding steel, nitrided	
NV (nitr.):	through-hardening and case-hardening steel, nitrided	
NV (nitrocar.):	through-hardening and case-hardening steel, nitrocarburized	

TABLE 5
Determination of Life Factor for Tooth Root Bending Stress, Y_N

(Ref. 4-3-1-A1/23.21)

Material	Number of load cycles	Life factor Y_N
V, GGG (perl., bain.), GTS (perl.)	N_L ≤10^4 , static	2.5

Material	Number of load cycles	Life factor Y_N
	N_L = 3 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
Eh, IF (root)	N_L ≤10^3 , static	2.5
	N_L = 3 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
St, NT, NV (nitr.), GG, GGG (ferr.)	N_L ≤10^3 , static	1.6
	N_L = 3 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0
NV (nitrocar.)	N_L ≤10^3 , static	1.0
	N_L = 3 x 10^6	1.0
	N_L = 10^{10} Optimum lubrication, material, manufacturing, and experience	0.85 1.0

Notes:

1. Abbreviations of materials are as explained in and of this Appendix.
2. $$N_L = n \cdot 60 \cdot HPD \cdot DPY \cdot YRS$$

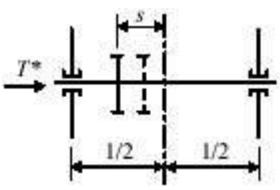
   n = rotational speed, rpm.

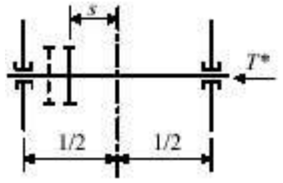
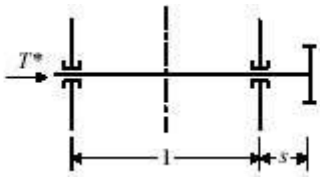
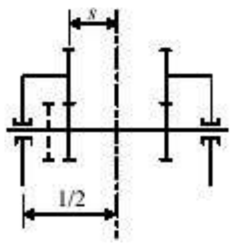
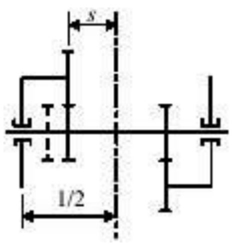
   HPD = operation hours per day

   DPY = days per year

   YRS = years (normal life of craft=25 *years*)

TABLE 6
Constant *K'* for the Calculation of the Pinion Offset Factor γ

Factor K'		Figure	Arrangement
With stiffening [1]	Without stiffening [1]		
0.48	0.8	a)	 with $s/\ell < 0.3$

Factor K'		Figure	Arrangement
With stiffening [1]	Without stiffening [1]		
-0.48	-0.8	b)	 with $s/\ell < 0.3$
1.33	1.33	c)	 with $s/\ell < 0.3$
-0.36	-0.6	d)	 with $s/\ell < 0.3$
-0.6	-1.0	e)	 with $s/\ell < 0.3$

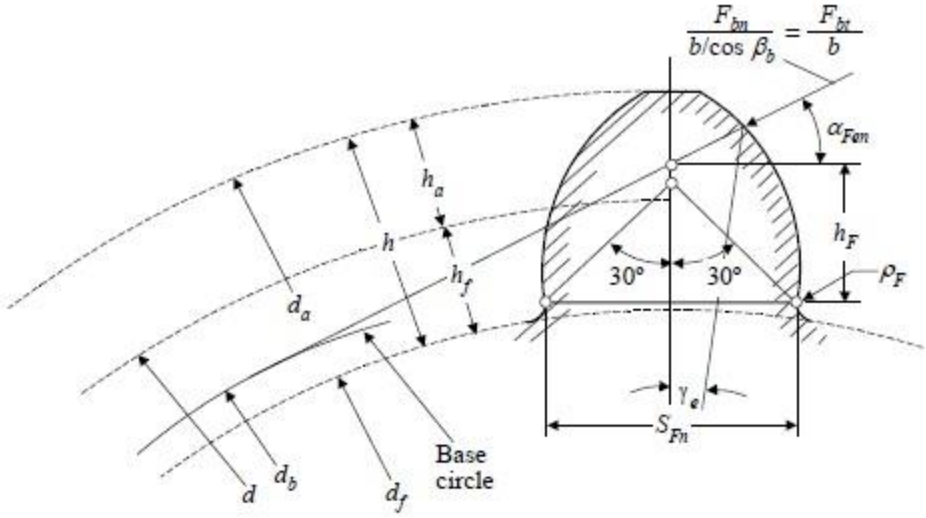
1. When $d_1/d_{sh} \geq 1.15$, stiffening is assumed; when $d_1/d_{sh} \geq 1.15$, there is no stiffening. Furthermore, scarcely any or no stiffening at all is to be expected when a pinion slides on a shaft and feather key or a similar fitting, nor when normally shrink fitted.

T^* is the input or output torqued end, not dependent on direction of rotation.

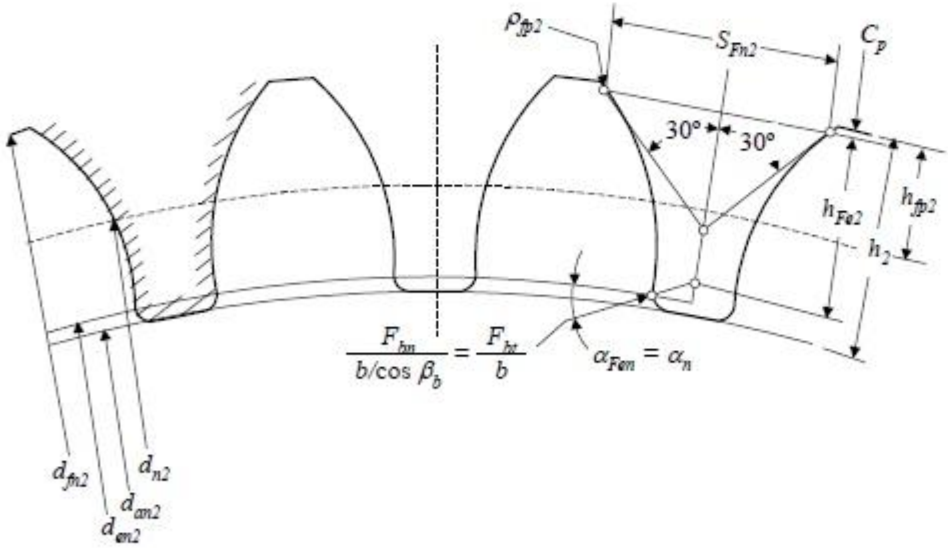
Dashed line indicates the less deformed helix of a double helical gear.

Determine t_{sh} from the diameter in the gaps of double helical gearing mounted centrally between bearings.

FIGURE 1
Tooth in Normal Section



External cylindrical gears



Internal cylindrical gears

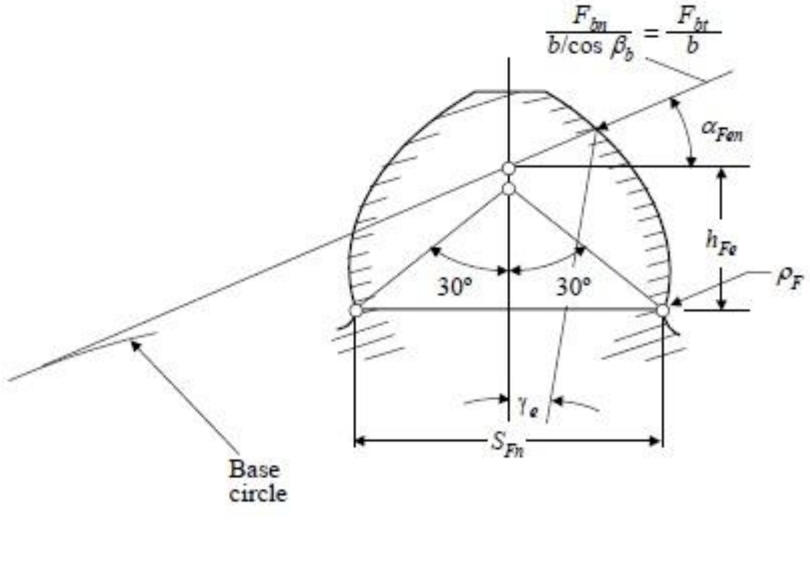
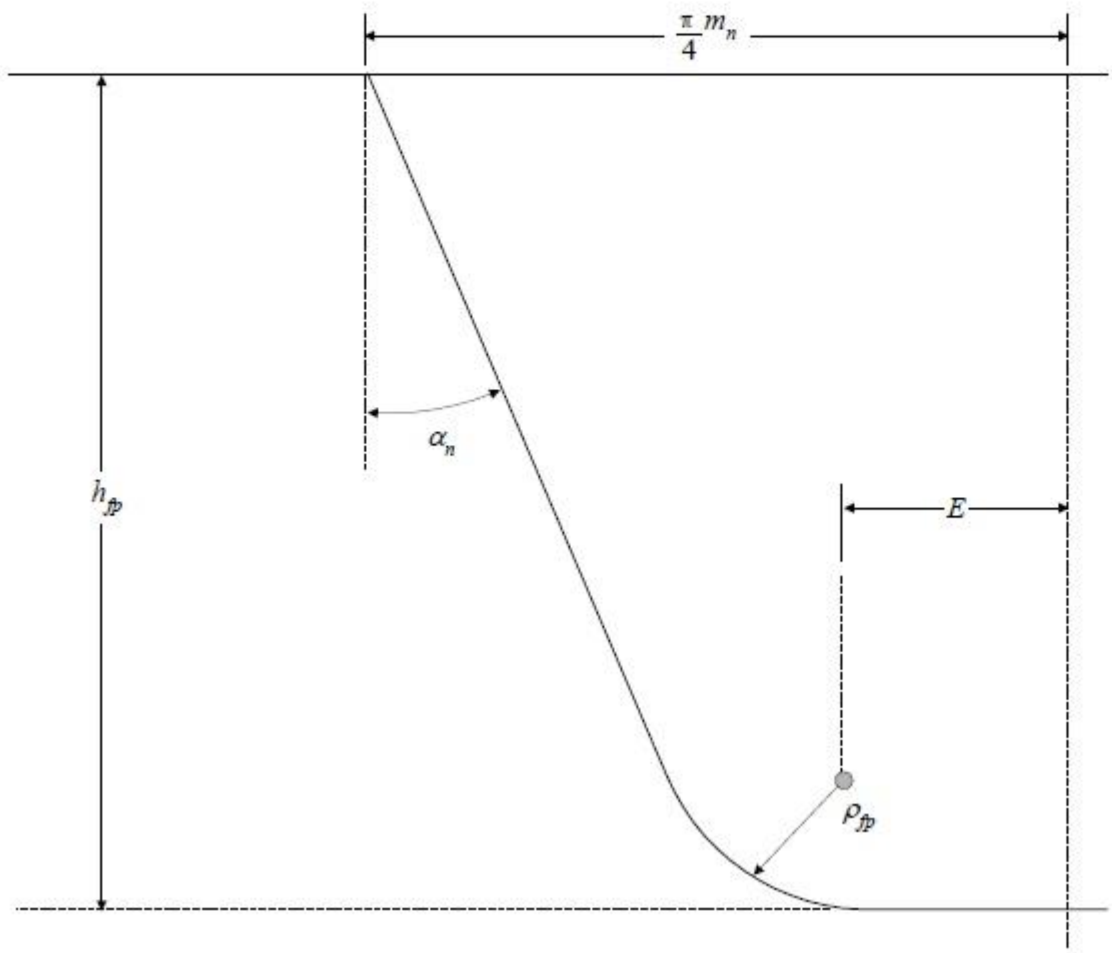
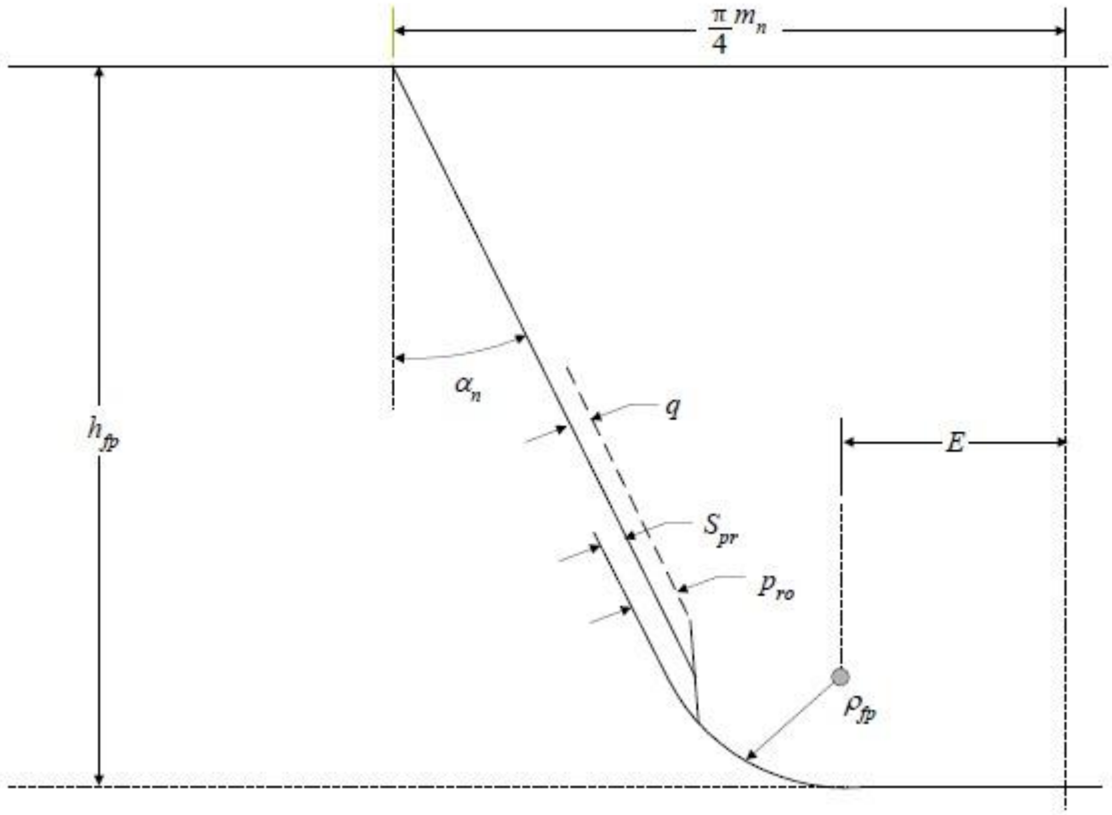


FIGURE 2
Dimensions and Basic Rack Profile of the Tooth (Finished Profile)



Without undercut



With undercut

FIGURE 3

Wheel Blank Factor C_R, Mean Values for Mating Gears of Similar or Stiffer Wheel Blank Design

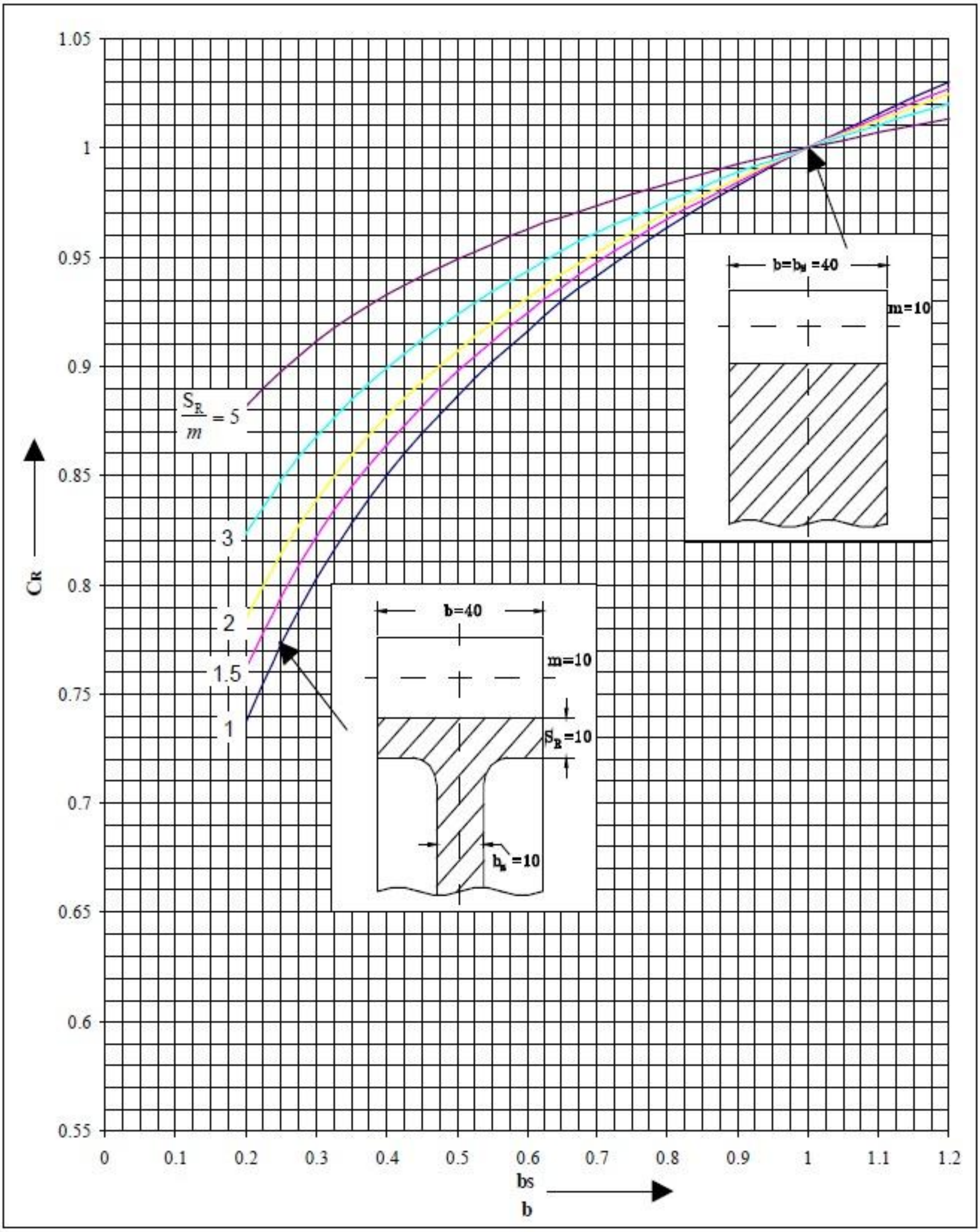


FIGURE 4
Bevel Gear Conversion to Equivalent Cylindrical Gear

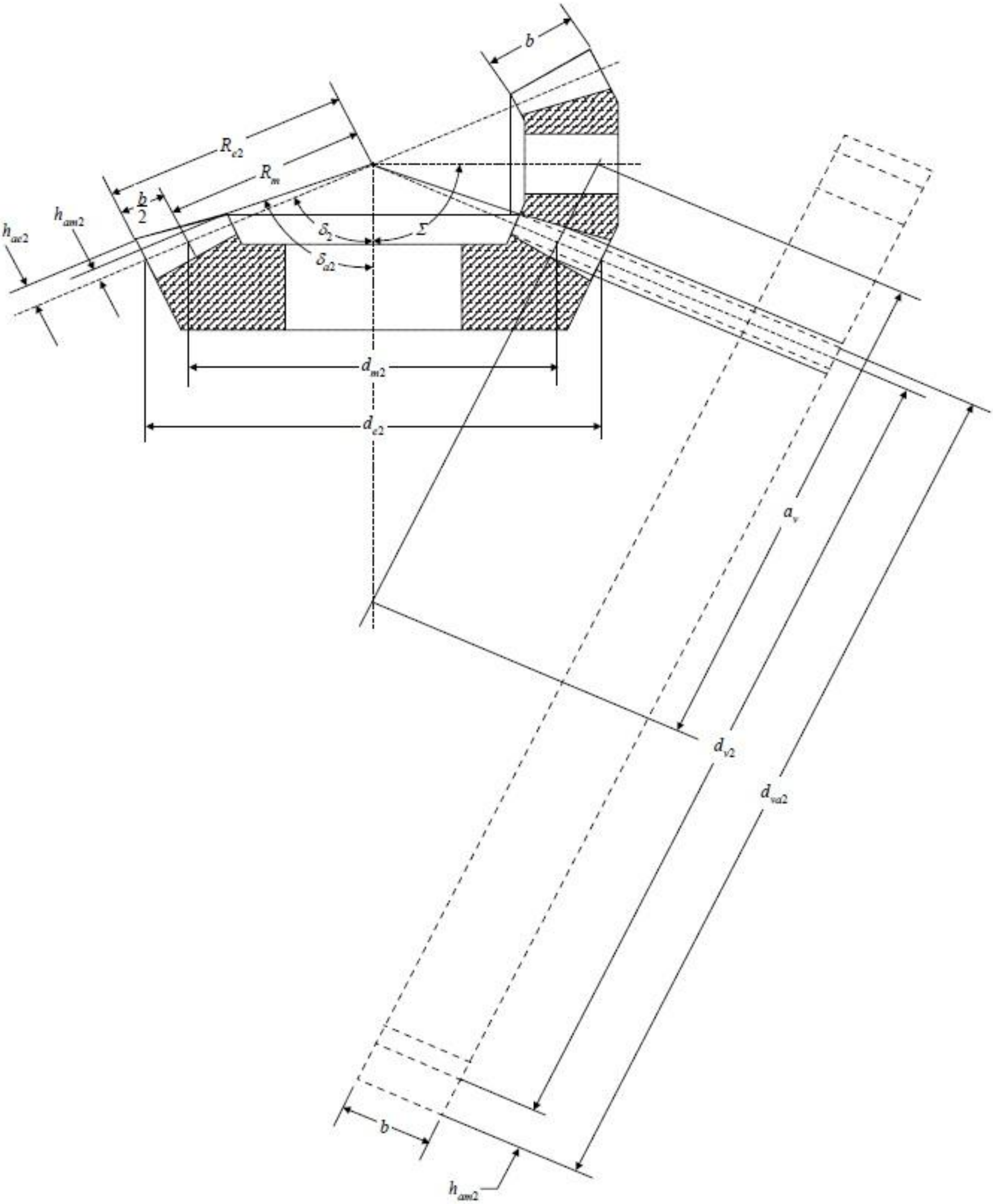
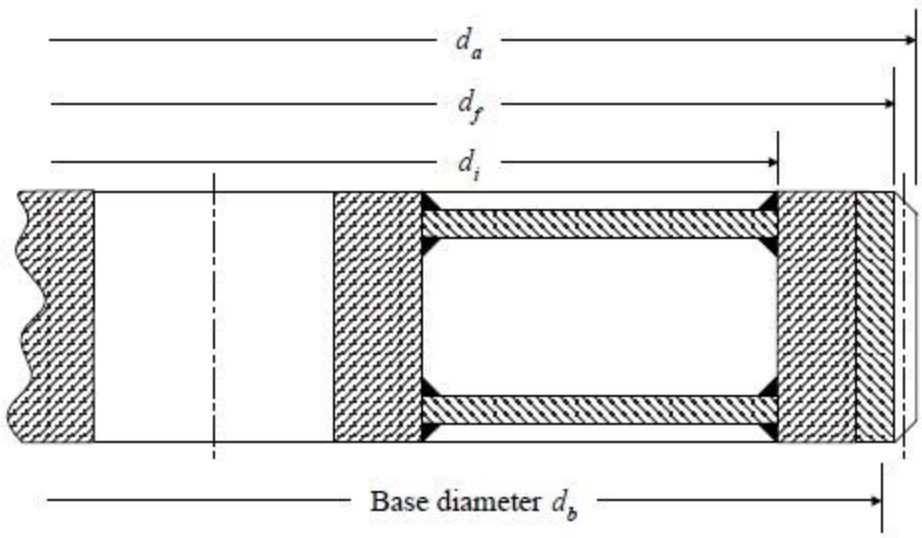


FIGURE 5
Definitions of the Various Diameters



Appendix 2 Gear Parameters

For purposes of submitting gear design for review, the following data and parameters may be used as a guide.

Gear manufacturer .... Year of build ....

Shipyard .... Hull number ....

Ship's name .... Shipowner ....

GENERAL DATA (1)

Gearing type ....

.... (non reversible, single reduction, double reduction, epicyclic, etc.)

Total gear ratio ....

Manufacturer and type of the main propulsion plant (or of the auxiliary machinery) ....

....

Power(2) .... kW, (PS, hp) Rotational speed (2) .... RPM

Maximum input torque for continuous service .... N-m, (kgf-m, lbf-ft)

Maximum input rotational speed for continuous service .... RPM

Type of coupling: (stiff coupling, hydraulic or equivalent coupling, high-elasticity coupling, other couplings,

quill shafts, etc.) ....

Specified grade of lubricating oil ....

Expected oil temperature when operating at the classification power (mean values of temperature at inlet and

outlet of reverse and/or reduction gearing) ....

Value of nominal kinematic viscosity, ν , at 40°C or 50°C of oil temperature .... (mm^2/s)

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Transmitted rated power, input in the gear drive – kW (mhp, hp) (3)	P				
Rotational speed, input in the gear drive (RPM) (2)	n				
Nominal transverse tangential load at reference cylinder – N, (kgf, lbf) (4)	F_t				
Number of teeth of pinion	z_1				
Number of teeth of wheel	z_2				
Gear ratio	u				
Center distance – mm (in.)	a				
Facewidth of pinion – mm (in.)	b_1				
Facewidth of wheel – mm (in.)	b_2				
Common facewidth – mm (in.)	b				
Overall facewidth, including the gap for double helical gears – mm (in.)	B				

1 The manufacturer can supply values, if available, supported by documents for stress numbers σ_{Hlim} and σ_{FE} involved in the formulas for gear strength with respect to the contact stress and with respect to the tooth root bending stress. See 4-3-1-A1/21 and 4-3-1-A1/23.

2 Maximum continuous performance of the machinery for which classification is requested.

3 It is intended the power mentioned under note (2) or fraction of it in case of divided power.

4 The nominal transverse tangential load is calculated on the basis of the above mentioned maximum continuous performance or on the basis of astern power when it gives a higher torque. In the case of navigation in ice, the nominal transverse tangential load is to be increased as required by 4-3-1-A1/9.

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Is the wheel an external or an internal teeth gear?	-				
Number of pinions	-				
Number of wheels	-				
Is the wheel an external or an internal teeth gear	-				
Is the pinion an intermediate gear?	-				
Is the wheel an intermediate gear?	-				
Is the carrier stationary or revolving (star or planetcarrier)?(5)	-				
Reference diameter of pinion – mm (in.)	d_1				
Reference diameter of wheel – mm (in.)	d_2				
Working pitch diameter of pinion – mm (in.)	d_{w1}				
Working pitch diameter of wheel – mm (in.)	d_{w2}				
Tip diameter of pinion – mm (in.)	d_{a1}				

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS Gear drive designation		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Tip diameter of wheel – mm (in.)	d_{a2}				
Base diameter of pinion – mm (in.)	d_{b1}				
Base diameter of wheel – mm (in.)	d_{b2}				
Tooth depth of pinion – mm (in.)(6)	h_1				
Tooth depth of wheel – mm (in.)(6)	h_2				
Addendum of pinion – mm (in.)	h_{a1}				
Addendum of wheel – mm (in.)	h_{a2}				
Addendum of tool referred to normal module for pinion	h_{a01}				
Addendum of tool referred to normal module for wheel	h_{a02}				
Dedendum of pinion – mm (in.)	h_{f1}				
Dedendum of wheel – mm (in.)	h_{f2}				
Dedendum of basic rack [referred to m_n (= h_{a01})] for pinion	h_{fp1}				
Dedendum of basic rack [referred to m_n (= h_{a02})] for wheel	h_{fp2}				
Bending moment arm of tooth – mm (in.)(7)	h_F				
Bending moment arm of tooth – mm (in.)(8)	h_{F2}				
Bending moment arm of tooth – mm (in.)(9)	h_{Fa}				
Width of tooth at tooth-root normal chord – mm (in.)(7,9)	S_{Fn}				
Width of tooth at tooth-root normal chord – mm (in.)(8)	S_{Fn2}				
Angle of application of load at the highest point of single tooth contact (degrees)	α_{Fen}				
Pressure angle at the highest point of single tooth contact (degrees)	α_{en}				

5 Only for epicyclic gears.

6 Measured from the tip circle (or circle passing through the point of beginning of the fillet at the tooth tip) to the beginning of the root fillet

7 Only for external gears.

8 Only for internal gears.

9 Only for bevel gears.

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS (Gear drive designation)		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Normal module – mm (in.)	m_n				
Outer normal module – mm (in.)	m_{na}				

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS (Gear drive designation)		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Transverse module	m_t				
Addendum modification coefficient of pinion	x_1				
Addendum modification coefficient of wheel	x_2				
Addendum modification coefficient refers to the midsection(9)	x_{mn}				
Tooth thickness modification coefficient (midface)(9)	x_{sm}				
Addendum of tool – mm (in.)	h_{a0}				
Protuberance of tool – mm (in.)	P_{r0}				
Machining allowances – mm (in.)	q				
Tip radius of tool – mm (in.)	ρ_{a0}				
Profile form deviation – mm (in.)	f_{fa}				
Transverse base pitch deviation – mm (in.)	f_{pb}				
Root fillet radius at the critical section – mm (in.)	ρ_F				
Root radius of basic rack [referred to m_n (= ρ_{a0})] – mm (in.)	ρ_{fp}				
Radius of curvature at pitch surface – mm (in.)	ρ_c				
Normal pressure angle at reference cylinder (degrees)	α_n				
Transverse pressure angle at reference cylinder (degrees)	α_t				
Transverse pressure angle at working pitch cylinder (degrees)	α_{tw}				
Helix angle at reference cylinder (degrees)	β				
Helix angle at base cylinder (degrees)	β_b				
Transverse contact ratio	ϵ_α				
Overlap ratio	ϵ_β				
Total contact ratio	ϵ_γ				
Angle of application of load at the highest point of single tooth contact (degrees)(9)	α_{Fan}				
Reference cone angle of pinion (degrees)(9)	δ_1				
Reference cone angle of wheel (degrees)(9)	δ_2				
Shaft angle (degrees)(9)	$\sum$				
Tip angle of pinion (degrees)(9)	$\delta_{\alpha 1}$				
Tip angle of wheel (degrees)(9)	$\delta_{\alpha 2}$				
Helix angle at reference cylinder (degrees)(9)	β_m				

CHARACTERISTIC ELEMENTS OF PINIONS AND WHEELS (Gear drive designation)		First gear Drive	Second gear drive	Third gear drive	Fourth gear drive
Cone distance pinion. wheel – mm (in.)[9]	R				
Middle cone distance – mm (in.)[9]	R_m				

9 Only for bevel gears.

MATERIALS

First gear drive

Pinion: Material grade or specification ....

Complete chemical analysis ....

....

Minimum ultimate tensile strength[10] .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength[10].. N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... % Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Specified surface roughness RZ or Ra relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures

proposed to ensure the securing of rims.[11] ....

Wheel: Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength[10] .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength[10].. N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC)

....

Depth of hardened layer versus hardness values (if possible in diagram)

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth)

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.(11) ....

10 Relevant to core of material

11 In case of shrink-fitted pinions, wheel rims or hubs

Second gear drive

<u>Pinion:</u> Material grade or specification ....

Complete chemical analysis

....

....

Minimum ultimate tensile strength (10) .... N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) .... N/mm², (kgf/mm², lbf/in.²)

Elongation (A_5) .... % Hardness (HB, HV10 or HRC)...................

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth)

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any

....

Specified grade of accuracy (according to ISO 1328)

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims. (11)

....

Wheel: Material grade or specification ....

Complete chemical analysis

....

....

Minimum ultimate tensile strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC) ....

Depth of hardened layer versus hardness values (if possible in diagram)

....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth)

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet

....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any

....

Specified grade of accuracy (according to ISO 1328)

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims. (11)

....

10 Relevant to core of material

11 In case of shrink-fitted pinions, wheel rims or hubs

Third gear drive

Pinion: Material grade or specification

....

Complete chemical analysis

....

....

Minimum ultimate tensile strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) ....N/mm², (kgf/mm², lbf/in.²

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC)

....

Depth of hardened layer versus hardness values (if possible in diagram)

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth)

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet

....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any

....

Specified grade of accuracy (according to ISO 1328)

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims. (11)

....

Wheel: Material grade or specification

....

Complete chemical analysis

....

....

Minimum ultimate tensile strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Minimum yield strength (10) ....N/mm², (kgf/mm², lbf/in.²)

Elongation (A_5)...................................... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC)

....

Depth of hardened layer versus hardness values (if possible in diagram)

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth)

....

....

Specified surface roughness R_Z or R_a relevant to tooth flank and root fillet

....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any

....

Specified grade of accuracy (according to ISO 1328)

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims. (11)

....

10 Relevant to core of material

11 In case of shrink-fitted pinions, wheel rims or hubs

Fourth gear drive

__**Pinion:**__ Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength(10) .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength[10].. N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Specified surface hardness (HB, HV10 or HRC)..

Depth of hardened layer versus hardness values (if possible in diagram) ....

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness RZ or Ra relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.[11] ....

<u>Wheel:</u> Material grade or specification ....

Complete chemical analysis ....

....

....

Minimum ultimate tensile strength[10] .... N/mm², (kgf/mm², lbf/in²)

Minimum yield strength[10] .... N/mm², (kgf/mm², lbf/in²)

Elongation (A_5) .... %; Hardness (HB, HV10 or HRC) ....

Heat treatment ....

Description of teeth surface-hardening

....

....

Finishing method of tooth flanks (hobbed, shaved, lapped, ground or shot-peened teeth) ....

....

....

Specified surface roughness R_z or R_a relevant to tooth flank and root fillet ....

Amount of tooth flank corrections (tip-relief, end-relief, crowning and helix correction) if any ....

....

Specified grade of accuracy (according to ISO 1328) ....

....

Amount of shrinkage with tolerances specifying the procedure foreseen for shrinking and measures proposed to ensure the securing of rims.(11) ....

VARIOUS ITEMS (indicate other elements, if any, in case of particular types of gears)

....

....

Date,..

1 General

1.1 Construction and Installation

1.1.1 Internal Combustion Engines and Associated Reduction Gears (2023)

Internal combustion engines of 100 kW [135 horsepower (hp)] and over and associated reduction gears are to be constructed in accordance with Part 4, Chapters 2 and 3 of the *Marine Vessel Rules,* except the design of a unit which has demonstrated satisfactory service experience for the intended application will be specially considered. Engines of less than 100 kW (135 hp) and associated reduction gears are to function as intended, and will be accepted subject to a satisfactory performance test conducted to the satisfaction of the Surveyor after installation. For all engines, their mounting in yachts is to be in accordance with the engine manufacturer's recommendations. Particular attention is to be given to proper mounting in fiberglass yachts.

For engines driving generators, refer to the applicable requirements of 4-6-4/3.17 and 4-6-4/3.19.

Additional requirements for exhaust emission abatement equipment connected to internal combustion engines or boilers are provided in Part 6, Chapter 3 of the *Marine Vessel Rules*.

1.1.2 Reduction Gears for Windlass

Windlass reduction gears of 100 kW [135 horsepower (hp)] and over and associated reduction gears are to be constructed in accordance with Section 4-3-1 of the *Marine Vessel Rules*. Reduction gears of less than 100 kW [135 horsepower (hp)] will be accepted based on compliance with a recognized national standard.

1.3 Piping Systems

In addition to requirements for the specific system in this section, piping systems are to comply with the applicable requirements in Part 4, Chapter 4.

1.5 Pressure Vessels and Heat Exchangers

Pressure vessels and heat exchangers are to be in accordance with the applicable requirements of Part 4, Chapter 4 of the *Marine Vessel Rules*.

1.7 Torsional Vibration Stresses

Refer to 4-3-1/27.

1.9 Crankcase Ventilation

1.9.1 General

Provision is to be made for ventilation of an enclosed crankcase by means of a small breather or by means of a slight suction not exceeding 25.4 mm (1 in.) of water. Crankcases are not to be ventilated by a blast of air. Otherwise, the general arrangements and installation are to be such as to preclude the possibility of free entry of air to the crankcase.

1.9.2 Piping Arrangement

Crankcase ventilation piping is not to be directly connected with any other piping system. Crankcase ventilation pipes from each engine are normally to be led independently to the weather and fitted with corrosion-resistant flame screens. However, crankcase ventilation pipes from two or more engines may lead to a common oil mist manifold.

Where a common oil mist manifold is employed, the vent pipes from each engine are to be led independently to the manifold and fitted with a corrosion-resistant flame screen within the manifold. The arrangement is not to violate the engine manufacturer's recommendations for crankcase ventilation. The common oil mist manifold is to be accessible at all times under normal conditions and effectively vented to the weather. Where venting of the manifold to the weather is accomplished by means of a common vent pipe, the location of the manifold is to be as close as practicable to the weather such that the length of the common vent pipe is no greater than one deck height. The clear open area of the common vent pipe is not to be less than the aggregate cross-sectional area of the individual vent pipes entering the manifold, and the outlet to the weather is to be fitted with a corrosion-resistant flame screen. The manifold is also to be fitted with an appropriate draining arrangement.

1.11 Warning Notices

Suitable warning notices are to be attached in a conspicuous place on each engine and are to caution against the opening of a hot crankcase for a specified period of time after shutdown based upon the size of the engine, but not less than 10 minutes in any case. Such notice is also to warn against restarting an overheated engine until the cause of overheating has been remedied.

1.13 Bedplate

The bedplate or crankcase is to be of rigid construction, oiltight, and provided with a sufficient number of bolts to secure the same to the yacht's structure. The structural arrangements for supporting and securing the main engines are to be submitted for approval. Refer to 3-2-6/1.1.3 for structural requirements. For welded construction, see also Chapter 4 of the ABS *Rules for Materials and Welding (Part 2)*.

3 Fuel Oil Pumps and Oil Heaters

3.1 Transfer Pumps

Refer to 4-4-4/3.

3.3 Booster Pumps (for Yachts Over 47.5 meters (150 feet) Rule Length) *(2017)*

3.3.1 Standby Pump, Single Engine Installation.

An independently driven standby pump is to be provided for each service pump, booster pump and other pumps serving the same purpose.

The capacity of the pumps, with any one pump out of service, is to be sufficient for continuous operation at rated power.

3.3.2 Standby Pump, Multiple Engine Installation.

For yachts fitted with two or more propulsion engines, the provision of a common standby pump (for each service pump, booster pump, etc.) capable of serving all engines sufficient for continuous operation at rated power will suffice rather than providing individual standby pumps for each engine.

3.3.3 Attached Pumps.

For multiple engine installations, engines having service, booster or similar pumps attached to and driven by the engine may, in lieu of the standby pump, be provided with a complete pump carried on board as a spare. The spare pump, upon being installed, is to allow the operation of the engine at rated power.

The spare pump need not be carried, provided that, in the event of the loss of one engine, at least forty percent of the total rated propulsion power remains.

3.5 Heaters

When fuel oil heaters are required for main engine operation, at least two heaters of approximately equal size are to be installed. The combined capacity of the heaters is to be not less than required to supply the main engine(s) at full power.

5 Fuel Oil Pressure Piping

Pipes from booster pumps to injection systems are to be at least standard seamless steel. Pipes conveying heated oil are to be at least standard seamless or electric-resistance-welded steel. ERW pipe is to be straight seam and fabricated with no filler metal (e.g., ABS Grade 2 or 3 ERW). Valves and fittings may be screwed in sizes up to and including 60 mm O.D. (2 in. N.P.S.), but screwed unions are not to be used on pressure lines in sizes 33 mm O.D. (1 in.) and over. Valves are to be so constructed as to permit packing under pressure.

7 Fuel Oil Injection System

7.1 General

Strainers are to be provided in the fuel oil injection pump suction line.

For main propulsion engines, the arrangement is to be such that the strainers may be cleaned without interrupting the fuel supply to the engine. However, where multiple engines are provided, a dedicated simplex strainer may be fitted for each engine, provided the yacht can maintain at least one-half of the design speed or seven knots, whichever is less, while operating with one engine temporarily out of service until its strainer can be cleaned.

For auxiliary engines, the arrangement is to be such that the strainers may be cleaned without undue interruption of power necessary for propulsion. Multiple auxiliary engines, each fitted with a separate strainer and arranged such that changeover to a standby unit can be accomplished without loss of propulsion capability, will be acceptable for this purpose.

Where strainers are fitted in parallel to enable cleaning without disrupting the oil supply, means are to be provided to minimize the possibility of a strainer being opened inadvertently. Strainers are to be provided with suitable means for venting when being put in operation and being depressurized before being opened. Strainers are to be so located that in the event of leakage, oil cannot be sprayed on to the exhaust manifold or surfaces with temperatures in excess of 220°C (428°F).

The injection lines are to be of seamless drawn pipe. Fittings are to be extra heavy. The material used may be either steel or nonferrous, as approved in connection with the design. Also refer to 4-4-4/3.7.

7.3 Piping Between Injection Pump and Injectors

7.3.1 Injection Piping

All external high pressure fuel delivery lines between the high-pressure fuel pumps and fuel injectors are to be protected with a jacketed piping system capable of containing fuel from a high-pressure line failure. A jacketed pipe incorporates an outer pipe into which the high-pressure fuel pipe is placed, forming a permanent assembly. Metallic hose of an approved type may be accepted as the outer pipe, where outer piping flexibility is required for the manufacturing process of the permanent assembly. The jacketed piping system is to include means for collection of leakages and arrangements are to be provided for an alarm to be given of a fuel line failure.

7.3.2 Fuel oil Return Piping

When the peak to peak pressure pulsation in the fuel oil return piping from the injectors exceeds 20 bar (20.5 kgf/cm^2, 285 psi), jacketing of the return pipes is also required.

7.3.3 High Pressure Common Rail System

Where a high pressure common rail system is fitted to an engine, the high pressure common rail is to be in accordance with Section 4-4-1 of the *Marine Vessel Rules* for pressure vessels, or a recognized standard as listed in 4-4-1/1.5 of the *Marine Vessel Rules*. Alternatively, the design may be verified by certified burst tests. Components are to be made of steel or cast steel. Components made of steel, other than cast steel, are to withstand not less than 4 times the maximum allowable working pressure. The cast steel common rails are to withstand not less than 5 times the maximum allowable working pressure. The use of non-ferrous materials, cast iron and nodular iron is prohibited. Materials are to comply with Chapter 3 of the ABS *Rule for Materials and Welding (Part 2)*.

The high pressure common rail system is required to be properly enclosed and provided with arrangement for leak collection and alarm in case of a failure of high pressure common rail system (see 4-2-1/7.3.1).

7.5 Piping Between Booster Pump and Injection Pumps

Spray shields are to be fitted around flanged joints, flanged bonnets and any other flanged or threaded connections in fuel oil piping systems under pressure exceeding 1.8 bar (1.84 kgf/cm^2, 26 psi) which are located above or near units of high temperature, including boilers, steam pipes, exhaust manifolds, silencers or other equipment required to be insulated by 4-4-4/1.1.2, and to avoid as far as practicable oil spray or oil leakage into machinery air intakes or other sources of ignition. The number of joints in such piping systems is to be kept to a minimum.

9 Lubricating Oil Systems

9.1 General

The following requirements are applicable for main and auxiliary diesel engines and for reduction gears associated with diesel propulsion. See also 4-1-1/17 and 4-4-4/9.

9.3 Low Oil Pressure Alarms, Temperature and Level Indicators *(2017)*

An alarm device with audible and visual signals for failure of the lubricating oil system is to be fitted for propulsion and auxiliary engines having a rated power greater than 37 kW (50 hp). Pressure and temperature indicators are to be installed in lubricating oil systems indicating that the proper circulation is being maintained.

Where gravity tanks are utilized for lubrication for reduction gears, these tanks are to be provided with a low-level alarm and the overflow lines from these tanks to the sump tanks are to be fitted with a sight flow glass. Level gauges are to be provided for all gravity tanks and sumps.

9.5 Drain Pipes

Lubricating oil drain pipes from the sump to the drain tank are to be submerged at their outlet ends. No interconnection is to be made between the drain pipes from the crankcases of two or more engines.

9.7 Lubricating Oil Pumps (for Yachts Over 47.5 meters (150 feet) Rule Length)

For each pressurized lubricating oil system essential for operation of the propulsion engine and/or gear, one independently driven standby pump is to be provided in addition to the necessary pumps for normal operation. Where the size and design of an engine is such that lubrication before starting is not necessary and an attached lubricating pump is normally used, an independently driven standby pump is not required if a complete duplicate of the attached pump is carried as a spare.

The spare pump(s) need not be provided for multiple-engine installations provided that, in the event of the loss of one engine, at least forty percent of the total rated propulsion power remains.

9.9 Filters

Oil filters are to be provided. In the case of main propulsion engines which are equipped with full-flow-type filters, the arrangement is to be such that the strainers may be cleaned without interrupting the oil supply. However, where multiple engines are provided, a dedicated simplex strainer may be fitted for each engine provided the yacht can maintain at least one-half of the design speed or 7 knots, whichever is less, while operating with one engine temporarily out of service until its filter can be cleaned.

For auxiliary engines, the arrangement is to be such that the filters may be cleaned without undue interruption of power necessary for propulsion. Multiple auxiliary engines, each fitted with a separate filter and arranged such that changeover to a standby unit can be accomplished without loss of propulsion capability, will be acceptable for this purpose.

The arrangement of the valving is to be such as to avoid release of debris into the lubricating-oil system upon activation of the relieving mechanism.

Where filters are fitted in parallel to enable cleaning without disrupting the oil supply, means are to be provided to minimize the possibility of a filter under pressure being opened inadvertently. Filters are to be provided with suitable means for venting when being put in operation and being depressurized before being opened. Valves and cocks with drain pipes led to a safe location are to be used for this purpose. Filters are to be so arranged as to prevent, in the event of leakage, spraying of oil onto the exhaust manifold and surfaces with temperatures in excess of 220°C (428°F).

9.11 Lubricating Oil Coolers

For all types of propulsion plants, one or more oil coolers with means for controlling the lubricating oil temperature are to be provided. Indicators are to be fitted by which the pressure and temperature of the water inlet and oil outlet may be determined. See also 4-2-1/11.7 for cooling water requirements.

11 Cooling Water Systems

11.1 General

Means are to be provided to ascertain the temperature of the circulating water at the return from each engine and to indicate that the proper circulation is being maintained. Drain cocks are to be provided at the lowest point of all jackets. For relief valves, see 4-4-1/9.15.

11.3 Sea Suctions

At least two independent sea suctions are to be provided for supplying water to the engine jackets or to the heat exchangers. The sea suctions are to be located so as to minimize the possibility of blanking off the cooling water.

11.5 Strainers

Where seawater is used for direct cooling of the engines, unless other equivalent arrangements are specially approved, suitable strainers are to be fitted between the sea valves and the pump suctions. The strainers are to be either of the duplex type or otherwise arranged so they can be cleaned without interrupting the cooling water supply.

11.7 Circulating Water Pumps (for Yachts Over 47.5 meters (150 feet) Rule Length)

There are to be at least two means for supplying cooling water to main and auxiliary engines, compressors, coolers, reduction gears, etc. One of these means is to be independently driven and may consist of a connection from a suitable pump of adequate size normally used for other purposes, such as a general service pump, or in the case of fresh-water circulation, one of the yacht's fresh-water pumps. Where, due to the design of the engine, the connection of an independent pump is impracticable, the independently driven stand-by pump will not be required if a complete duplicate of the attached pump is carried as a spare. For multiple propulsion engines using identical attached pumps, only one complete pump needs to be carried as a spare.

The spare pump need not be provided for multiple-engine installations provided that, in the event of the loss of one engine, at least forty percent of the total rated propulsion power remains.

13 Starting Systems

13.1 Starting Air Systems

The design and construction of all air reservoirs are to be in accordance with the applicable requirements of Part 4, Chapter 4 of the *Marine Vessel Rules*. The piping system is to be in accordance with the applicable requirements of Part 4, Chapter 4 of these Rules. The air reservoirs are to be so installed as to make the drain connections effective under extreme conditions of trim. Compressed air systems are to be fitted with relief valves and each air reservoir which can be isolated from a relief valve is to be provided with its own safety valves or equivalent. Connections are also to be provided for cleaning the air reservoir and pipe lines.

All discharge pipes from starting air compressors are to be led directly to the starting air reservoirs, and all starting pipes from the air reservoirs to main or auxiliary engines are to be entirely separate from the compressor discharge piping system.

13.3 Starting Air Capacity

Yachts having main engines arranged for air starting are to be provided with at least two starting-air reservoirs of approximately equal size. The total capacity of the starting-air reservoirs is to be sufficient to provide, without recharging the air reservoirs, at least the number of starts stated below.

If other compressed air systems, such as control air, are supplied from starting air reservoirs, the capacity of the air reservoirs is to be sufficient for continued operation of these systems after the air necessary for the required number of consecutive starts has been used.

13.3.1 Diesel Propulsion

The minimum number of consecutive starts (total) required to be provided from the starting-air reservoirs is to be based upon the arrangement of the engines and shafting systems as indicated in the following table.

	Single Propeller Yachts		*Multiple Propeller Yachts*	
Engine Type	*One engine coupled to shaft directly or through reduction gear*	*Two or more engines coupled to shaft through clutch and reduction gear*	*One engine coupled to each shaft directly or through reduction gear*	*Two or more engines coupled to each shaft through clutch and reduction gear*
Reversible	12	16	16	16
Non-reversible	6	8	8	8

For arrangements of engine and shafting systems which differ from those indicated in the table, the capacity of the starting-air reservoirs will be specially considered based on an equivalent number of starts.

13.3.2 Diesel-electric Propulsion

The minimum number of consecutive starts required to be provided from the starting-air reservoirs is to be determined from the following equation:

$$S = 6 + G(G - 1)$$

where

S = total number of consecutive starts

G = number of engines necessary to maintain sufficient electrical load to permit yacht transit at full seagoing power and maneuvering. The value of G need not exceed 3.

13.5 Starting Air Compressors (1 July 2021)

For yachts having internal-combustion engines arranged for air starting, there are to be two or more air compressors, at least one of which is to be driven independently of the main propulsion unit, and the total capacity of air compressors driven independently of the main propulsion unit is to be not less than 50% of the total required.

The total capacity of the air compressors is to be sufficient to supply within one hour the quantity of the air needed to satisfy 4-2-1/13.3 by charging the reservoirs from atmospheric pressure. Where fitted, topping up compressors may be included in the capacity calculations.

The total capacity, *V*, required by 4-2-1/13.3 is to be approximately equally divided between the number of compressors fitted, *n*, excluding an emergency compressor, where fitted. Where a topping-up compressor is fitted, the capacity of each remaining air compressor is approximately to be at least (V-capacity of topping-up air compressor)/(n-1).

The arrangement for dead ship air starting is to be such that the necessary air for the first charge can be produced onboard without external aid. See 4-1-1/19.

13.7 Protective Devices for Starting-air Mains

In order to protect starting air mains against explosions arising from improper functioning of starting valves, an isolation non-return valve or equivalent is to be installed at the starting air supply connection to each engine. Where engine bores exceed 230 mm (9 $^{1}/_{16}$ in.), a bursting disc or flame arrester is to be fitted in way of the starting valve of each cylinder for direct reversing engines having a main starting manifold or at the supply inlet to the starting-air manifold for non-reversing engines.

The above requirement is applicable to engines where the air is directly injected into the cylinder. It is not intended to apply to engines utilizing air start motors.

13.9 Electrical Starting

13.9.1 Main Engine

Where the main engine is arranged for electric starting, at least two separate batteries (or separate sets of batteries) are to be fitted. The arrangement is to be such that the batteries (or sets of batteries) cannot be connected simultaneously in parallel. Each battery (or set) is to be capable of starting the main engine when in cold and ready to start conditions. The combined capacity of the batteries is to be sufficient without recharging to provide within 30 minutes the number of starts of main engines as required for air starting in 4-2-1/13.3, and if arranged also to supply starting for auxiliary engines, the number of starts required in 4-2-1/13.9.2. See also 4-2-1/13.9.3.

13.9.2 Auxiliary Engine

Electric starting arrangements for auxiliary engines are to have at least two separate batteries (or separate sets of batteries) or may be supplied by separate circuits from the main engine batteries when such are provided. Where one auxiliary engine is arranged for electric starting, one battery (or set) may be accepted in lieu of two separate batteries (or sets). The capacity of the batteries for starting the auxiliary engines is to be sufficient for at least three starts for each engine.

13.9.3 Other Requirements

The starting batteries (or set of batteries) are to be used for starting and for engine's own control and monitoring purpose only. When the starting batteries are used for engine's own control and monitoring purpose, the aggregate capacity of the batteries is to be sufficient for continued operation of such system in addition to the required number of starting capacity. Provisions are to be made to continuously maintain the stored energy at all times. See also 4-6-2/5.19 and 4-6-3/3.7.

13.11 Hydraulic Starting

Hydraulic oil accumulators for starting the main propulsion engines are to have sufficient capacity without recharging for starting the main engines, as required in 4-2-1/13.3.

15 Engine Exhaust Systems

15.1 General **(2020)**

The exhaust pipes are to be water-jacketed or effectively insulated. Engine exhaust systems are to be so installed that the yacht's structure cannot be damaged by heat from the systems. Exhaust pipes of several engines are not to be connected together but are to be run separately to the atmosphere unless arranged to prevent the return of gases to an idle engine. Exhaust lines which are led overboard near the waterline are to be protected against the possibility of the water finding its way inboard. The exhaust gas system is to be designed such that the back-pressure across the piping is within the allowable limits stated by the engine and fired equipment manufacturer under all expected operating conditions.

15.3 Exhaust System Materials

Materials used in the exhaust system are to be resistant to saltwater corrosion, galvanically compatible to each other and resistant to exhaust products. Plate flanges will be considered where the specified material is suitable for exhaust piping pressures and temperatures.

15.5 Exhaust Gas Temperature

15.5.1 Propulsion Engines with Bores Exceeding 200 mm (7.87 in.)

Propulsion engines with bores exceeding 200 mm (7.87 in.) are to be fitted with a means to display the exhaust gas temperature of each cylinder.

15.5.2 Wet Exhaust Systems

For wet exhaust systems, a high temperature indicator or alarm is to be fitted in the wet side of the exhaust system.

15.7 Exhaust Emission Abatement Systems **(2023)**

Where a vessel is fitted with an exhaust emission abatement system and the optional vessel notations detailed under 6-3-1/9.3 through 6-3-1/9.9 of the *Marine Vessel Rules* are not requested, the installed exhaust emission abatement system is to comply with the minimum requirements prescribed in 6-3-1/15 TABLE 1 of the *Marine Vessel Rules* and is to be verified by an ABS Surveyor during installation. This is applicable to new construction and existing vessel conversions.

17 Couplings

17.1 Flexible Shaft Couplings

Details of the various components of flexible couplings for main propulsion machinery and ship's service generator sets are to be submitted for approval.

17.1.1 Design

Flexible couplings intended for use in propulsion shafting are to be of approved designs. Couplings are to be designed for the rated torque, fatigue and avoidance of overheating. Where elastomeric material is used as a torque-transmitting component, it is to withstand environmental and service conditions over the design life of the coupling, taking into consideration the full range of maximum to minimum vibratory torque. Flexible coupling design will be evaluated, based on submitted engineering analyses.

17.1.2 Torsional Displacement Limiter

Flexible couplings with elastomer or spring type flexible members, whose failure will lead to total loss of propulsion capability of the yacht, such as that used in the line shaft of a single propeller yacht, are to be provided with a torsional displacement limiter. The device is to lock the coupling or prevent excessive torsional displacement when a pre-determined torsional displacement limit is exceeded. Operation of the yacht under such circumstances may be at reduced power. Warning notices for such reduced power are to be posted at all propulsion control stations.

17.1.3 Barred Range

Conditions where the allowable vibratory torque or the allowable dissipated power may be exceeded under the normal operating range of the engine are to be identified and are to be marked as a barred range in order to avoid continuous operation within this range.

17.1.4 Diesel Generators **(2022)**

Flexible couplings for diesel generator sets are to be capable of absorbing short time impact torque due to electrical short-circuit conditions up to 6 (six) times the nominal torque, or the couplings are to be evaluated for capability to absorb the transient torsional vibration peak torque for each application.

17.3 Flanged Couplings and Coupling Bolts

Refer to 4-3-1/19.5 for flanged couplings.

Elongation for auxiliary machinery coupling bolts made of steel having an ultimate tensile strength over 690 N/mm^2 (70 kgf/mm^2, 100,000 psi) will be subject to special consideration. Also refer to 4-3-1/19.1 and 4-3-1/19.3.

19 Testing of Pumps Associated with Engine and Reduction Gear Operation

Pumps associated with engine and reduction gear operation (oil, water, fuel) utilized with engines having bores exceeding 300 mm (11.8 in.) are to be provided with certificates issued by the Surveyor. The following tests are to be conducted to the Surveyor’s satisfaction:

19.1 Pumps Hydrostatic Tests

Independently-driven pumps are to be hydrostatically tested to 4 bar (4 kgf/cm^2, 57 psi), but not less than 1.5 P, where P is the maximum working pressure in the part concerned.

19.3 Capacity Tests

Tests of pump capacities are to be conducted to the satisfaction of the Surveyor with the pump operating at design conditions. Capacity tests will not be required for individual pumps produced on a production line basis, provided the Surveyor is satisfied from periodic inspections and the manufacturer’s quality assurance procedures that the pump capacities are acceptable.

21 Trial

Before final acceptance, the entire installation is to be operated in the presence of the Surveyor to demonstrate its ability to function satisfactorily under operating conditions and its freedom from harmful vibration at speeds within the operating range. See also 4-1-1/31.